ALL GROUND-WATER HYDROLOGY WORK IS MODELING

A Model is a representation of a system.

Modeling begins when one formulates a concept of a hydrologic system,
continues with application of, for example,
Darcy's Law or the
Theis equation to the problem,
and may
culminate in a complex numerical simulation.

MODELS can be used BENEFICIALLY and for DECEPTION

GROUND WATER MODELING

WHY MODEL?

- To make predictions about a ground-water system's response to a stress
- To understand the System
- To design field studies
- Use as a thinking tool

Characterize the system

Governing equation of Ground Water Flow:

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right)-W=S_s \frac{\partial h}{\partial t}$$

Geometry

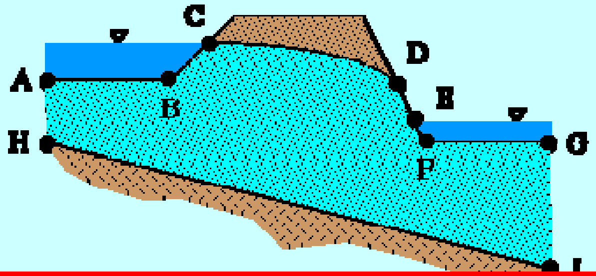
Material Properties (K, S, T, Φ_e, D, R, etc)

Boundary Conditions (Head, Flux, Concentration etc)

Stresses (changing boundary conditions)

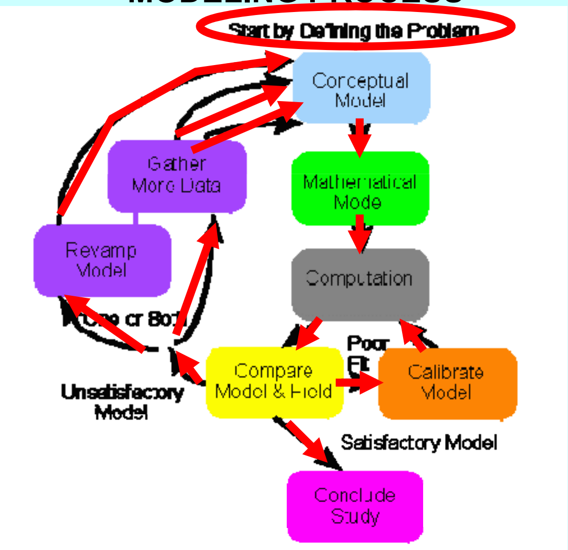
Boundary Types

infinite source/sink	**Specified Head/Concentration:** a special case of constant head (ABC, EFG) **Constant Head /Concentration:** could replace (ABC, EFG)
calculated head maintains flux	**Specified Flux:** could be recharge across (CD) **No Flow (Streamline):** a special case of specified flux (HI)
infinite source/sink	**Head Dependent Flux:** could replace (ABC, EFG)
adjusts position	**Free Surface:** water-table, phreatic surface (CD)
adjusts length	**Seepage Face:** h = z; pressure = atmospheric at ground surface (DE)



What about A-H and G-I?
Every boundary must be defined. There aren't any open ended options.
If you are using a model and not required to enter a condition, then some condition is assumed, most likely flux=0.

MODELING PROCESS



ALL IMPORTANT MECHANISMS & PROCESSES MUST BE INCLUDED IN THE MODEL, OR RESULTS WILL BE INVALID.

TYPES OF MODELS

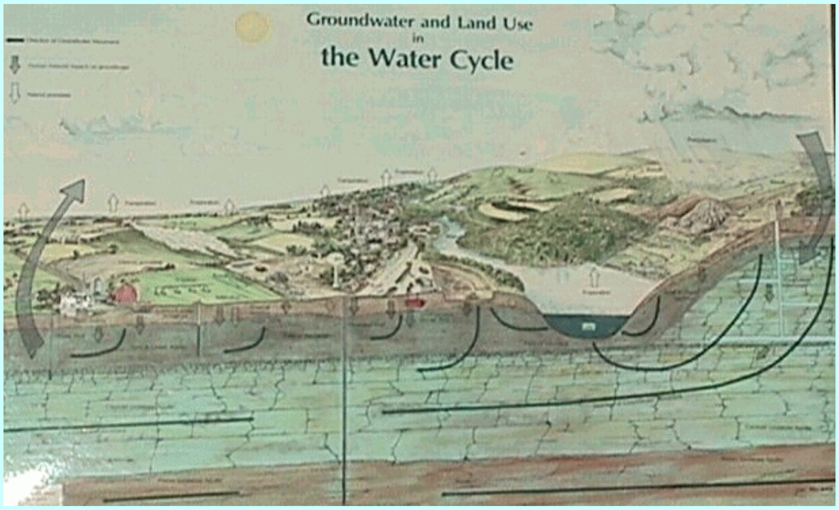
CONCEPTUAL
PHYSICAL
ANALOG
EMPIRICAL
GRAPHICAL
MATHEMATICAL
  - **SIMPLE - ANALYTICAL**
  - **COMPLEX - NUMERICAL**

CONCEPTUAL MODEL

Geometry
Material Properties
Boundary Conditions
General Flow Patterns
System Stresses (usually BCs)

