

# Using Dynamic DCF and Real Option Methods for Economic Analysis in NI43-101 Technical Reports

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## **Abstract**

The introduction of Dynamic Discounted Cash Flow (“Dynamic DCF”) and Real Options (“RO”) analysis into a NI43-101 report is an intriguing new avenue for improving the communication of project risk and the understanding of how a project’s risk profile influences project economics. However, these techniques have not found wide-spread use in NI43-101 and other public technical reports to date. This is beginning to change as in June 2010 Ivanhoe and Entree Gold both published NI43-101 reports for the Oyu Tolgoi project containing Dynamic DCF and RO analysis. In addition, Canadian securities regulatory authorities have recently published amendments to their NI43-101 rules to make it clear companies can provide more sophisticated economic and risk analyses in these reports.

This paper proposes an extended valuation and risk management framework that includes these methods and discusses how they can be used to support the objectives of a NI43-101 technical report. It highlights how these techniques can complement and extend conventional evaluation analyses through a review of the Dynamic DCF and RO analysis contained in the Ivanhoe NI43-101 report. A set of best practice guidelines are then proposed to ensure that Dynamic DCF and RO analysis is presented in a professional and objective manner that benefits investors and is consistent with a conventional DCF analysis.

## Introduction

Public mining companies have the responsibility of informing their investors about significant new information affecting their exploration and development stage projects. The release of new information to investors related to the quality and quantity of the mineral resource is controlled by securities regulations. These regulations include the National Instrument 43-101 *Standards of Disclosure for Mineral Projects* (“NI43-101”; Canadian Securities Administrators, 2005, 2011) rules in Canadian financial markets, the *Australasian Code for Reporting of Exploration Results, Mineral Resources and Ore Reserves* (Joint Ore Reserves Committee, 2004) prepared by the Australasian Joint Ore Reserves Committee and used in Australian financial markets, and the *South African Code for Reporting of Exploration Results, Mineral Resources and Mineral Reserves* (South African Mineral Resource Committee, 2009) used in South African financial markets. These codes all have a common goal of ensuring that new information about mineral resources is released through public technical reports in a manner that is transparent, professional, and of a consistent standard for all public companies. In this paper, we focus on public technical reports issued under NI43-101 rules (“NI43-101 Technical Reports”).

An important focus of NI43-101 Technical Reports is the technical information generated by exploration activities and how this information is processed to arrive at an estimate of the quality and quantity of mineral resource. Reporting on this technical information includes discussing the characteristics of the exploration program, the structural interpretation of geological samples, sampling and assaying techniques, statistical analysis methods, and the approach to resource modelling. A key requirement for NI43-101 Technical Report technical sections is to ensure that investors have the information available to judge the robustness of the resource quality and quantity estimate.

A further requirement under NI43-101 guidelines is that the technical report must inform the investor about the economic viability of the resource given a suitable project design and using reasonable assumptions about the current and future economic environment. This is an important requirement as it underlies the actual definition of a mineral resource: a mineral occurrence can only be declared a resource if it can be demonstrated that there is a reasonable prospect of economic extraction.

It is notable that the analysis supporting conclusions about the possibility of economic extraction is often not performed to the same level of sophistication as the technical analysis supporting conclusions about the quantity and quality of mineral resources. This is unfortunate because there are numerous techniques that may be used to extend the conventional economic analysis provided in NI43-101 Technical Reports such that an investor is provided with a better understanding of a project’s economic prospects.

## Current economic analysis techniques in NI43-101 technical reports

The predominant economic analysis technique used in NI43-101 Technical Reports is the static Discounted Cash Flow (“Static DCF”) method. This technique estimates the future net cash flows from the project using annual single-point forecasts of production and economic variables such as future metal prices, waste and ore production amounts, metal grades, recoveries, prices and amounts of consumables, labour, and services. These forecasts are then used to construct an annual expected project net cash flow equal to revenues less capital and operating

costs, government and third party royalties, corporate income taxes, cost of transporting output to market, insurance, smelting and refining charges, and other deductions.

This expected net cash flow is then used to calculate a project Internal Rate of Return (“IRR”) or Net Present Value (“NPV”) as an indication of project viability.

Project IRR represents the annualized effective compounded return rate on capital invested in the project. This is also the discount rate at which the NPV of expected project cash flows is zero. Generally, a project with an IRR that is higher than its opportunity cost of capital is thought to be an attractive investment.

Project NPV represents the value created or destroyed by investing in a project. The calculation of a Static DCF NPV requires estimating net annual cash flows and then discounting each annual cash flow for the value effects of uncertainty and time to determine a cash flow present value. NPV is the sum of these present values. The value effect of uncertainty and time is recognised by summarising their impact into a single constant risk-adjusted rate that is used in the discounting process. This discount rate is likely used for a broad class of investment projects regardless of the actual uncertainty characteristics of the particular project. A positive Static DCF NPV suggests that a project is an attractive investment.

There are three potential shortcomings of a Static DCF NPV calculation:

- 1) **Static DCF ignores randomness in cash flow variables:** The reliance on only the expectations of uncertain cash flow variables such as metal price excludes a more detailed description of their randomness in the cash flow analysis. Static DCF analysis may use sensitivity analysis that varies a single variable at a time by a set percentage to gain insight into how NPV changes with random outcomes for that variable. However, this type of analysis is very limited compared to the insights into cash flow randomness that can be done with more complete methods of analysis.
- 2) **Static DCF ignores the effects of contingent cash flows and flexibility:** Projects incorporate contingencies that cause the structure of cash flow to change with variations in the project environment. For example, both production policy and sliding scale royalty rates may change with variation in metal prices, leading to changes in cash flow structure. A static DCF model is unable to recognize the contingent nature of cash flows and so may provide an erroneous estimate of project expected net cash flow. A cash flow estimation method that recognizes contingent cash flow structures and corrects this error may be necessary to generate a reasonable cash flow estimate.
- 3) **Static DCF risk adjustments do not recognize the dynamic variation of cash flow risk through time:** The use of a single discount rate implies that project cash flow uncertainty increases through time in a regular manner. However, most mine valuation professionals would agree that the cash flow uncertainty changes in a dynamic and erratic manner due to changes in metal grades and prices, operating costs, mining method, exhaustion of tax shields, and tax and royalty rates among other things. A risk adjustment method that responds to changes in cash flow uncertainty would be preferred.

The economic analysis provided in a NI43-101 Technical Report can be extended with better numerical methods and concepts from finance theory to correct the problems with a Static DCF analysis. This assertion does not mean that Static DCF models should not be used in NI43-101 Technical Reports. Indeed, it is most likely that Static DCF analysis will continue to be the predominant method employed in these reports, with more involved numerical and financial

analysis being only used when there is an important project characteristic that would not be fully recognized by a Static DCF model.

However, it is important for mining professionals to realize that NI43-101 rules do not prohibit the use of more advanced economic analysis methods in these technical reports. The only restriction is that analytic techniques being used are considered acceptable by industry professionals and that the analysis is conducted by someone with professional experience in the proposed analysis. In Canada, select advanced valuation techniques such as RO NPV have been recognized as professionally acceptable by the Canadian Institute of Mining, Metallurgy, and Petroleum in their Standards and Guidelines for the Valuation of Mineral Properties ("CIMVal Standards", Canadian Institute of Mining, Metallurgy, and Petroleum, Special Committee on Valuation of Mineral Properties, 2003) and so may be used when necessary for economic analysis in NI43-101 Technical Reports. Indeed, the main challenge for the inclusion of these techniques in these reports is finding qualified professionals to conduct the analysis.

