Extracting the Green's function from measurements of the energy flux

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Abstract: Existing methods for Green's function extraction give the Green's function from the correlation of field fluctuations recorded at those points. In this work it is shown that the Green's function for acoustic waves can be retrieved from measurements of the integrated energy flux through a closed surface taken from three experiments where two time-harmonic sources first operate separately, and then simultaneously. This makes it possible to infer the Green's function in acoustics from measurements of the energy flux through an arbitrary closed surface surrounding both sources. The theory is also applicable to quantum mechanics where the Green's function can be retrieved from measurement of the flux of scattered particles through a closed surface.

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1. Introduction

Extracting the Green's function from field fluctuations is a technique that has gone through a rapid growth.[1–7] Traditionally the Green's function is retrieved from cross correlation of measured field fluctuations. In this work we present a different method that allows extraction of the Green's function of acoustic waves by considering a sequence of three experiments with time-harmonic sources at two locations, $\mathbf{r}_A$ and $\mathbf{r}_B$. In these experiments one first measures the total energy flux through a closed surface surrounding the sources when each of the sources is used separately, and then measures the total energy flux when both sources are used simultaneously. We show that these measurements can be used to determine the Green's function $G(\mathbf{r}_A, \mathbf{r}_B)$. Our method extracts the Green's function between two source locations, as shown by Curtis *et al.*,[8] the key difference being that we show that measurements of the total energy flux through a closed surface are sufficient to extract the Green's function of acoustic waves. In holography, phase information is restored from intensity measurements of the interference of a reference wave with waves reflected off an object.[9,10] In our work the phase information is retrieved from the energy flux through a closed surface of waves that are excited by different time-harmonic sources.

This concept can also be applied to quantum mechanics. Existing methods for the retrieval of the phase of the wave function[11–14] are based on intensity measurements taken at the location where the phase is retrieved. In the method proposed here,

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the amplitude and phase of the Green's function is inferred from measurements of the average flux of scattered particles through a closed surface.

2. Retrieving the green's function of acoustic waves from the energy flux

The theory presented here is valid for time-harmonic fields with time dependence $e^{-i\omega t}$ that are excited by volume injection sources $q(\mathbf{r}, t) = q(\mathbf{r})e^{-i\omega t}$. The pressure p and particle velocity $\mathbf{v}$ satisfy for such sources the following equation of motion and constitutive relation

$$\nabla p - i\omega\rho\mathbf{v} = 0, \tag{1}$$

$$(\nabla \cdot \mathbf{v}) - i\omega\kappa p = -i\omega q. \tag{2}$$

Both the compressibility $\kappa(\mathbf{r})$ and mass density $\rho(\mathbf{r})$ can be arbitrary functions of space, but must be real in the following derivation. This means that the theory is not valid for attenuating media.

We derive an expression for the energy current by forming the combination $\mathbf{v}^*(\mathrm{E1}) + p(\mathrm{E2})^* + \mathbf{v}(\mathrm{E1})^* + p^*(\mathrm{E2})$, where (E2)* denotes, for example, the complex conjugate of Eq. (2). Forming this combination, integrating over volume, averaging over time, and applying Gauss' law gives

$$-\frac{4i}{\omega}\Phi = \int_V \left\{ q^* p - q p^* \right\} dV, \tag{3}$$

where Φ is the time-averaged energy flux through ∂V that is given by[15]

$$\Phi = \frac{1}{4}\oint_{\partial V} \langle p\mathbf{v}^* + p^*\mathbf{v} \rangle \cdot d\mathbf{S}, \tag{4}$$