CONCEPTUAL MODEL + FLOW EQUATIONS
= QUANTITATIVE MODEL OF FLOW SYSTEM

CONCEPTUAL MODEL

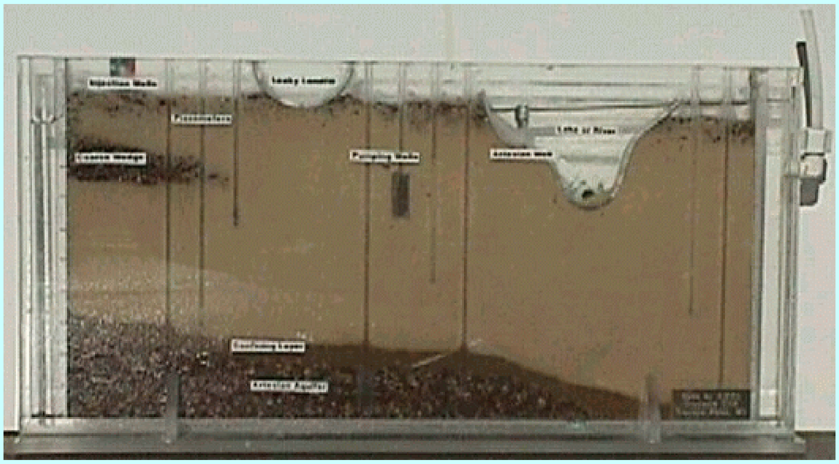


PHYSICAL MODEL

Geometry

Materials

Boundary Conditions

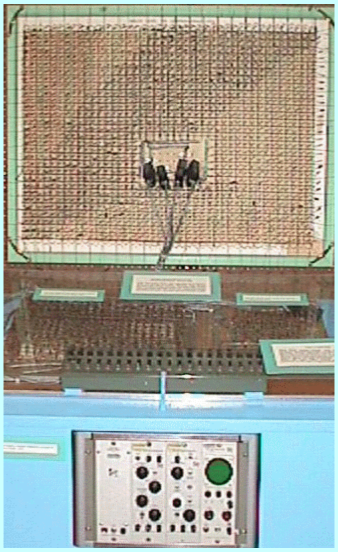


ANALOG MODEL

Geometry

Material Properties

Boundary Conditions



Electrical analog model of the Champaign-Urbana Illinois area ground-water system (circa 1960)

The top panel is a circuit of resistors and capacitors representing the regional model

Measuring the voltage at various locations in the circuit is equivalent to measuring head in the aquifer

The middle level includes a local model of a portion of the regional model at both the same scale and twice the scale

The lower area includes the controls for imposing current on the model

These models are very difficult to calibrate because each change of material properties involves removing and re-soldering the resistors and capacitors

EMPIRICAL MODEL

A Mathematical Fit to Data Unrelated to Process Equations

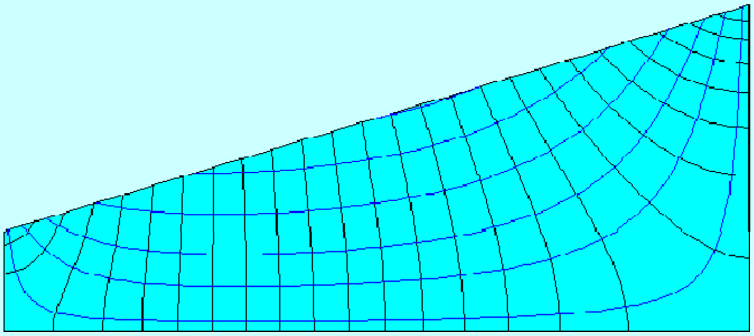
e.g. Manning's Equation

$$V = \frac{1.49\, R^{2/3}\, S^{1/2}}{n}$$

where: V = average velocity in fps
R = hydraulic radius (flow area [ft^2]/wetted perimeter[ft])
S = slope of energy gradient
n = Manning friction factor

GRAPHICAL MODEL - FLOW NET

Geometry
Material Properties
Boundary Conditions



ANALYTICAL MODEL

Closed form algebraic solution

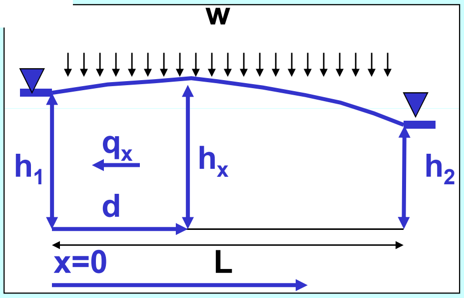
Geometry
Material Properties
Boundary Conditions

Recall Dupuit, Flow to fixed heads with recharge:

$$h_x = \sqrt{h_1^2 - \frac{\left(h_1^2 - h_2^2\right)x}{L} + \frac{w}{K}\left(L - x\right)x}$$

$$q_x = \frac{K\left(h_1^2 - h_2^2\right)}{2L} - w\left(\frac{L}{2} - x\right)$$

$$d = \frac{L}{2} - \frac{K}{w}\frac{h_1^2 - h_2^2}{2L}$$



A steady state solution to the flow equations in 1D with Boundary Conditions:

Bottom, no-flow (fixed flux = 0) head is calculated such that flow will parallel boundary

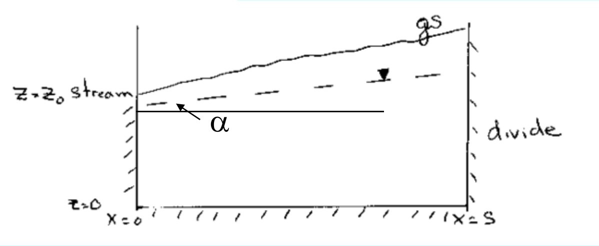
Top, fixed flux = recharge head/gradient are calculated to accommodate that recharge - e.g. High recharge >> High heads

Sides, h1 and h2 are fixed heads flux is calculated to accommodate those heads - e.g. a high h1 will shift the divide to the left of the problem domain and produce large influx that joins the recharge and discharges to the right, if h2 is very low, that influx will be even higher

Recall Toth developed a solution for Steady State flow in a 2D section from a divide to a stream

he solved the Laplace Equation

$$\frac{\partial^2 h}{\partial x^2} - \frac{\partial^2 h}{\partial z^2} = 0$$



boundaries

left $\frac{\partial h}{\partial x}(0,z) = 0$ right $\frac{\partial h}{\partial x}(s,z) = 0$

lower $\frac{\partial h}{\partial z}(x,0) = 0$

upper water table $h(x, z_o) = z$

What are the Boundary Conditions?