There have been recent changes to NI 43-101 rules in order to accommodate analysis that goes beyond a Static DCF model in scope. The changes are largely in Form F1, the form that specifies the content of the technical report. Formerly the technical report form specified annual cash flow forecasts -- a requirement that in practice was met by a standard Static DCF calculation -- and "sensitivity analyses with variants in metal prices, grade, capital and operating costs" (Canadian Securities Administrators, 2005). The form also called for a discussion of the payback period and of expected mine life. The requirement applied to properties in production and to "development properties" -- defined under the old rule as properties being prepared for production whose economic viability has been demonstrated by a feasibility study. While it was possible to argue that a project at the pre-feasibility stage, or in earlier stages of economic assessment, was not a "development property", the rule's requirement that a technical report "not omit any material scientific and technical information" meant that any technical report supporting disclosure of economic information should include a similar discussion of the project's economics.

Under the changes to Form F1 (now grouped under the form's Item 22), the technical report must include annual cash flow forecasts and an annual production schedule, using either mineral reserves or, in the case of preliminary economic assessments, mineral resources. There must also be a discussion of NPV, IRR, and the payback period. The uncertainties in the Static DCF economic analysis are covered under the form's Item 22 (e), which requires the technical report include:

*"...sensitivity or other analyses using variants in commodity price, grade, capital and operating costs, or other significant parameters, as appropriate, and discuss the impact of the results."*

This change to the rules recognizes that single-variable sensitivity analyses superimposed on Static DCF calculations are a useful, but limited, way of describing the uncertainties surrounding a mineral project. "Other analyses" could include Dynamic DCF or RO NPV calculations that incorporate uncertainty analysis techniques such as Monte Carlo simulation or risk adjustments tuned to unique project uncertainty characteristics. The requirement in the revised Form F1 applies explicitly to all "advanced properties" -- defined in the rule as a property with mineral reserves or a property with mineral resources whose potential economic viability is supported by at least a preliminary economic assessment. With that change, the old definition of "development property" no longer had a function in the rule and was removed.

The parts of the technical report that deal with the uncertainty of the economic model are still subject to the general requirement (in s. 2.1 of NI 43-101) that all scientific and technical information disclosed by an issuer on a material mineral project be the responsibility of a qualified person (“QP”). While the 2011 revisions to NI 43-101 did provide some additional flexibility for the QP to rely on other experts for matters that are not within their realm of scientific or engineering practice, that does not extend to the economic analysis or to studies of uncertainties in it.

It should be made clear that demonstrating the economic viability of a resource in a NI 43-101 Technical Report is not considered a formal business valuation. Formal valuations are only required under Multilateral Instrument 61-101, Protection of Minority Security Holders in Special Transactions, which is a rule applying to takeover bids, adopted only in the provinces of Ontario and Quebec. The Companion Policy for this rule refers to general business valuation standards as a "reasonable approach" to disclosure. MI 61-101 is silent on the specifics of mineral property valuations, but an approach incorporating the CIMVal Standards will normally meet similar business valuation standards. The CIMVal Standards recognize RO analysis as a primary valuation method for properties with mineral reserves and in some cases for properties with resources.

## **An extended evaluation and risk management framework**

This section provides a brief overview of an extended evaluation and risk management framework (“Extended Evaluation Framework”) for mine project evaluation that might be included in a technical report. This framework divides the cash flow modelling process into two parts. The first part translates the project into a qualitative cash flow model description which highlights important characteristics of the business environment, the project, and the participation terms of project stakeholders. The framework’s second part comprises the analytical processes used to estimate cash flows, calculate economic benefits, and assess risk.

It is important to remember that any project cash flow model built within the Extended Evaluation Framework is still just an approximate model estimating project cash flows generated within a highly complex project and economic environment. Cash flow modelling within this framework can produce substantial amounts of information about a project’s economic and risk characteristics that are both informative and helpful when making an investment decision. However, as with any economic analysis, these results should still be treated with some caution and tempered with industry experience.

### **Describing the project with a cash flow model**

Translating a project’s development and production plan into a cash flow model consists of two components. The first component is a description of the most important uncertainties associated with the project and the business environment. The second component is an outline of project structure which encompasses not only the project’s cash flow consequences but also possible sources of management flexibility and the division of the underlying project cash flow between equity, creditors, and government. **Figure 1** provides a representation of these two components.

Given the complexity of mining projects, a cash flow model will by necessity be only a crude representation of the project’s full cash flow generating capabilities since it is not possible or even desirable to model every project uncertainty, management flexibility, or financing and

taxation term. An important function of the valuation professional is to identify the most important project characteristics that influence value and risk while ignoring secondary influences so that the interpretation of results is transparent and manageable.

### **Description of uncertainty**

A mining project is exposed to many uncertainties, including economic and financial uncertainties (e.g. metal prices, foreign exchange rates), technical or physical uncertainties (e.g. metal grades, resource tonnage, processing efficiencies), or uncertainties linked to the political or regulatory environment (e.g. environmental regulations, cancelling of mining licenses due to social pressure).

The prime focus of modelling a particular uncertainty is finding a set of conditional probability distributions that describe the possible future random outcomes for this variable. A further consideration is how each uncertainty is resolved during the project, as this may affect the manner in which the project is managed and valued. Uncertainty can be resolved at one distinct point in time through an event such as the awarding of a mining license when an exploration license expires or partially resolved in discrete steps such as through a multi-year exploration or commercial research program. There are also uncertainties that are never fully resolved such as future commodity prices that have their forecasts revised as new information is received on a continual basis from financial markets.

A final aspect of modelling project uncertainty involves the relationship between the various uncertainties and also their interaction with the project and business environment. Some of these uncertainties may be independent of each other, such as metal concentration and metal price. Other project uncertainties may display some correlation, such as copper and gold prices. For valuation and risk discounting purposes, it is also important to understand the relationship between an individual uncertainty and the overall financial markets as this can influence how cash flow discounting is performed.

### **Project structure**

Project structure refers to the design and operational details, management's ability to change development and operation, and the terms of finance and taxation that combine to influence the generation of project cash flow. Static DCF models tend to be concerned with only the timing of cash flows and their composition. However, when modelling project uncertainty there may be reason to consider a more detailed description of structure that incorporates flexibility and contingent tax and financing payoffs.

At some projects, managers may be able to change project structure and cash flow when the arrival of new information causes them to update their expectations about the future. This ability to modify project cash flows is called management flexibility. It increases value as it allows managers to mitigate possible losses in adverse conditions, enhance economic gains in advantageous situations and manage risk through operational decisions as new information is received. The actual value benefit can be large or small depending on the flexibility available, the costs of that flexibility, and the characteristics of underlying project uncertainty.

The presence of contingent financing and taxation terms may also require an extended modelling of cash flow structure as these terms can complicate the division of cash flow between equity, creditors, and government. Contingent terms introduce complexity because

they alter stakeholder cash flow distributions in some situations (e.g. high metal price scenarios) but not in others (e.g. low metal price scenarios). For example, project financing may have payouts that are linked to commodity prices through hedging arrangements (e.g. “costless” collars) while tax regimes have contingent features such as restrictions on tax loss carry forwards, depreciation and net profit royalties.

