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Steering and focusing diffusive fields using synthetic aperture

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Abstract – Although beam steering and focusing have been used for waves in many important ways, the application of these concepts to diffusive fields has not been wide spread because of the common belief that diffusion lacks directionality and therefore can neither be steered nor focused. We use the similarities between diffusion and waves and prove that diffusive fields can be steered and focused both in the frequency domain and in the time domain. This finding has the potential of extending the use of diffusive fields as a diagnostic tool in science.

Introduction. – In general, diffusive fields (such as diffusing chemicals, heat, low-frequency electromagnetics in conductive media, light in highly scattering media) are fields whose propagation is governed by the diffusion equation or that have a diffusive character. An example of the latter are low-frequency electromagnetic fields in conductive media [1]. These diffusive fields have been used in important ways in modern science [2,3]. Based on the similarities of wave propagation and diffusion, many of the wave concepts have been extended to diffusion and show significant improvement in diffusive fields applications. Examples of such similarities are the interference [2,4–7], refraction [8] and scattering [9,10] of diffusive fields. Beam steering and focusing have been widely used for waves [11–19] either by using a physical array that steers or focuses waves, or by using data processing to achieve this (referred to as the synthetic-aperture technique in refs. [11–14]). These techniques have seldom been discussed for diffusive fields because it is commonly believed that such fields lack directionality [3,20]. In this letter, we use the similarities between the Green's functions of waves and diffusion in a homogeneous space to show that diffusive fields can be steered and focused.

As with the applications involving waves, beam steering and focusing for diffusive fields can extend the use of these fields and open new research directions. For example, it is shown in ref. [21] that anomalies in the electromagnetic field due to the presence of the submarine hydrocarbon reservoir are dramatically increased by applying beam steering to diffusive electromagnetic fields.

Similarities between diffusion and waves. – Although diffusion and wave propagation are two different fundamental physical processes, the analogy in the mathematical description of these two processes is known [22–25]. The 3D diffusion equation in a homogeneous medium, under the Fourier convention $f(t)=\int F(\omega)e^{i\omega t}\,\mathrm{d}\omega$, can be written in the frequency domain as

$$D\nabla^2 G(\mathbf{r},\mathbf{r}_s,\omega)-i\omega G(\mathbf{r},\mathbf{r}_s,\omega)=-\delta(\mathbf{r}-\mathbf{r}_s), \qquad (1)$$

where D is the diffusivity of the medium, δ the Dirac delta function, ω the angular frequency, and $G(\mathbf{r},\mathbf{r}_s,\omega)$ the Green's function at position $\mathbf{r}$ for a source at $\mathbf{r}_s$. In this frequency domain expression, the source term $\delta(\mathbf{r}-\mathbf{r}_s)$ is frequency independent, hence it corresponds to a delta function in time after the Fourier transform. The homogeneous equation $D\nabla^2 u(\mathbf{r},\omega)-i\omega u(\mathbf{r},\omega)=0$ has plane-wave solution in the frequency domain,

$$u(\mathbf{r},\omega)=e^{-ik\hat{\mathbf{n}}\cdot\mathbf{r}}e^{-k\hat{\mathbf{n}}\cdot\mathbf{r}}, \qquad (2)$$

where $k=\sqrt{\omega/(2D)}$. Equation (2) shows that at a single angular frequency ω, diffusion can be treated as damped wave propagation [3]. Because of the used Fourier convention, this solution is multiplied by $e^{i\omega t}$ to give an outgoing

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propagation. Term $e^{-ik\hat{\mathbf{n}}\cdot\mathbf{r}}$, therefore describes the propagation of the field in the $\hat{\mathbf{n}}$ direction and $e^{-k\hat{\mathbf{n}}\cdot\mathbf{r}}$ defines the decay of the field in the $\hat{\mathbf{n}}$ direction. Note that as with monochromatic wave propagation, diffusion admits solutions with a specific direction of propagation.

A transform used often to convert a diffusive field into a lossless wave is $i\omega' = \sqrt{i\omega}$ [20,25]. This transform is referred to as the q-transform or diffusive-to-propagation mapping. In practice, this transform usually is not stable. In this letter, we use another approach to implement diffusion as damped and dispersive-wave propagation. Following the definition of the phase velocity $v = \omega/k = \sqrt{2D\omega}$ and the attenuation coefficient $\alpha = \sqrt{\omega/(2D)}$, one can view diffusion as a dispersive wave both in phase velocity and attenuation. With this representation, a traditional Fourier transform can be used to transfer the field to the time domain.