His result:

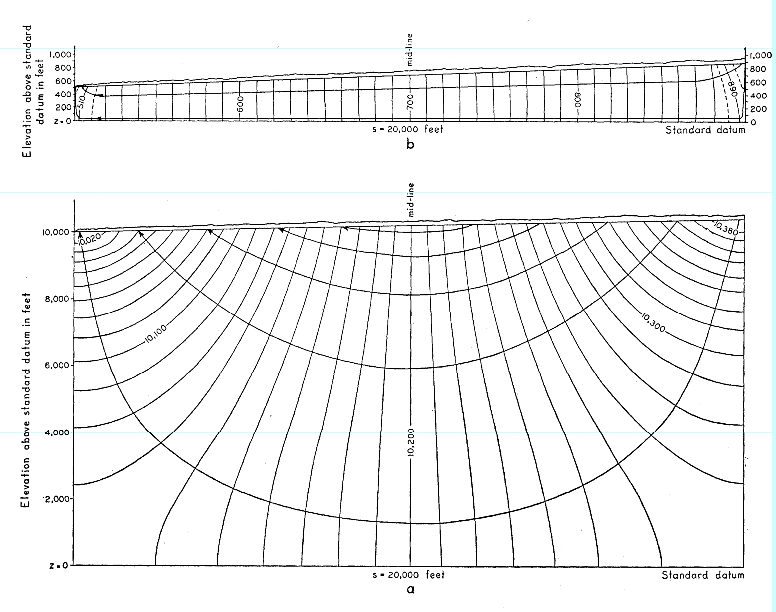


Fig. 3. Two-dimensional theoretical potential distributions and flow patterns for different depths to the horizontal impermeable boundary.

Theis Equation

Drawdown given Q T S r t

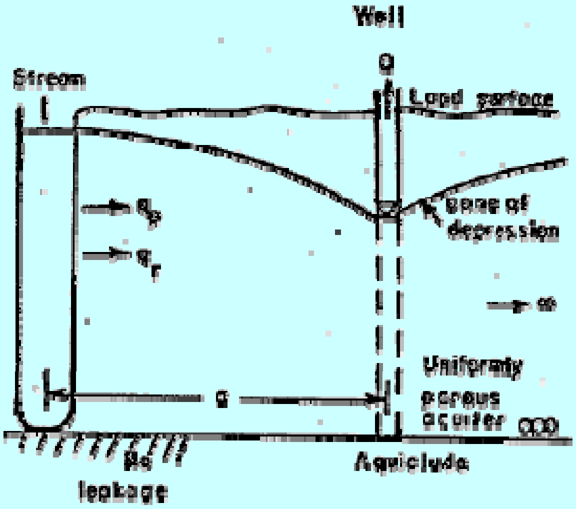
$$s = h_o - h = \frac{Q}{4\pi T} W(u) \qquad u = \frac{r^2 S}{4Tt} \quad or \quad \frac{r^2}{t} = \frac{4T}{S} u$$

s = drawdown [L]
h_o = initial head @ r [L]
h = head at r at time t [L]
t = time since pumping began [T]
r = distance from pumping well [L]
Q = discharge rate [L^3/T]
T = transmissivity [L^2/T]
S = Storativity [ ]

What are the Boundary Conditions?

$$W(u) = \int_u^{\infty} \frac{e^{-u}}{u} du = \left[-0.5772 - \ln u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} + \ldots\ldots\right]$$

Pumping near a stream, we are interested in depletion of stream flow



q_p [L^3/T] = rate of stream depletion at time t measured from start of pumping

$$q_p = Q \operatorname{erfc}\left(\frac{a}{\sqrt{4tT/S}}\right)$$

a = perpendicular distance between well and stream [L]
t = time since pumping began [T]
T = aquifer transmissivity (K*thickness) [L^2/T]
S = aquifer storage coefficient [dimensionless]
Q = pumping rate [L^3/T]
erfc = the complimentary error function [dimensionless]

v_p [L^3] is the total volume of stream depletion since pumping began

$$v_p = Qt\left[\left(\frac{a^2}{2tT/S}+1\right)\operatorname{erfc}\left(\frac{a}{\sqrt{4tT/S}}\right) - \left(\frac{a}{\sqrt{4tT/S}}\right)\left(\frac{2e^{-a^2/(4tT/S)}}{\sqrt{\pi}}\right)\right]$$

Qt = is the total volume pumped since pumping began [L^3]

Rate and volume of stream depletion after pumping stops is determined by the superposition where drawdown is summed with drawup from an injection image well that starts when pumping stops

t = time from start to stop of pumping [T]

t' = time since pumping stopped [T]

q_r [L^3/ T]is the residual rate of stream depletion at time t + t'

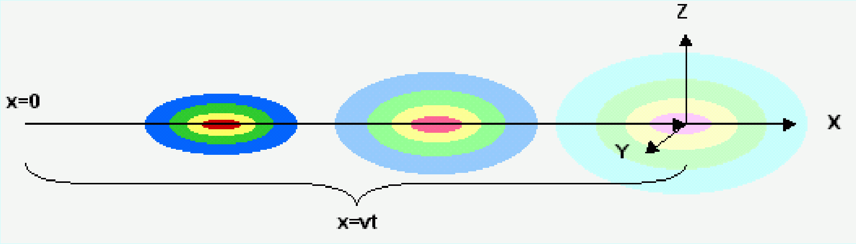
$$q_r = Q \operatorname{erfc}\left(\frac{a}{\sqrt{4(t+t')\,T/S}}\right) - Q \operatorname{erfc}\left(\frac{a}{\sqrt{4(t')\,T/S}}\right)$$