A complete model of project structure needs to fully articulate the dynamic interaction of equity, creditor, and government interests in order to correctly estimate each stakeholder’s cash flow under various possible random project outcomes.

## **Project cash flow calculation, value estimation and risk assessment**

Once the details of a cash flow model have been decided, a three-part analytical process is followed to estimate cash flow, analyse the project’s economic benefits, and describe its uncertainty and risk characteristics. **Figure 2** outlines this three-part analytical process. The actual details of this process will be determined by the objective of the evaluation exercise, the availability of information, and resources available to perform the evaluation.

### **Numerical method**

The standard approach for estimating mine project cash flows is the Static DCF method, where expected or median outcomes for uncertain project variables such as metal price, ore grades, and processing are used in a simple and well-known set of multiplicative and additive calculations to arrive at an expected net cash flow estimate. It is important to recognize that a static cash flow estimation technique will generate erroneous cash flow expectations when there is uncertainty, flexibility, and contingent tax and financing terms that create expectations over a non-linearity. In some situations, this error may be minor and can be ignored. However, static cash flow error can be large in other situations which may result in incorrect conclusions about a project’s economic viability.

There are several advanced numerical methods that can be used to avoid static cash flow error. Among these techniques, binomial trees (or lattices) are popular as cash flow estimation tools since these methods are more visual and can be implemented relatively easily in a spreadsheet. However, binomial trees have difficulty correctly estimating the effects of path-dependent cash flow structures such as tax-loss carry forwards or net profit royalties that allow the recovery of an initial investment. For this reason, Monte Carlo simulation may be preferred as a method of modelling uncertainty.

A more powerful numerical approach finding increased use in the mining industry is based on the Longstaff-Schwartz algorithm (Longstaff and Schwartz, 2001). This flexible simulation technique uses Monte Carlo simulation to generate a large number of random project cash flows based on the underlying project uncertainty model. Then, it uses dynamic programming combined with regression techniques to estimate cash flow consequences of various project designs and operating decisions. When matched with either the Dynamic DCF or RO valuation methods it can identify those designs and decisions that maximize expected project value within the valuation framework. Natural resource project examples of using this approach include Blais *et al* (2004), Sabour and Poulin (2006), and Laughton *et al* (2008).



## Estimate of project value and return

The project cash flow estimate generated by the chosen numerical method is used to determine whether participating in a project is expected to have a positive net benefit for a potential stakeholder. There are two primary measures of whether project participation is advantageous. These two measures are IRR and NPV. IRR was briefly discussed in the introduction and will not be considered further.

NPV is the expected net benefit or loss to a participant from making an investment in a project after accounting for the risk-adjusted opportunity cost of capital. The natural resource industries primarily use the DCF method to calculate project NPV. A distinguishing feature of a Static and Dynamic DCF NPV is that adjustments for the value effects of uncertainty and time are applied on an aggregate basis to the project expected net cash flow stream using a constant risk-adjusted discount rate ("RADR"). That RADR is applied via the bond pricing formula  $1/(1+\text{RADR})^t$ . The choice of RADR may be based on a financial market model of asset returns such as the Capital Asset Pricing Model ("CAPM") or the Weighted Average Cost of Capital of the investor. One discount rate is often used across a broad class of investment projects regardless of the actual uncertainty characteristics of each particular project. The primary difficulty with DCF NPV is reconciling the risk adjustment applied to a cash flow with the cash flow's actual uncertainty and risk characteristics.

The RO method is an alternative approach to calculating project NPV that is being investigated by some participants in the natural resource industries. The RO NPV method is derived from the same finance theory as the DCF method, with additional influences from the development of financial option pricing theory by Black, Scholes, and Merton during the early 1970s (Black and Scholes, 1973; Merton, 1973). The term Real Option was suggested as a means of identifying the application of option theory to real assets as opposed to financial assets. The RO method described in the next few paragraphs may seem novel to some mining professionals but it can be considered a certainty equivalent approach to calculating an Expected Net Present Value which is an accepted method for calculating fair value under IFRS 13 by the International Accounting Standards Board (IFRS Foundation, 2011).

The distinguishing feature of the RO method is its risk adjustment approach. This may be surprising to some as many RO publications state that the explicit recognition of flexibility is the main difference between the DCF and RO NPV methods. However, RO's recognition of flexibility is not unique, as modifications to the Static DCF method for flexibility (*i.e.* Dynamic DCF) have been proposed since at least the mid-1960s (Magee, 1964), although they are only infrequently used.

The RO method takes the underlying static or dynamic cash flow model and applies risk adjustments to the primary commercial sources of cash-flow uncertainty, such as input and output prices. Measures of the estimated risk associated with these uncertainties are developed based on capital market information and its supporting finance theory. If the prices of uncertain inputs and outputs are observable in well-developed financial markets, the forward price curve for the particular input or output may be used as risk-adjusted expected prices in the cash flow calculation because a commodity's forward price may be interpreted as a risk-adjusted expected price (Geman, 2005; Markert and Zimmermann, 2008). Alternatively, a corporate forecast price may be risk adjusted using a model such as the CAPM to estimate risk-adjusted expected prices if corporate guidelines require the use of a forecast price or there is a lack of market information.

Risk-adjusted cash flows for the project are then estimated using these risk-adjusted forecasts. Under an assumption that this process has dealt with all uncertainties, the risk-adjusted cash flow is valued by discounting it for the time value of money (i.e. at the risk-free rate) based on the long-term yield of a publicly-traded government bond.

A Real Option model can be further supplemented with a residual project risk factor applied to the risk-adjusted cash flow to account for aspects of technical and commercial project uncertainty that have not been explicitly recognized under the RO methodology (Smith and McCardle, 1998, 1999). Support for the use of a residual risk premium is also found in recent accounting guidelines requiring the use of a counter-party risk premium when estimating the fair value of a forward contract and other derivative financial instruments (CICA Emerging Issues Committee, 2009). These guidelines recognize that not all the uncertainty and risk attached to the payout of a forward contract or other derivative is recognized by a standard derivative valuation model. One method of recognizing additional contract risks such as counterparty default is to supplement the discounting process in the standard model with an additional risk premium.

The difference between DCF and RO risk-adjustment may appear small but it has important implications for project evaluation. The key benefit of the RO approach is its ability to recognize the complex variation of cash flow risk over the project life due to the characteristics of individual project uncertainties, project structure and management flexibility, and the terms of financing and taxation.

In contrast, the DCF method's approach to risk adjustment implies that cash flow uncertainty increases in a regular manner over the life of the project. This may be an appropriate simplification for some project evaluations but it may also produce misleading results for other evaluations. Situations where a DCF risk adjustment may cause problems include:

- 1) long-life projects where the short-term price of inputs or outputs fluctuates around a long-term equilibrium level.
- 2) projects where input or output prices do not exhibit reversion (i.e. they fluctuate in a manner similar to stock prices) and management has the flexibility to respond to this uncertainty.
- 3) projects in which the structure of royalty, taxation, and financing cash flows change significantly with output price levels or are dependent on the path of past prices.

This list does not imply the DCF NPV method is inappropriate in these cases. It suggests evaluation situations where a valuation professional should consider whether the DCF risk-adjustment approach is suitable.