In the next section, we construct synthetic elongated sources (*i.e.* the white bar in fig. 1) by adding point sources. The field from a point source is given by the Green's function of eq. (1),

$$G(\mathbf{r}, \mathbf{r}_s, \omega) = \frac{1}{4\pi D\,|\mathbf{r}-\mathbf{r}_s|} e^{-ik|\mathbf{r}-\mathbf{r}_s|} e^{-k|\mathbf{r}-\mathbf{r}_s|}, \qquad (3)$$

where $\mathbf{r}_s$ is the source position [26]. The diffusive field generated by the synthetic line source is steered and focused by applying appropriate phase shift and energy compensation. Both frequency domain and time domain examples are shown in the next section.

Numerical examples. –

Frequency domain. In many applications of diffusive fields, the analysis is carried out in the frequency domain. This is the case, for example, in controlled source electromagnetics for hydrocarbon exploration [27,28]. The frequency domain steering and focusing method in this section can also be adapted to time domain monochromatic fields such as a single-frequency modulated light in strongly scattering media [2].

We show a numerical example for a three-dimensional homogeneous medium with the diffusivity $D = 2.4 \times 10^5\,\mathrm{m}^2/\mathrm{s}$, which corresponds to the diffusivity of low-frequency electromagnetic fields in conductive sea water. The frequency of the field used in this section is 0.25 Hz. The field from a 10 km linear synthetic source, as illustrated by the white bars in fig. 1, is defined as

$$G_A(\mathbf{r}, \omega) = \int_{-L/2}^{L/2} e^{-i\Delta\phi(\omega,x)} e^{-A(\omega,x)} G(\mathbf{r}, x, \omega)\,\mathrm{d}x, \qquad (4)$$

where x is the individual point source location, L the length of the synthetic source, $\Delta\phi(x)$ the phase shift for the source at location x and $A(x)$ an energy compensation coefficient for the source at x. The phase shift $\Delta\phi(x)$ controls the interference of the field from different sources. Because of the strong decay of the field, the contribution from each individual source to the total field can be very different at a specific point depending on the distance between the individual source and this observation point. For this reason, an exponential term $e^{-A(x)}$ is needed to compensate for the diffusive loss. We choose the energy compensation coefficient $A(x)$ to be the same as $\Delta\phi(x)$, because the attenuation coefficient in eq. (3) has the same value as the wave number ($\sqrt{\omega/(2D)}$).

For steering the field, we use a linear phase shift $\Delta\phi(x) = c_1 k\Delta x$, where $\Delta x = |x + L/2|$ is the distance between each source and the left edge of the synthetic source, $c_1 = \sin\theta$ is a coefficient to control the steering angle θ. We next steer the field at approximately 45 degrees to the right side of the source using $c_1 = 0.7$. Because of the rapid decay of the diffusive field, no feature can be visualized with a linear scale. In order to visualize the oscillation feature of the diffusive field, we define the following transformation:

$$G_T = \mathtt{Re}(G_A)/\left|G_A\right|, \qquad (5)$$

where $\mathtt{Re}(G_A)$ is the real part of G_A. The upper and middle panels of fig. 1 show the new dimensionless field G_T, whose amplitude varies from -1 to 1. The upper panel shows the field with zero steering ($c_1 = 0$). In the middle panel, the field is steered at approximately 45 degrees and the constant phase fronts are tilted by the steering ($c_1 = 0.7$). The lower panel of fig. 1 is the ratio of the field amplitude with phase steering to the field amplitude without the phase steering, defined as

$$R = \frac{\left|\int_{-L/2}^{L/2} e^{-i\Delta\phi(x)} e^{-\Delta\phi(x)} G(\mathbf{r}, x, \omega)\,\mathrm{d}x\right|}{\left|\int_{-L/2}^{L/2} e^{-\Delta\phi(x)} G(\mathbf{r}, x, \omega)\,\mathrm{d}x\right|}. \qquad (6)$$

This ratio is the largest at an angle of 45 degrees from the synthetic source. This example illustrates that we can indeed steer a frequency domain diffusive field at a desired angle.

We next apply a phase shift to the individual sources in order to focus the diffusive field at the pre-selected target. One thing worth mentioning here is that the "focusing" and "diffive field" are used in a different sense in refs. [29–32] (focusing of light in disordered media). When light travels through disordered media, on the microscopic scale the light still propagates like a wave. References [29–32] show that the transmitted light can be focused by deconvolving the input waveform with the normal output wavefront. In the context of our work, we study diffusive fields such as the electromagnetic field in a homogeneous conductive medium. There is no multiple wave scattering even on the microscopic scale. We focus the field by modulating spatially distributed individual sources.