V_P [L^3] is the total volume of stream depletion since pumping began (note the t in the Qt_s of the second term is t')

V_r [L^3] accounts for depletion after pumping stops

$$\begin{aligned} v_r = {} & Q\,(t+t')\left[\left(\frac{a}{2(t+t')\,T/S}+1\right)\operatorname{erfc}\left(\frac{a}{\sqrt{4(t+t')\,T/S}}\right)\right. \\ & \left. - \left(\frac{a}{\sqrt{4(t+t')\,T/S}}\right)\left(\frac{2e^{-a^2/[4(t+t')\,T/S]}}{\sqrt{\pi}}\right)\right] \\ & - Qt_s\left[\left(\frac{a^2}{2t'\,T/S}+1\right)\operatorname{erfc}\left(\frac{a}{\sqrt{4t'\,T/S}}\right)\right. \\ & \left. - \left(\frac{a}{\sqrt{4t'\,T/S}}\right)\left(\frac{2e^{-a^2/(4t'\,T/S)}}{\sqrt{\pi}}\right)\right] \end{aligned}$$

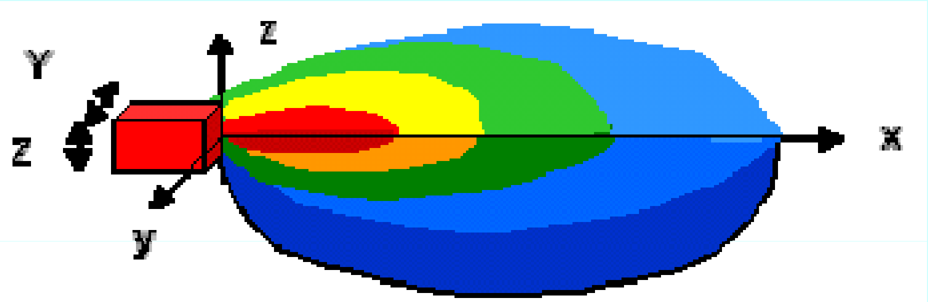
Recall Analytical Transport models, e.g. for 1D slug source:



$$C(x=\bar{v}t+X, y=Y, z=Z) = \frac{M}{8(\pi t)^{\frac{3}{2}}\sqrt{D_x D_y D_z}}\, e^{-\frac{X^2}{4D_x t}-\frac{Y^2}{4D_y t}-\frac{Z^2}{4D_z t}}$$

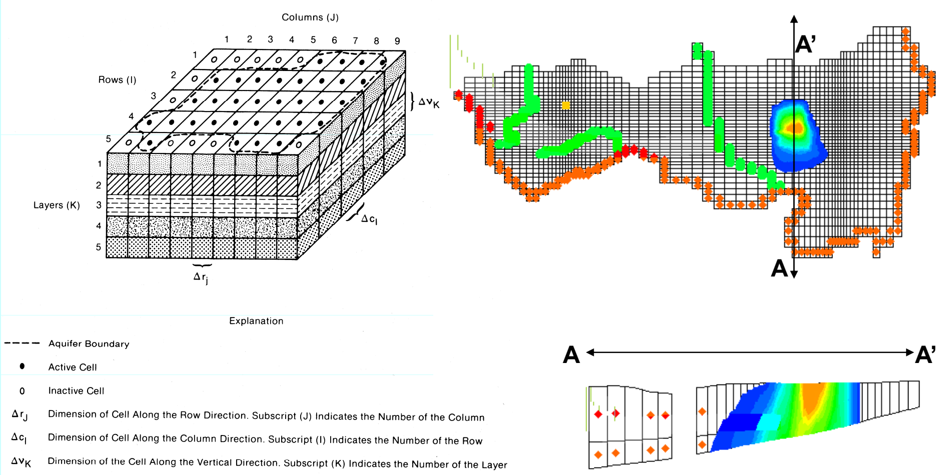
What are the boundary conditions in this case?
What type of conditions are needed that we did not need before?
What properties are needed here that weren't needed in the previous models we have discussed today?

Analytical Solution for transport in
1D flow field
continuous source
3D spreading



What are the boundary conditions in this case?

NUMERICAL MODELING DISCRETIZES THE SYSTEM
e.g. Finite Difference Modeling



Each block has only ONE AVERAGED value
of each Property, Boundary Condition, State Variable
To get more detail, Use more blocks
The model on the right represents vertical variation of head and concentration using only 2 points. How many does the model on the left use?

NUMERICAL FLOW MODELING

DISCRETIZE

Write equations of GW Flow between each node
Darcy's Law
Conservation of Mass

Define Material Properties
Boundary Conditions
Initial Conditions
Stresses (varying conditions)

At each node either H or Q is known, the other is unknown
n equations & n unknowns
solve simultaneously with matrix algebra

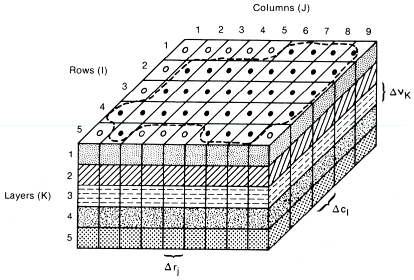
Result H at each known Q node
Q at each known H node