## **Project risk assessment**

Traditionally, quantitative risk assessment within a Static DCF model has been limited to scenario and sensitivity analyses. Scenario analysis re-calculates project cash flows and NPVs for a number of alternative project scenarios while sensitivity analysis changes a particular input variable over the life of the project. Both techniques provide an indication of the variability in project NPV to changes in input assumptions. However, they do not provide any guidance on project risk since they do not define nor provide a summary measure of project risk.

The use of Monte Carlo simulation combined with risk management concepts from the finance industry has extended the ability of valuation professionals to perform project risk analysis and communicate the results in a more concise manner. Monte Carlo simulation allows the

reporting of not only expected cash flow but also cash flow confidence boundaries and measures of cash flow uncertainty. Further analysis can then be performed on histograms of cumulative project cash flow to highlight potential cash flow losses or gains from investing in the project as well as how management flexibility or financial strategies may limit the losses and enhance the gains (Samis and Davis, 2009). Lastly, the information from simulation can be used to estimate the probability of certain events occurring, such as closing early due to low metal prices or the development of sub-economic resources if metal prices increase given the uncertainty assumptions used in the cash flow model. This is particularly useful as a means of probabilistically estimating reserves and resources.

The use of quantitative financial risk management concepts during project evaluation are still a novel application in the mining industry. Although it is not clear at this time how financial risk management concepts can be adapted for analysing mining projects, the range and depth of risk management applications in the financial industry provide useful tools (and warnings) for mining project risk assessment and decision making.

## **The Oyu Tolgoi Project – an example of Dynamic DCF / Real Option analysis**

Two NI43-101 Technical Reports have been published during the last two years that include Dynamic DCF and RO analyses as a complement to Static DCF analysis and as a means of recognizing specific value-influencing project characteristics. The first was Ivanhoe Mines' NI43-101 Technical Report for the Oyu Tolgoi Project ("OT NI43-101 Report"; Ivanhoe, 2010) which performed these analyses on a "no-flexibility" basis (i.e. the effects of management flexibility were ignored) for the Life-of-Mine Sensitivity Case. This report was followed shortly afterwards by a similar report published by Entree Gold for their interest in the same project. We provide a short review of the analyses in the Ivanhoe report to illustrate how these techniques can be used to communicate the impact of specific project characteristics on economic viability.

The Oyu Tolgoi Project is one of the world's largest undeveloped copper-gold projects. It is located in Mongolia, approximately 550 km south of the capital, Ulaanbaatar. It is being jointly developed by Ivanhoe Mines, Rio Tinto, and the Government of Mongolia. The OT NI43-101 Report predicts the initial project development phase will last to December 2013 followed by an expansion development phase that continues to 2020. The project's production horizon lasts from 2014 to 2071 during which an estimated 3 billion tonnes of Run-of-Mine ("ROM") ore, assuming resources are converted to reserves, will be processed. This ore will produce 50 billion pounds ("lb") of copper, 25 million troy ounces ("oz") of gold, 161 million oz of silver, and 136 million lb of molybdenum after adjustments for recoveries, smelter / refinery deductions, and other factors. Over the life of the project, average annual after-tax operating cash flow is estimated to be US\$978 million in real 2010 Dollar terms with an average profit margin of 43 per cent given a forecast long-term copper price of US\$2.00/lb and gold price of US\$850/oz. Capital costs in real terms for the initial development and expansion phases were estimated to be US\$7.2 billion while sustaining capital was projected at US\$11.4 billion over the project's production horizon.

The primary problem confronting a valuation professional when dealing with a project like the Oyu Tolgoi Project is how to recognize the benefits of forecasted strong positive cash flows over a very long production horizon against the cost of large amounts of capital initially incurred to bring the project into production. Static DCF models have difficulty fully communicating the

uncertainty and risk characteristics of this cash flow pattern generated by a base metal mine. Using Dynamic DCF and RO methods within an extended evaluation framework provides a more complete set of tools with which to perform an economic analysis.

The Dynamic DCF and RO analyses in the OT NI43-101 Report are broken into four sections. These include:

- 1) a description of metal price uncertainty;
- 2) an overview of cash flow uncertainty characteristics when there is no management flexibility;
- 3) a statement of the Dynamic DCF and RO evaluation results and sensitivity to uncertainty model parameters; and
- 4) an explanation for the differences between the DCF and RO NPV results.

### **Description of metal price uncertainty**

The Dynamic DCF and RO analyses in the OT NI43-101 Report incorporate explicit models of copper, gold, and molybdenum price uncertainty, while uncertainties related to capital and operating costs and technical characteristics are not modelled. In this paper, we discuss only the copper price model because copper revenues comprise 80 per cent of overall project revenues. Copper price uncertainty is modelled with a one-factor reverting lognormal stochastic process. Reversion is the tendency for a metal spot price to fluctuate randomly around a long-term equilibrium level. When a metal price exhibits reversion, prices that are much higher or lower than the long-term equilibrium price are hard to sustain as it is assumed that supply/demand forces in the economy move metal prices back to the equilibrium level.

This type of price model also recognizes that market participants can update their price forecasts in response to new price information. This characteristic is illustrated in **Figure 3**, where a single simulated copper price scenario in constant monetary terms is displayed as a solid black line. Note that this price scenario is just one of many generated during the Monte Carlo simulation process. Figure 3 also provides a snapshot of forecast prices and the associated level of price uncertainty at various project times in constant monetary terms. At the start of the project, price expectations are represented by the dashed purple line. These expectations decline from the market spot price on 31 December 2009 towards the long-term Ivanhoe forecast of \$2.00/lb.

The uncertainty associated with this forecast is illustrated by 90 per cent / 10 per cent confidence boundaries which are demarcated by the dotted light purple lines. These boundaries provide a price range within which 80 per cent of metal prices are expected to fall at a particular time. The confidence boundaries for copper price stabilize at \$3.00/\$1.16 per lb due to the effect of reversion.

The updating of forecasts and uncertainty are shown by the blue and green lines in Figure 3. These lines depict the model's revision of copper price expectations and associated confidence boundaries based on the simulated spot price at a select point in time. They can be re-drawn at any point along the simulated path. The dashed blue line and its accompanying dotted light blue lines outline the revised metal price expectations and confidence boundaries for 31 December 2019 when simulated copper price is \$3.19/lb as of that date. The dashed green and dotted light green lines present revised price expectations and confidence boundaries based on that simulated outcome. For both lines, copper price expectations revert back towards their

long-term equilibrium levels while long-term confidence boundaries after 2040 are unaffected by short-term price changes because of price reversion.

Price model inputs such as price volatility and reversion strength were estimated by econometric statistical techniques from historic price data. The assumption of historic copper price reversion was tested and supported by a statistical test. Price medians in the model were set so that the metal price models had long-term expectations equal to Ivanhoe's long-range price forecasts.

### **Cash flow expectations and uncertainty characteristics**

The static cash flow model underlying Ivanhoe's economic analysis of the Oyu Tolgoi Project was rebuilt to facilitate the simulation of metal price uncertainty and the effect of this uncertainty on project cash flows. Metal prices were simulated in constant dollar terms and then inflated at a 2 per cent rate into nominal terms to calculate cash flow. Price-dependent calculations such as government taxes and royalties and the Entrée cash flow distribution were adapted to allow for the wide range of metal prices generated by simulation while still reflecting the original terms. Cash flows were then deflated into real terms for various reports and graphs.