When $(x_f,\ z_f)$ is the designed focal point in the (x, z)-plane, the distance from the focal point to the nearest end $(x_e,\ z_e)$ of the synthetic source is $r = \sqrt{(x_e - x_f)^2 + (z_e - z_f)^2}$. The phase shift

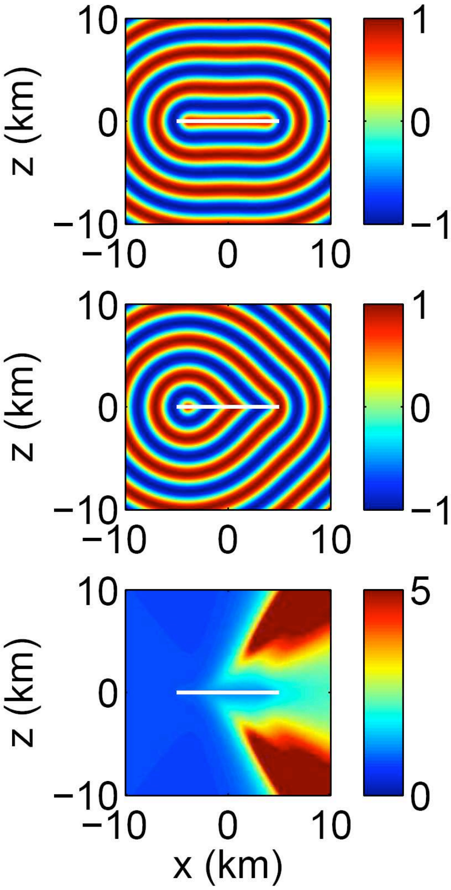

Fig. 1: Upper panel: transformed field G_T with zero steering ($c_1 = 0$); Middle panel: transformed field G_T after steering at an angle of 45 degrees ($c_1 = 0.7$). Lower panel: ratio of the field amplitude $|G_A|$ with steering to the amplitude without steering. The white bar in the two panels illustrates the size of the elongated synthetic source.

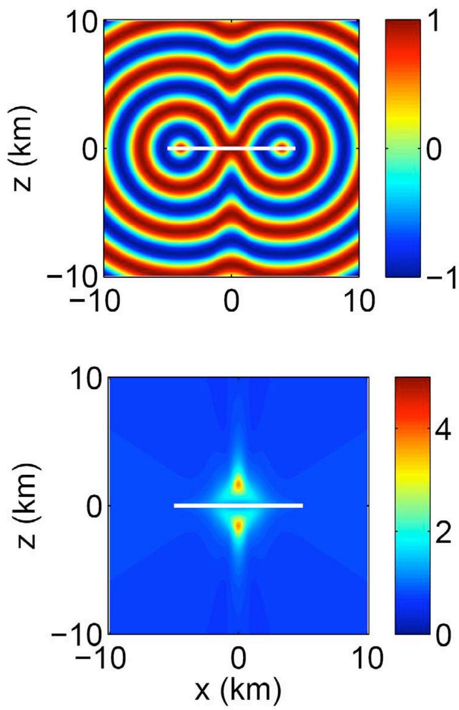

Fig. 2: Upper panel: transformed field G_T using eq. (5) after focusing at ($x = 0\,\mathrm{km}$, $z = 1\,\mathrm{km}$). Lower panel: ratio of the field amplitude $|G_A|$ with focusing to the amplitude without focusing. The white bar in the two panels illustrates the size of the elongated synthetic source.

for each point source is defined as $\Delta\phi(x) = k(\sqrt{(x-x_f)^2 + (z-z_f)^2} - r)$. In fig. 2 we show an example of focusing with the focal point at ($x = 0\,\mathrm{km}$, $z = 1\,\mathrm{km}$). As in the example in fig. 1, the real part of G_A is transformed using eq. (5) and the field G_T is shown in the upper panel. The lower panel is the ratio of $|G_A|$ after and before focusing, as defined in eq. (6). The lower panel of fig. 2 shows that with this phase shift the diffusive field indeed focuses at the designed location. Note that "focusing" here means the field at the focal point increased (3 times in fig. 2), though it may still be weaker than the field at other locations because the diffusion counteracts the focusing. The relationship between field focusing and the synthetic-source properties (*e.g.* size and distance to the focal point) is described in the literature on focusing techniques for waves [16,17,33].

Time domain. For waves in non-dispersive media, the steering and focusing described in the previous section can be also achieved by employing the individual sources with a different time delay [17,33], but because of the dispersive character of diffusive fields, this time delay cannot be applied to diffusive fields. Following eq. (4), the steering in the frequency domain for a source at location x can be represented by a multiplication by $S(\omega,\theta,x) = e^{-\sqrt{i\omega/D}(x+L/2)\sin\theta}$. In the time domain, this multiplication corresponds to a temporal convolution with

$$s(t,\theta,x) = H(t)\frac{(x+L/2)\sin\theta}{\sqrt{4\pi D t^3}} e^{(x+L/2)^2\sin^2\theta/(4Dt)}, \quad (7)$$

where $H(t)$ is the Heaviside function (which makes $s(t,\theta,x)$ zero for the negative time). Contrary to non-dispersive waves, the steering is no longer a time shift factor, but a full time convolution operator acting on all times for the source switching time to the time instant when steering is desired.