and $\langle \cdots \rangle$ denotes time averaging. Because p^* and $\mathbf{v}$ vary with time as $e^{i\omega t}$ and $e^{-i\omega t}$, respectively, the integrand in expression (4) does not depend on time. The time-averaging thus is an average over measurements taken over a given time-interval. Such averaging suppresses the influence of additive noise, but no further time-averaging of the theory is needed. When the excitation is given by a superposition of point sources at locations $\mathbf{r}_n$ with complex spectrum W_n, the excitation is given by $q(\mathbf{r}, t) = \Sigma_n W_n \delta(\mathbf{r} - \mathbf{r}_n)e^{-i\omega t}$ and the response is $p(\mathbf{r}, t) = \Sigma_n W_n G(\mathbf{r}, \mathbf{r}_n)e^{-i\omega t}$, with G the Green's function. For simplicity of notation we suppress the frequency dependence of G. From Eq. (3) and reciprocity ($G(\mathbf{r}_n, \mathbf{r}_m) = G(\mathbf{r}_m, \mathbf{r}_n)$), the energy flux then satisfies

$$-\frac{4i}{\omega}\Phi = \sum_{n,m} W_n W_m^* \left\{ G(\mathbf{r}_n, \mathbf{r}_m) - G^*(\mathbf{r}_n, \mathbf{r}_m) \right\}. \tag{5}$$

We consider the three experiments shown in Fig. 1. First, a time-harmonic source with complex spectrum W is present at location $\mathbf{r}_A$. Then the experiment is repeated with a source with the same spectrum at location $\mathbf{r}_B$, and then these two time-harmonic sources are used simultaneously at locations $\mathbf{r}_A$ and $\mathbf{r}_B$. For each source configuration the flux through ∂V is denoted by Φ_A, Φ_B, and Φ_{AB}, respectively. In the experiment in the left panel of Fig. 1, Eq. (5) gives

$$-\frac{4i}{\omega|W|^2}\Phi_A = \left\{ G(\mathbf{r}_A, \mathbf{r}_A) - G^*(\mathbf{r}_A, \mathbf{r}_A) \right\}. \tag{6}$$

The right hand side is, strictly speaking, given by the time average, but since we have used that the Green's function is given by $G(\mathbf{r}, \mathbf{r}_A, \omega)e^{-i\omega t}$, the frequency domain Green's function $G(\mathbf{r}, \mathbf{r}_A, \omega)$ does not depend on time and the time averaging can be

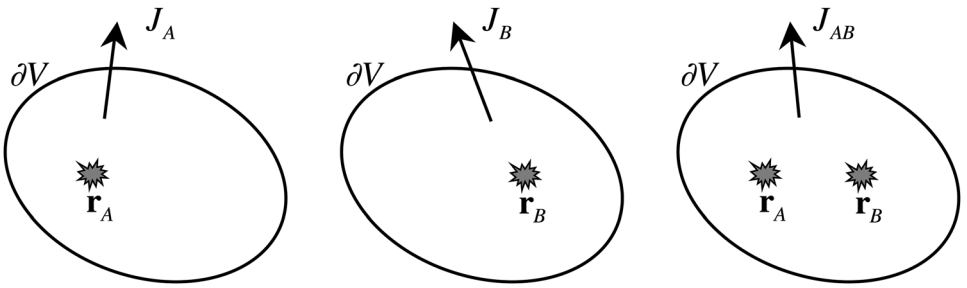

FIG. 1. Three source configurations and their energy currents. The currents in the three configurations correspond to fluxes Φ_A, Φ_B, and Φ_{AB}, respectively.

omitted. The right hand side of Eq. (6) is equal to $2i\mathrm{Im}(G(\mathbf{r}_A, \mathbf{r}_A))$, with Im denoting the imaginary part. Since the imaginary part of the Green's function satisfies a homogeneous equation, it is finite at the source,[16,17] and the right hand side of Eq. (6) therefore is finite. The experiment in the middle panel of Fig. 1 gives the same result as expression (6) but for the source at $\mathbf{r}_B$

$$-\frac{4i}{\omega|W|^2}\Phi_B = \Big\{G(\mathbf{r}_B,\mathbf{r}_B) - G^*(\mathbf{r}_B,\mathbf{r}_B)\Big\}. \tag{7}$$