**Figure 4** presents the expected real after-tax operating cash flow for the Oyu Tolgoi project and the associated 10 per cent and 90 per cent confidence boundaries. After-tax operating cash flow is defined as revenue less direct and indirect costs, taxes, royalties, the Entrée distribution, and miscellaneous charges but does not include capital expenditures or working capital. Based on the information in the OT NI43-101 Report, annual average after-tax operating cash flow is US\$978 million, though annual expected amounts vary greatly. The cash flow confidence boundaries demarcate a range in which 80 per cent of the cash flows are estimated to fall for a specific time and provide some indication of cash flow uncertainty. The level trend in the uncertainty of cash flow amounts between 2025 and 2050 is due to the combination of similar annual production amounts and copper price reversion.

It is important to realize that the project cash flow expectations and uncertainty characteristics in Figure 4 along with the addition of capital expenditure and working capital underlie both of the Dynamic DCF and RO NPV calculations. These two NPV techniques start with the same project cash flow and apply different risk adjustment methods to arrive at what are most likely different NPVs.

### **Dynamic DCF and RO evaluation results**

The primary economic analysis section in the OT NI43-101 Report used the Static DCF method to estimate the project's NPV at US\$5.6 billion. This calculation was done in constant dollars with a real 8 per cent risk-adjusted discount rate.

The Dynamic DCF and RO analysis conducted in the report differed from the conventional Static DCF calculation as follows:

- 1) A nominal 10.2 per cent risk-adjusted discount rate was used in the Dynamic DCF NPV calculation which is a discrete discounting adjustment applied to the real 8 per cent DCF discount rate to recognize a 2 per cent inflation rate.
- 2) The RO NPV calculation used a residual risk premium of 2.2 per cent to risk-adjust cash flows for non-metal price uncertainty. This premium was estimated by splitting the risk rate in the real DCF discount rate into revenue and non-revenue components. Risk-adjusted

RO cash flows were discounted using a nominal time discount (risk-free) rate of 4.7 per cent.

- 3) The Capital Asset Pricing Model and financial market data was used to estimate individual risk adjustments for gold, copper, and molybdenum prices in the RO calculation as a proxy for how markets would evaluate these risks.
- 4) Initial prices for copper and molybdenum for the cash flow analysis were set to market prices as of December 31, 2009 and allowed to revert back to the Ivanhoe long-term metal price forecast instead of starting at the long-term forecast price.
- 5) Cash flow estimates were generated with simulation in order to remove possible cash flow estimation errors linked to static cash flow calculations and to generate cash flow uncertainty information.

Over the project life, the Dynamic DCF and RO analyses estimate cumulative net cash flow at US\$38.4 billion. The Static DCF calculation estimated this figure to be \$37.7 billion. The difference in the two estimates is due to the Dynamic DCF and RO analyses starting from the actual current market prices for copper and molybdenum and small non-linear cash flow effects related to expectations over taxes and royalties.

The NPVs from Dynamic DCF and RO calculations in the supplemental analysis were US\$6.0 billion and US\$7.6 billion respectively. The difference between the two estimates is the result of risk adjustment differences. The particular risk associated with price reversion is recognized by RO through an explicit risk adjustment factor to the copper and molybdenum revenue streams. The RO NPV is higher than the DCF NPV in this particular instance due to what turns out to be a lower risk adjustment. This outcome may not occur for other projects.

### **Discounting and risk adjustment differences between Dynamic DCF and RO NPV results**

The OT NI43-101 Report provides a comparison of the of the discounting and risk adjustments applied to the project after-tax operating cash flow by the Dynamic DCF and RO NPV methods. This comparison is made to give valuation professionals an understanding of how well each NPV method adapts discounting and risk adjustments to changes in cash flow uncertainty during the project. Insight into these adaptations may assist a valuation professional to choose between DCF and RO NPV results if it is important to have a detailed correspondence between risk adjustment and the level of cash flow uncertainty.

The difference between Dynamic DCF and RO NPV results is attributed in the OT NI43-101 Report to an overall difference in effective discounting. The report relies on annual RO and Dynamic DCF cash flow discount factors ("CFDF") for the after-tax operating cash flow to explain the NPV difference.

A CFDF is defined as the ratio of a cash flow's present value to its expected value and reflects adjustments for both cash flow uncertainty through a risk adjustment and the time value of money. This ratio estimates the amount the market would be willing to pay now for a dollar of uncertain project cash flow at a specific future time. Dynamic DCF and a RO CFDFs will likely differ because each NPV method adjusts for risk in a different manner.

**Figure 5** presents the Dynamic DCF and RO CFDFs for the Oyu Tolgoi Project and reveals why the RO NPV is higher. In this figure, RO CFDFs, demarcated by the dashed green line, indicate that the RO method tends to value cash flow more highly than the Dynamic DCF method whose CFDFs are delineated by a solid grey line. For example, for after-tax operating cash flow

occurring in 2039, the RO method estimates that each \$1 of expected cash flow has a present value of \$0.103 while the Dynamic DCF method estimates that each \$1 of expected cash flow in the same year has a present value of \$0.058. This graph shows that the RO CFDFs vary dynamically from year-to-year due to changes in cash flow uncertainty caused by changes in operating leverage (change in unit cost) while the DCF CFDFs decrease in a steady manner related to the functional form of the discounting formula.

The analysis of Dynamic DCF and RO discounting in the OT NI43-101 Report goes further by considering how the price risk adjustment effectively applied by each NPV method responds to changes in cash flow uncertainty. This is done by removing the adjustments for the time value of money and residual risk from the CFDFs to generate Cash Flow Price Risk Discount Factors (“CF-PRDF”) and comparing the modified Dynamic DCF and RO factors to changes in cash flow uncertainty.

**Figure 6** presents this detailed risk adjustment analysis. In this figure, after-tax operating cash flow uncertainty is tracked by the dash-dot blue line against the left-hand Y-Axis using the cash flow Coefficient of Variation (“CoV”). Cash flow CoV is defined as the standard deviation of the uncertain cash flow divided by its expected amount. A larger CoV indicates a relatively larger amount of uncertainty. The cash flow CoV line shows that annual after-tax operating cash flow uncertainty varies within a range of just under 40 per cent to 100 per cent before 2063 and then increasing markedly after production is sourced increasingly from the lower-grade Heruga Deposit. This graph shows that the variation of cash flow uncertainty is dynamic and non-constant over the life of the Project. Uncertainty increases or decreases and displays sudden large variations as year-to-year unit costs change due to variations in ore grade or changes in mining methods. The gradual trend of increasing cash flow uncertainty between the years 2020 to 2060 is due to the interaction between copper price reversion and a gradual decline in ore grades.

The right-hand Y-Axis in **Figure 6** plots the pattern of RO and Dynamic DCF Cash Flow-Price Risk Discount factors (“CF-PRDF”). These CF-PRDFs represent an estimate of the amount the market would be willing to pay now on a metal-price risk adjusted but not time discounted or residual risk-adjusted basis for a dollar of a future cash flow.