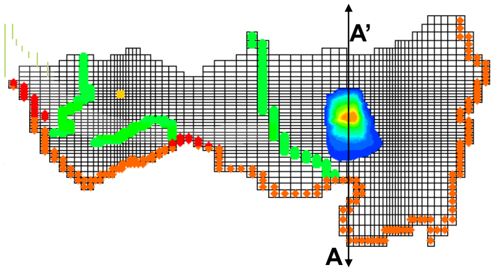
Calibrate Steady State
Transient

Validate
Sensitivity
Predictions

Similar Process for Transport Modeling only Concentration and Flux is unknown

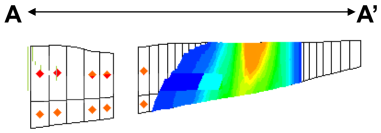
NUMERICAL MODELING DISCRETIZES THE SYSTEM
e.g. Finite Difference Modeling





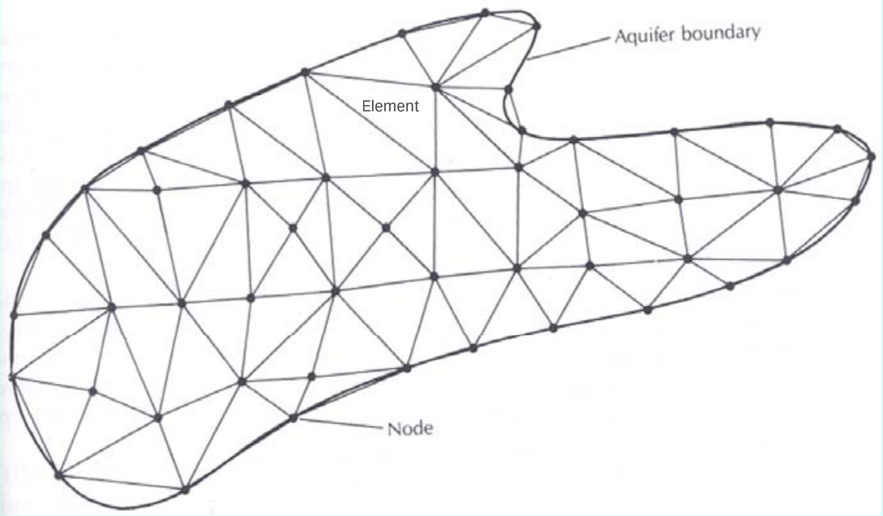
Explanation

- - - - Aquifer Boundary
• Active Cell
0 Inactive Cell
Δr_J Dimension of Cell Along the Row Direction. Subscript (J) Indicates the Number of the Column
Δc_I Dimension of Cell Along the Column Direction. Subscript (I) Indicates the Number of the Row
Δv_K Dimension of the Cell Along the Vertical Direction. Subscript (K) Indicates the Number of the Layer



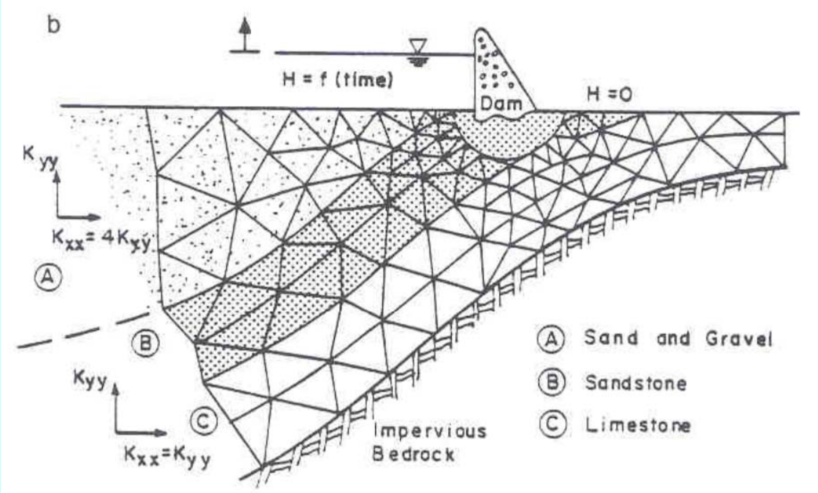
Each block has only ONE AVERAGED value
of each Property, Boundary Condition, State Variable
To get more detail, Use more blocks

NUMERICAL MODELING – Finite Element Modeling



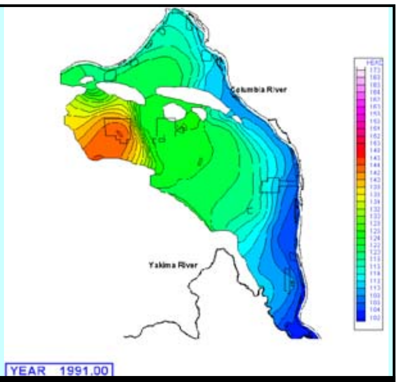
NUMERICAL MODELING – Finite Element Modeling

More flexibility in designing Grid



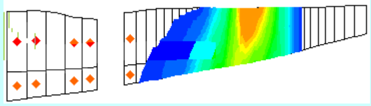
Examples of Model Results with links to animations

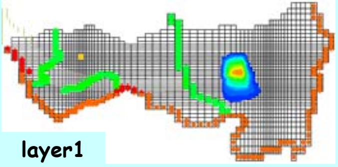
Flow Model
Hanford Reservation
Waste Water Disposal
http://inside.mines.edu/~epoeter/_GW/24Modeling/flow_43_96.avi

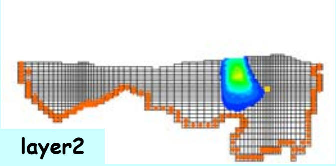


Flow and Transport Model
East Texas Land Fill Plume
http://inside.mines.edu/~epoeter/_GW/24Modeling/etex2.avi