In the experiment in the right panel of Fig. 1, Eq. (5) reduces to

$$\begin{aligned}-\frac{4i}{\omega|W|^2}\Phi_{AB} = \Big\{&G(\mathbf{r}_A,\mathbf{r}_A) + 2G(\mathbf{r}_A,\mathbf{r}_B) + G(\mathbf{r}_B,\mathbf{r}_B)\\ &- G^*(\mathbf{r}_A,\mathbf{r}_A) - 2G^*(\mathbf{r}_A,\mathbf{r}_B) - G^*(\mathbf{r}_B,\mathbf{r}_B)\Big\}.\end{aligned} \tag{8}$$

Subtracting Eqs. (6) and (7) from Eq. (8) gives

$$G(\mathbf{r}_A,\mathbf{r}_B) - G^*(\mathbf{r}_A,\mathbf{r}_B) = \frac{2i}{\omega|W|^2}(\Phi_A + \Phi_B - \Phi_{AB}). \tag{9}$$

The left hand side gives the imaginary part of $G(\mathbf{r}_A, \mathbf{r}_B)$ at angular frequency ω. One might think that this is not enough information to recover the full Green's function, but repeating the experiment for all frequencies ω of interest and Fourier transforming to the time domain changes the left hand side of Eq. (9) into $G(\mathbf{r}_A, \mathbf{r}_B, t) - G(\mathbf{r}_A, \mathbf{r}_B, -t)$. Since the Green's function is causal, $G(\mathbf{r}_A, \mathbf{r}_B, t)$ is only nonzero for $t>0$ and $G(\mathbf{r}_A, \mathbf{r}_B, -t)$ is only nonzero for $t<0$. By parsing these contributions the full Green's functions can be determined. The right hand side of Eq. (9) depends on the measured energy fluxes. This means that expression (9) can be used to determine the Green's function from the measurement of the integrated energy fluxes for the experiments in Fig. 1.

3. Experimental aspects

The measurements of Φ_A and Φ_B in Sec. 2 are presumed to be carried out with identical point sources at $\mathbf{r}_A$ and $\mathbf{r}_B$. Consider next a point source at $\mathbf{r}_A$ with spectrum $W_A = R_A e^{i\varphi_A}$ with positive amplitude R_A and phase φ_A, and a point source with spectrum $W_B = R_B e^{i\varphi_B}$ at $\mathbf{r}_B$. Repeating the derivation of the previous section generalizes Eq. (6) into

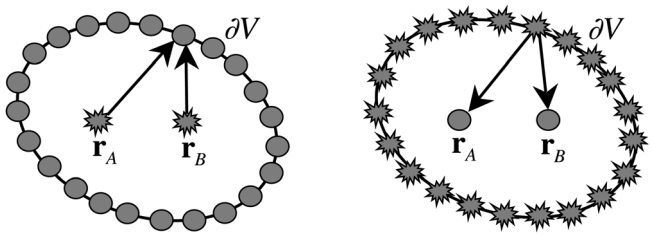

FIG. 2. Comparison of the traditional method for Green's function extraction (Ref. 27) from random sources (right panel), with the method for Green's function extraction proposed in this work (left panel). Sources are denoted by stars, receivers by circles.

$$-\frac{4i}{\omega}\Phi_A = \langle R_A^2 \rangle \Big\{ G(\mathbf{r}_A, \mathbf{r}_A) - G^*(\mathbf{r}_A, \mathbf{r}_A) \Big\}. \tag{10}$$

The phase shift $e^{i\varphi_A}$ cancels in the measurements of Φ_A, hence there is no need to control the phase of the source for the measurements with a single source. The situation is different for the measurement of Φ_{AB} for two simultaneous sources, because this measurement relies on the interference of waves excited at $\mathbf{r}_A$ and $\mathbf{r}_B$. Repeating the derivation leading to expression (9), and assuming that the amplitude and phase variations are independent, gives in this case

$$\langle R_A R_B \rangle \langle \cos(\varphi_A - \varphi_B) \rangle \Big\{ G(\mathbf{r}_A, \mathbf{r}_B) - G^*(\mathbf{r}_A, \mathbf{r}_B) \Big\} = \frac{2i}{\omega}(\Phi_A + \Phi_B - \Phi_{AB}). \tag{11}$$