The Dynamic DCF CF-PRDF and pattern of DCF metal price risk discounting is demarcated by the solid dark grey line. This line reflects a pattern of metal price risk adjustments associated with cash flow uncertainty that is continually increasing in a regular manner whereas the profile of cash flow CoVs suggests a cash flow uncertainty pattern that increases in a more erratic fashion. The Dynamic DCF CF-PRDFs do not appear particularly consistent with the pattern of cash flow uncertainty at the Oyu Tolgoi Project.

The dashed green line in **Figure 6** delineates the RO CF-PRDFs and shows the RO metal price risk adjustments respond in a manner that is more consistent with the erratic annual changes in after-tax operating cash flow uncertainty. Specifically, increases in cash flow uncertainty result in an increase in risk adjustment (*i.e.* a decline in the CF-PRDF and an increase in risk compensation) and a decline in uncertainty results in a corresponding decline in risk adjustment (*i.e.* an increase in CF-PRDF and a decline in risk compensation).

Choosing between the Dynamic DCF and RO NPV results is a matter of professional preference. However, **Figure 6** suggests that this choice could be made on how the risk adjustments of each NPV method respond to changes in cash flow uncertainty. A fundamental principle of finance theory is that investors are risk averse and require more compensation for

investing in a project with more uncertain cash flows. Risk compensation is determined by a cash flow risk discount whereby an investor would be willing to pay an amount that is smaller than the expected amount of an uncertain cash flow. This concept of risk/return implies that annual variations in cash flow uncertainty should be accompanied by changes in the amount an investor is willing to pay on a risk-adjusted basis per cash flow dollar. In the case of the Oyu Tolgoi Project, the pattern of RO CF-PRDFs is more consistent with the pattern of cash flow uncertainty than the Dynamic DCF CF-PRDFs, which suggests that RO NPV may be more representative of the project cash flow risk profile caused by reversion in copper price and changes in unit costs over time.

## **Some thoughts on best practice for Dynamic DCF / Real Option analysis in NI43-101 reports**

The use of Dynamic DCF and RO methods in a NI43-101 Technical Report to demonstrate the prospects for the economic extraction of a proposed resource requires some thought as to what is considered professionally acceptable analysis. A Static DCF model is a straightforward analytic framework which is broadly applied in a consistent manner across the mining industry. The use of the Extended Evaluation Framework with its advanced valuation theory and numerical methods requires a specialist skill set that is less common in industry. This increases the prospect of inconsistent and unprofessional Dynamic DCF and RO analysis being included in NI43-101 Technical Reports. Examples of poor professional practice using variations of the Extended Evaluation Framework in general mining valuation situations include the use of the Black-Scholes model based on superficial similarities between an undeveloped mining project and a financial call option, and the use of real option models where uncertainty is modelled in aggregate at the level of operating profit, rather than at individual cash flow elements, such as is discussed in Copeland and Antikarov (2001) and other standard RO textbooks. The worry here is that the use of these discredited approaches will lead to the production of economic information that is misleading and confusing (Borison 2005, Smith 2005).

This paper proposes some analytical elements of good professional practice when working with the Extended Evaluation Framework. These elements include:

- 1) A description of project uncertainty with statistical support for model assumptions;
- 2) Analysis of cash flow uncertainty characteristics when there is both no flexibility and management flexibility (if considered);
- 3) Details of the links between Static DCF analysis and the analysis conducted with the Extended Evaluation Framework; and
- 4) A sensitivity analysis of the economic input parameters for the Dynamic DCF and RO models.

### **Project uncertainty**

The introduction of explicit uncertainty models will require that each individual model be supported by some statistical justification. There is a tendency in the mining industry to use inappropriate models (e.g. modelling metal price density functions as triangular distributions) that are based on personal judgement and experiences in unrelated areas.

For commodity prices used in estimating revenues and costs, there are several useful models of price movements ranging from fairly simple one-factor stochastic models to more sophisticated three-factor models, in which information about price standard deviation, skewness, kurtosis,



and intertemporal correlation can be included. Statistical analysis of historic price data must be used to estimate model parameters and to demonstrate whether a particular type of model is appropriate for the commodity being modelled. Long-term metal price expectations in the uncertainty models should be consistent with those used in the base case Static DCF model, and any differences between this expectation and financial market trends should be discussed. Lastly, a qualitative description of each model, supported by graphs, should be included to illustrate the uncertainty characteristics in non-mathematical terms.

Technical uncertainty will be more difficult to integrate into the Extended Evaluation Framework given that there may be insufficient project information to develop a model (e.g. a grade uncertainty model at an early exploration stage). However, some late-stage projects may have large amounts of information from earlier exploration and engineering work that allows a qualified professional to develop a technical uncertainty model that is justified in a manner conceptually similar to models of economic uncertainty.

### **Cash flow uncertainty and flexibility**

Advanced numerical methods such as Monte Carlo simulation or lattice methods are able to assess the economic sensitivity of a resource to changes in its business and project environment. This information generated with these methods provides a richer description of cash flow uncertainty than the more commonly used sensitivity analysis involving spider plots or tornado diagrams.

Analysis of cash flow uncertainty must start with the base case development and production policy under the condition of no management flexibility as this is directly comparable to the Static DCF model used in earlier sections of the NI43-101 Technical Report. Analysis may include graphical representations of expected annual cash flows and confidence boundaries, relative measures of annual cash flow uncertainty such as CoV plots, histograms of cumulative cash flow, and other risk management tools such as conditional tails expectations. Cash flow uncertainty characteristics should be discussed in the context of project features such as linking increases or decreases in cash flow uncertainty to changes in metal mix, grades, mining method, depth of operation, or mining area.

Management flexibility has yet to be introduced into the economic analysis section of a NI43-101 Technical Report. However, flexibility analysis is a powerful method for describing the economic conditions in which sub-economic resources may be developed at a profit or the how a project will be managed in adverse conditions such as low metal prices. Management flexibility should be described where possible with a decision tree and details of each development and production alternative represented by the individual decision tree branches. The cash flow uncertainty characteristics of each operating alternative and the conditions under which each alternative would be selected should be discussed. Cash flow uncertainty would be articulated in a manner similar to that of no-flexibility analysis while selection of operating alternatives can be described in terms of policy plots that link price boundaries to changes in operating policy. Ultimately, a dynamic programming application such as a binomial lattice, partial differential equations, or Longstaff-Schwartz simulation will be needed to solve for the project value given the optimal operating policy under various random price outcomes.

Finally, the impact of flexibility should be considered on a project-wide basis to explain how flexibility affects value and risk through changing the histogram of cumulative net cash flows. The dynamic programming solution will also provide information as to how management is

predicted to react under various random outcomes, and this can be checked for realism and constraint violations.

### **Linking Static DCF and Dynamic DCF / RO analysis**

There will inevitably be differences between conventional Static DCF NPV and NPVs calculated using the Dynamic DCF and RO methods. These differences need to be explained to support the validity of results produced with the Extended Evaluation Framework, particularly given the mining industry's familiarity with the Static DCF method.

A starting point for explaining the differences in results is a component NPV build up highlighting how project NPV changes with the introduction of metal price expectations linked to a stochastic model, the effect of recognizing cash flow non-linearities with simulation, the use of RO risk-adjustments in place of DCF discounting, switching between constant dollar and nominal dollar cash flow calculations, and the value impact of various types of flexibility. The purpose of the build-up is to indicate the primary factors impacting the economic extraction of a resource which may be able to include among other considerations the presence of non-linear windfall taxes or sliding-scale royalties, the flexibility value of modular capital development or the contingent development of sub-economic resources, or the effect of price reversion on long-term cash flows. An effort should be made to explain how the risk adjustments profiles differ between Dynamic DCF and RO NPV calculations to assist with understanding the value change produced by switching between the two NPV methods.