North-South cross section through plume







MODFLOW

Block Centered 3D Finite Difference Ground Water Flow Model

Developed by McDonald & Harbaugh at USGS in 1983
- enhanced many times since then

Public Domain

Most widely used Saturated Porous Media Flow model

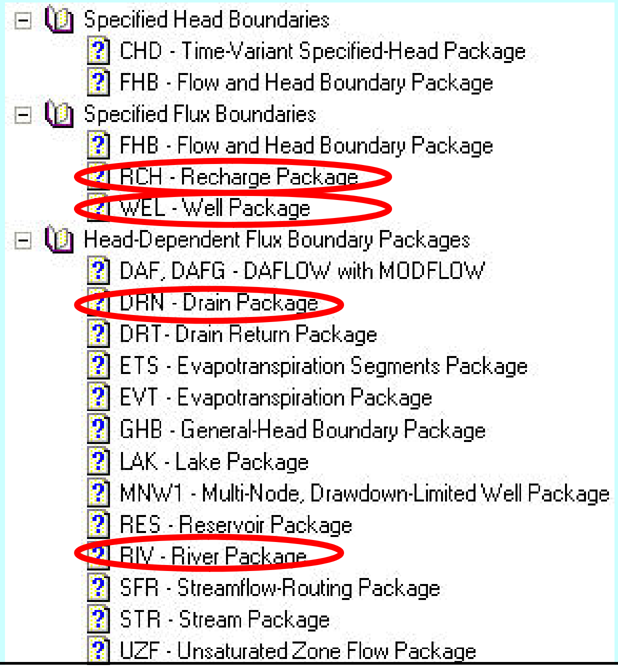
Many features available

MODFLOW:

Input Files → **MODFLOW Executable** → **With Packages provided, runs calculations Based on Model Set-up** → **Output**

MODFLOW uses text file input and output
GUI's can facilitate your work by creating the input files, running the program, and reading the output files, through a graphical interface
Graphical User Interface

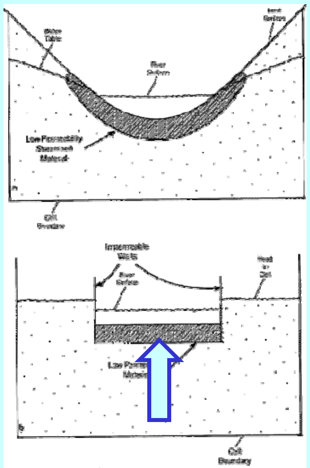
Some MODFLOW Boundary Condition Packages



- Specified Head Boundaries
  - CHD - Time-Variant Specified-Head Package
  - FHB - Flow and Head Boundary Package
- Specified Flux Boundaries
  - FHB - Flow and Head Boundary Package
  - RCH - Recharge Package
  - WEL - Well Package
- Head-Dependent Flux Boundary Packages
  - DAF, DAFG - DAFLOW with MODFLOW
  - DRN - Drain Package
  - DRT - Drain Return Package
  - ETS - Evapotranspiration Segments Package
  - EVT - Evapotranspiration Package
  - GHB - General-Head Boundary Package
  - LAK - Lake Package
  - MNW1 - Multi-Node, Drawdown-Limited Well Package
  - RES - Reservoir Package
  - RIV - River Package
  - SFR - Streamflow-Routing Package
  - STR - Stream Package
  - UZF - Unsaturated Zone Flow Package

MODFLOW Head-dependent Flux Boundary Condition Packages

RIVER package



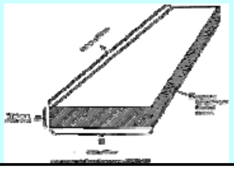
For each river reach in each cell:
MODFLOW requires that the user input Conductance, which is all of Darcy's Law except the head difference for Head Dependent Flux boundaries.

Q = KA dh/dl

Conductance = KA/thickness

then MODFLOW calculates the flow as:
Q = Conductance dh

Conductance of the river bed is calculated as:
Kv * Area(the plan view area,L*W) / thickness



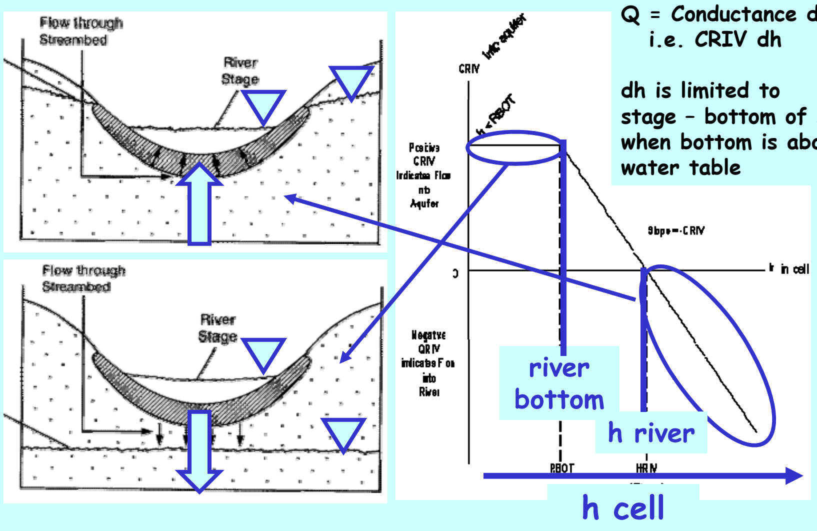
dh = (Stage-input-by-user − Head-in-cell-calculated-by-MODFLOW)

Unless head in cell is less than bottom of river, then

dh = (Stage-input-by-user − elevation-of-bottom-of-river)