Here we take the dispersive-wave view of diffusion and present a method to steer or focus diffusive fields by synthesizing these fields in the frequency domain following the method we presented in the previous section, and then Fourier-transform back to the time domain. The following example shows that this synthesized field indeed can be steered or focused in the time domain.

In this example, the medium is the same as the one we used in the above examples. The source signal

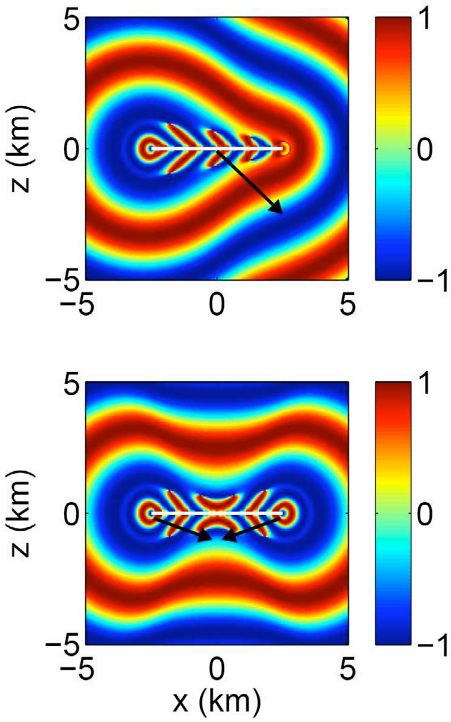

Fig. 3: Upper panel: the diffusive field at $t = 0.2\,\mathrm{s}$ with field steered at a steering angle of $\theta = 45$ degrees. Lower panel: the diffusive field at $t = 0.2\,\mathrm{s}$ with field focused at position ($x = 0\,\mathrm{km}$, $z = 1\,\mathrm{km}$). The white bar in the two panels illustrates the size of the synthetic source.

is a Gaussian wavelet with a frequency distribution $g = e^{-(f-f_0)^2/\sigma^2}$, where f is the frequency, central frequency $f_0 = 5\,\mathrm{Hz}$, $\sigma = \sqrt{5}\,\mathrm{Hz}$. A linear 5 long km synthetic source (the white bars in fig. 3) is constructed by adding individual sources in the line. Each frequency component is steered or focused using the method presented above (with same steering angle and focal point), but in this example the sources are not weighted. The Fourier transform of the modified field shows how the field propagates in time. Figure 3 is a snapshot at $t = 0.2\,\mathrm{s}$. The transformed field G_T is displayed using eq. (5). The white bar in both panels illustrates the extent of the synthetic source. The upper panel shows that the field propagates at a steering angle of $\theta = 45$ degrees to the right of the source as denoted by the black arrow. Because the attenuation increases with frequency, the high-frequency components are only visible close to the source array, while the low-frequency components travel further. In the lower panel, the field focuses at the depth of 1 km. The dashed arrows illustrate how the field propagates. A video of how the field is propagated in time can be seen at `www.mines.edu/~rsnieder/SAtimedomain`. Although the original waveform is not retained because of the dispersion in both velocity and attenuation, this example illustrates that the total field in the time domain indeed can be steered and focused.

Discussion and conclusion. – We have shown that diffusive fields can be treated as waves with a specific dispersion in both phase velocity and attenuation. In the frequency domain, diffusive fields can be steered and focused with proper phase shifts and amplitude weighting. In the time domain, the field cannot be steered or focused with simple time shifts to the individual sources. Instead, each frequency component needs to be treated separately in the frequency domain first. The Fourier transform of these treated frequency components shows a steering or focusing of the field in time. Alternatively one can convolve the source in the time domain with the operator of eq. (7).

The steering technique has significantly improved the sensitivity of the low-frequency electromagnetic field to the presence of the subsurface hydrocarbon reservoirs. To avoid duplicating published work we refer the reader to ref. [21]. The basic idea is that the radiation pattern and polarization of the low-frequency electromagnetic field (diffusive) are changed by field steering in a way to make it more sensitive to the presence of subsurface hydrocarbon reservoirs. In a field data example, as shown in ref. [21], the anomaly due to the presence of a hydrocarbon reservoir has been increased from 20% to more than 300% by applying field steering. The technique provided in this letter has the potential of extending or improving the use of other diffusive fields, in applications such as submarine communications, and medical imaging using diffusive light.

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