### **Sensitivity to changes in Dynamic DCF / RO uncertainty model parameters**

Dynamic DCF and RO models rely on the estimation of parameters from historic data that may not be considered appropriate for future economic and project conditions by some investors. Some of these parameters may include the price of risk in financial markets (a measure of investor risk aversion), commodity price volatility, correlation between commodity prices and financial markets, strength of reversion or even the presence of reversion. A sensitivity analysis should be provided that shows how project NPV varies with uncertainty model parameters. Providing this analysis will allow investors to assess the project's economic prospects under alternative market and technical circumstances.

## Conclusion

Techniques for evaluating mining project economics are continually improving and becoming more sophisticated. The analysis associated with the traditional Static DCF method is little changed from when mining engineers first used this method in the early 1900s. This paper provides an overview of how the Static DCF method can be adapted and extended by incorporating some of the advances in finance and economics that have been made in the past 40 years. The two specific approaches to evaluation that we describe are called Dynamic DCF and Real Option analyses. Both of these methods have the potential to estimate project cash flows given explicit models of project uncertainty and dynamic project structure such as management flexibility. However, the two analyses techniques vary in the manner in which a risk adjustment is applied. Dynamic DCF risk adjusts cash flows through discounting the net cash flow stream with a constant discount rate while the Real Option method risk adjusts individual sources of cash flow uncertainty (e.g. metal price) using a tailored risk adjustment before discounting the resulting risk-adjusted net cash flow for the time value of money. Risk adjustment differences between these techniques will produce different valuation results which will require explanation that will ultimately provide valuable information to investors interested in the project.

Recent amendments to the guidelines for NI43-101 Technical Reports allow economic analysis using a Static DCF model to be supplemented with analysis considering the economic impact of project uncertainty. We discuss an example of this supplemental analysis in which Ivanhoe Mines and Entrée Gold used the Dynamic DCF and Real Option methods in their respective NI43-101 reports for the Oyu Tolgoi Project to provide additional information about the characteristics of project risk and its effect on project value. This analysis showed that the Real Option method tends to vary its effective risk discounting commensurately with the variation of the net cash flow risk, in contrast to the Dynamic DCF approach which employs a risk adjustment that does not vary with changes in project risk over time.

There is now considerable latitude for qualified persons to use more sophisticated economic analyses in technical reports. We would even argue that best practice requires that the effort spent on economic analysis in technical reports be bolstered to match the effort and sophistication on the technical side. Since these economic techniques require skills that are new to the industry, the main difficulty in their implementation is finding a professional with the appropriate training to undertake the analysis and finding senior managers comfortable with interpreting and explaining the results. It is becoming increasingly evident that project evaluation requires mining professionals and senior managers that are familiar with both the technical aspects of mining and the financial aspects of commodity markets. However, the expense and effort required to gain this familiarity will be more than offset with an improved understanding of how the characteristics of project and market uncertainty combine with financing arrangements and management flexibility to affect project value and risk.

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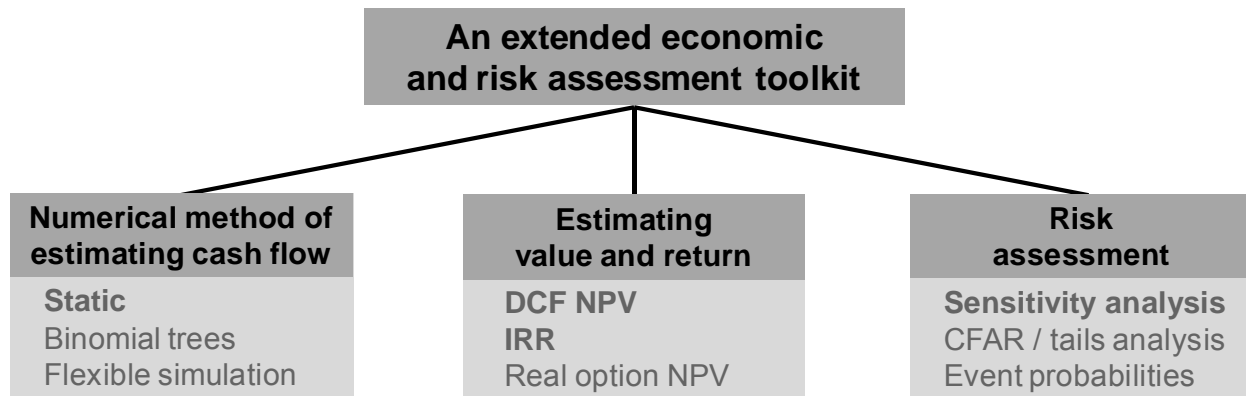
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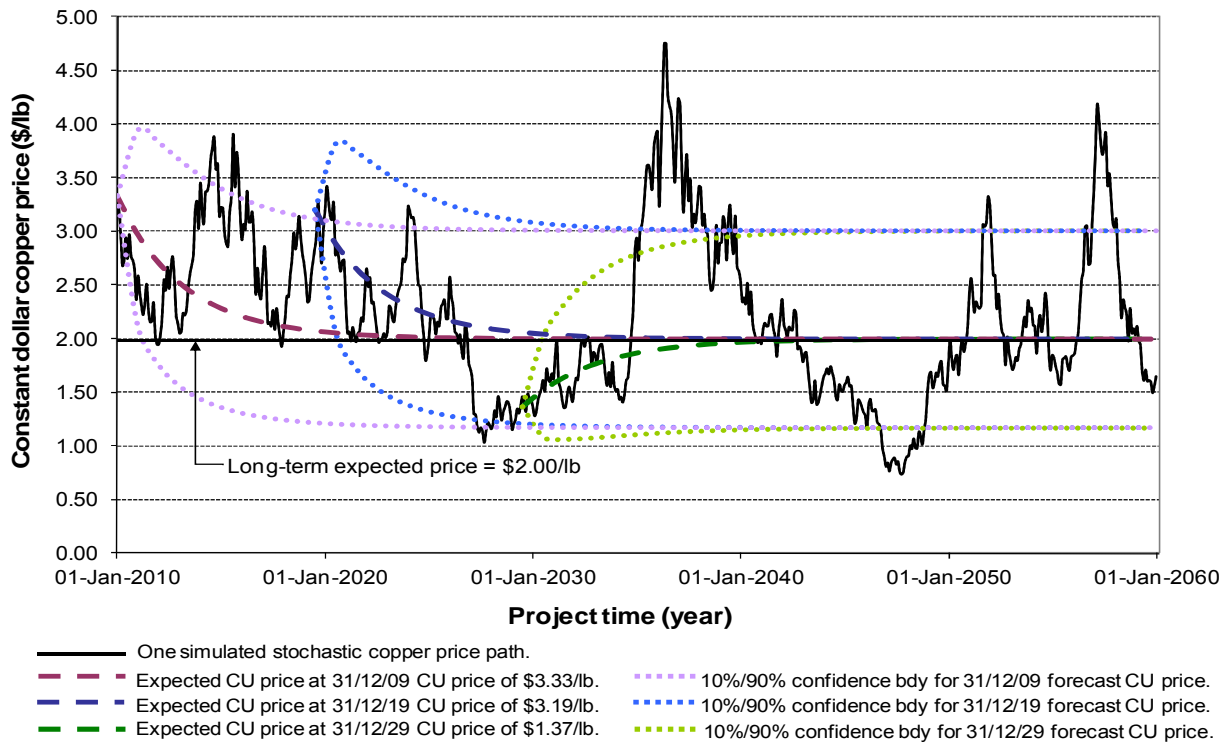
**Figure 1** Components of a project cash flow model