MODFLOW Head-dependent Flux Boundary Condition Packages

RIVER package



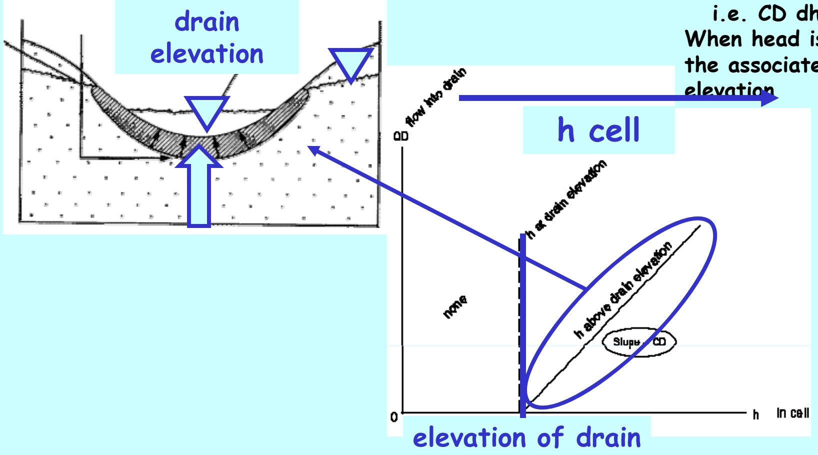
Q = Conductance dh
i.e. CRIV dh

dh is limited to
stage – bottom of sediment
when bottom is above the
water table

MODFLOW Head-dependent Flux Boundary Condition Packages

DRAIN package

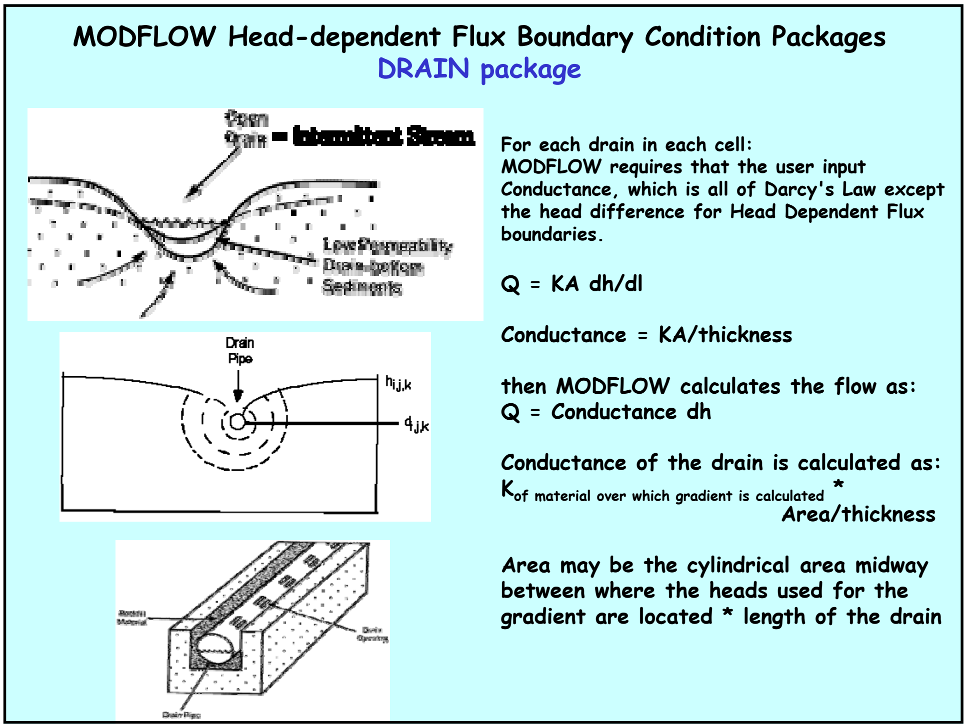
ONLY allows OUTFLOW



Q = Conductance dh
i.e. CD dh
When head is above
the associated
elevation

dh = (Drain-elevation-input-by-user − Head-in-cell-calculated-by-MODFLOW)

Unless head in cell is less than drain elevation, then dh is set to 0



MODFLOW Head-dependent Flux Boundary Condition Packages

DRAIN package

For each drain in each cell:
MODFLOW requires that the user input Conductance, which is all of Darcy's Law except the head difference for Head Dependent Flux boundaries.

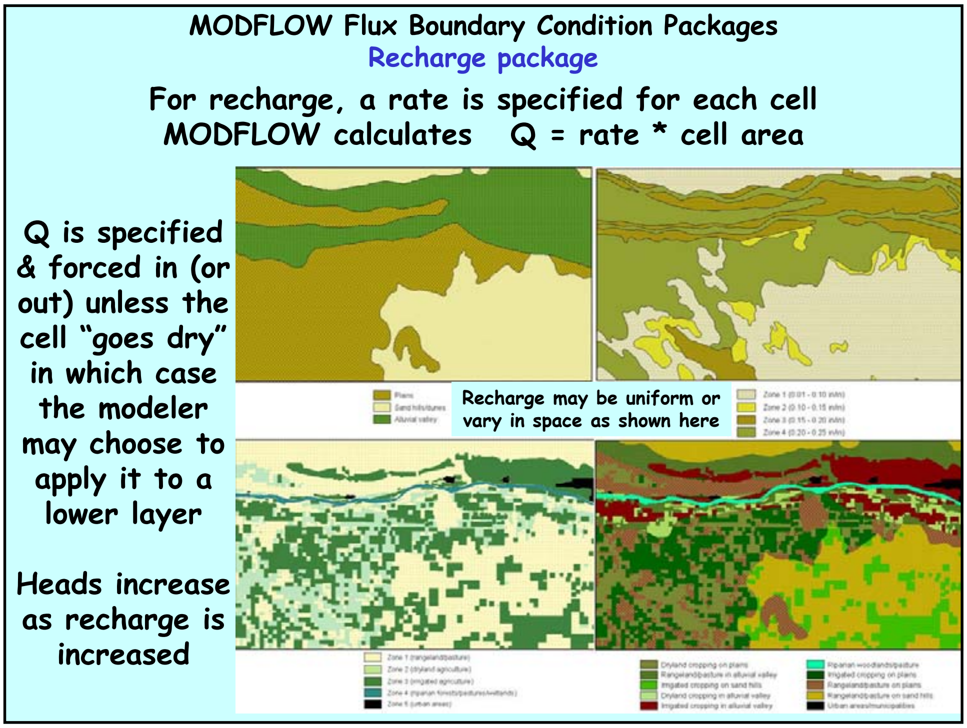
Q = KA dh/dl

Conductance = KA/thickness

then MODFLOW calculates the flow as:
Q = Conductance dh

Conductance of the drain is calculated as:
$K_{\text{of material over which gradient is calculated}}$ * Area/thickness

Area may be the cylindrical area midway between where the heads used for the gradient are located * length of the drain



MODFLOW Flux Boundary Condition Packages

Recharge package

For recharge, a rate is specified for each cell
MODFLOW calculates Q = rate * cell area

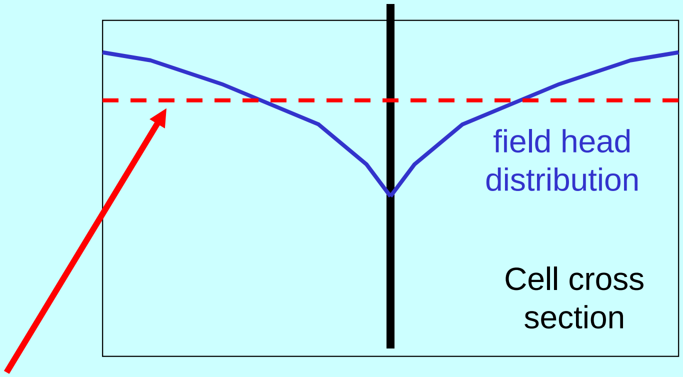
Q is specified & forced in (or out) unless the cell "goes dry" in which case the modeler may choose to apply it to a lower layer

Heads increase as recharge is increased

Recharge may be uniform or vary in space as shown here

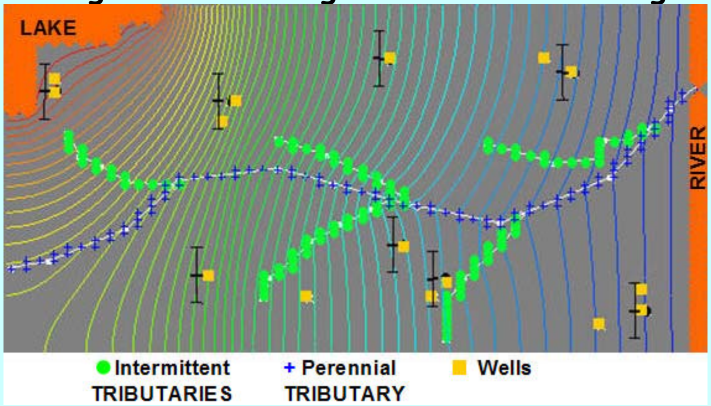
MODFLOW Flux Boundary Condition Packages
Well package

When pumping a well in a MODFLOW grid cell
Q is specified & withdrawn or injected unless the cell "goes dry"
Calculated drawdown represents the average drawdown in the cell, not the actual drawdown in the well



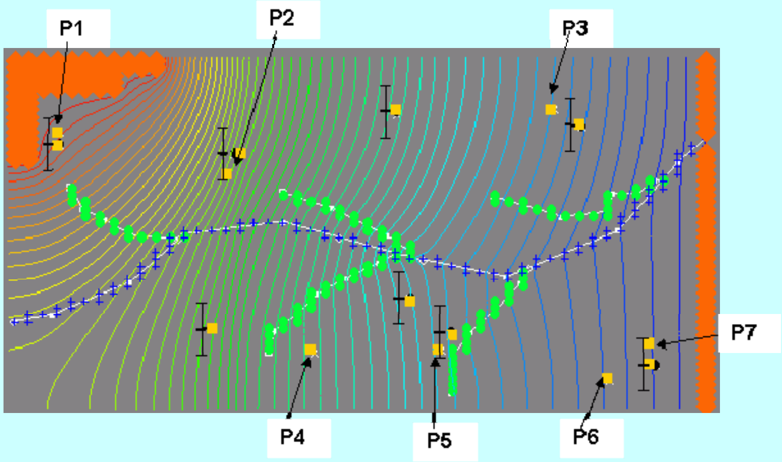
MODFLOW assigns one head value to each cell

Next we will use MODFLOW and GMS (Ground Water Modeling System) to get a feel for ground water modeling



The model will be used to design withdrawals from an existing well field to meet regulatory requirements on drawdown and change in stream flow. If the simulated head is lower than the desired head the error bar will drop down from the center, otherwise it will be above the center. If the difference is <= the specified 95% intervals the bar will be green, if the difference is greater than that but less than twice that it will be yellow and beyond that it will be red. This feature is usually used for calibrating a model but here it is a useful tool for accomplishing your design.

Locations of Production Wells
Where you can adjust withdrawal rate



Experiment with MODFLOW

Using the GMS (Ground Water Modeling System)

GUI (Graphical User Interface)

Download the Example Files and Associated Write-up on the class web page for this lecture

http://inside.mines.edu/~epoeter/_GW/24Modeling/Modeling.htm