<b>Components of a cash flow model</b>	
<u>Description of uncertainty</u>	<u>Project structure</u>
<b>1) Type</b> Economic / technical / political	<b>1) Cash flow</b> Timing / leverage / non-linearities
<b>2) Resolution / updating</b> Continuous / discrete / Bayesian	<b>2) Flexibility</b> Operations / development / other
<b>3) Interaction</b> Systemic / unsystemic / correlation	<b>3) Financing / taxation</b> Equity / creditors / government

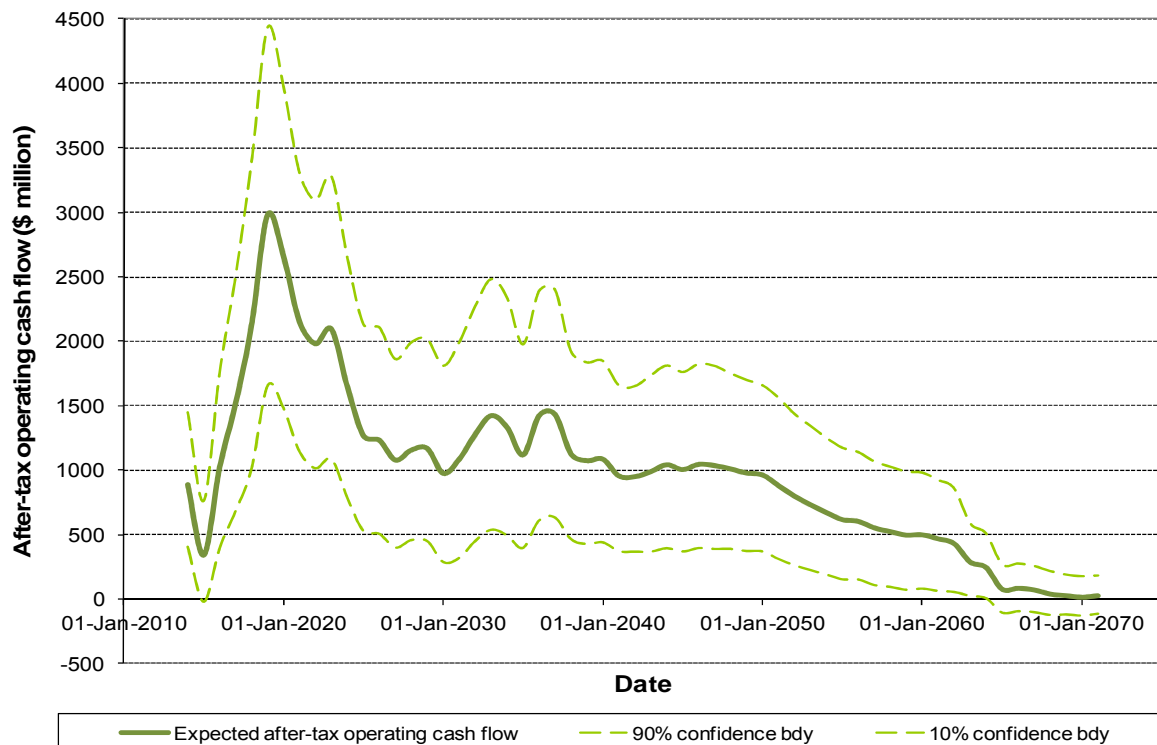
**Figure 2** An extended project evaluation and risk management analytical process



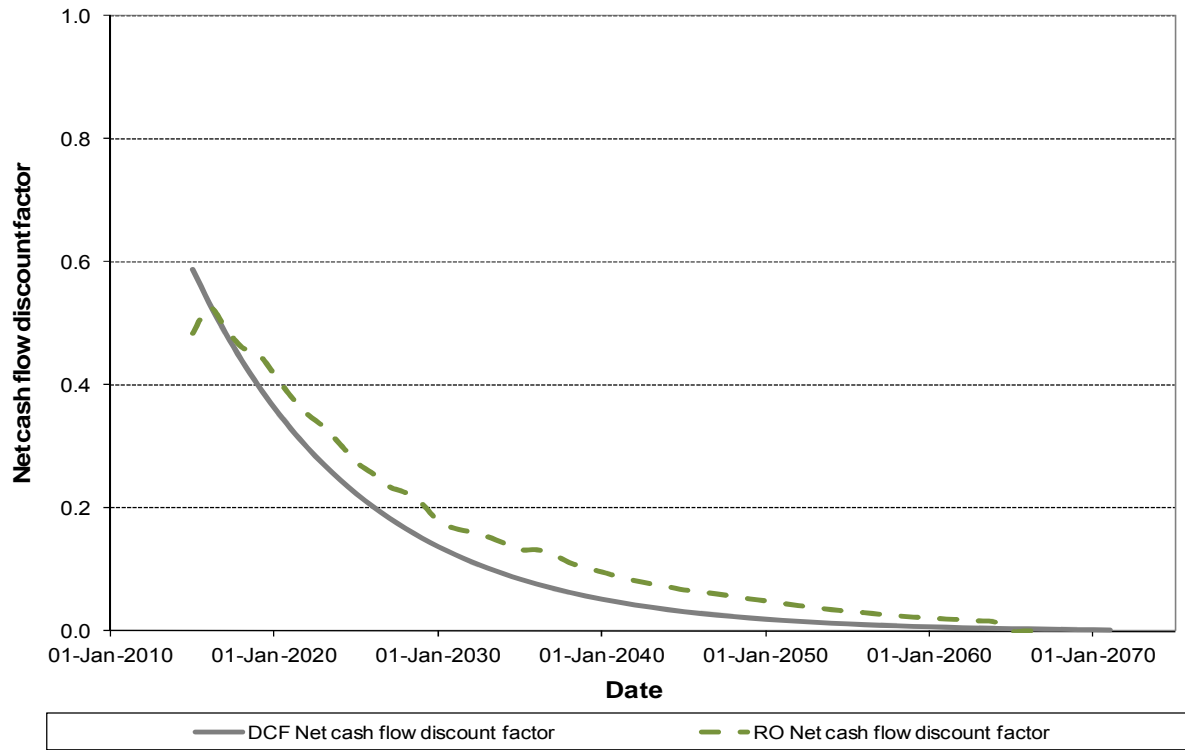
**Figure 3** A simulated price path with expectations and confidence boundaries for a one-factor stochastic copper price model



**Figure 4** Real after-tax operating cash flow expectations with 90 per cent / 10 per cent confidence boundaries



**Figure 5** Dynamic DCF and Real Option net cash flow discount factors



**Figure 6** Dynamic DCF and Real Option cash flow coefficient of variation and cash flow price risk discount factors