When the sources are incoherent during the averaging, $\langle \cos(\varphi_A - \varphi_B) \rangle = 0$ and the Green's function cannot be retrieved. Hence the sources must be coherent and have a constant phase *difference* when used simultaneously. This phase difference needs not vanish, as long as it is different from $\varphi_A - \varphi_B = \pm\pi/2$. Note that it is not necessary to know the absolute phase φ_A and φ_B of the sources, and that we did not assume that the phase of the sources in the three experiments of Fig. 1 is identical. A constant phase shift between the sources at $\mathbf{r}_A$ and $\mathbf{r}_B$ thus only gives an overall amplitude change, as do variations in amplitudes of the sources. Note that the amplitudes of the sources need not be equal, as long as their average product $\langle R_A R_B \rangle$ is known.

In experiments the source at $\mathbf{r}_A$ may not be a point source, but a finite real source distribution $q(\mathbf{r}) = S_A(\mathbf{r}_A - \mathbf{r})$ centered at $\mathbf{r}_A$. The waves radiated by a source of finite extent at $\mathbf{r}_A$ are given by $p(\mathbf{r}) = \int G(\mathbf{r}, \mathbf{r}') S_A(\mathbf{r}_A - \mathbf{r}') dV'$, and the right hand side of Eq. (3) contains $\int q^*(\mathbf{r}) p(\mathbf{r}) dV = \int\int S_A(\mathbf{r}_A - \mathbf{r}) G(\mathbf{r}, \mathbf{r}') S_A(\mathbf{r}_A - \mathbf{r}') dV dV'$. Suppose that the source at $\mathbf{r}_B$ is given by a distribution $S_B(\mathbf{r}_B - \mathbf{r})$, centered at $\mathbf{r}_B$ that is also real. Generalizing the derivation given earlier then gives

$$\iint S_A(\mathbf{r}_A - \mathbf{r}) \{ G(\mathbf{r}, \mathbf{r}') - G*(\mathbf{r}, \mathbf{r}') \} S_B(\mathbf{r}_B - \mathbf{r}') dV dV' = \frac{2i}{\omega}(\Phi_A^S + \Phi_B^S - \Phi_{AB}^S), \tag{12}$$

where Φ^S denotes the flux generated by the finite sources S_A and S_B. In this case a double convolution of the Green's function with the source functions is obtained. By deconvolution one can, in principle, obtain $G - G^*$. In practice this may not be possible for all frequencies; in that case one can only obtain a band-limited version of $G - G^*$.

4. Quantum mechanics

The treatment of Sec. 2 for acoustic waves can be extended to the Schrödinger equation. This application may be relevant because the wave function cannot be directly

measured,[18] but the expectation value of the probability density flux, as derived below, is an observable quantity.

Consider a wave function ψ that satisfies Schrödinger equation[18] $H\psi = i\hbar\partial\psi/\partial t$, where H denotes the Hamiltonian, and an unperturbed wave ψ_0 that satisfies $H_0\psi_0 = i\hbar\partial\psi_0/\partial t$, where H_0 does not need to denote a homogeneous medium. Decomposing the wave function $\psi = \psi_0 + \psi_S$ into unperturbed waves ψ_0 and scattered waves ψ_S, and decomposing the Hamiltonian as $H = H_0 + H'$, gives the following inhomogeneous Schrödinger equation for the scattered waves: $H\psi_S = i\hbar\partial\psi_S/\partial t + H'\psi_0$. For time-harmonic problems the wave function of the scattered waves thus satisfies

$$\nabla^2\psi_S(\mathbf{r}) + \left(k^2 - \frac{2m}{\hbar^2}V(\mathbf{r})\right)\psi_S(\mathbf{r}) = 4\pi q(\mathbf{r}), \tag{13}$$

where $\hbar$ is Planck's constant divided by 2π, m is the mass, ω is the angular frequency, $k = \sqrt{2m\omega/\hbar}$ is the wave number, and $V(\mathbf{r})$ is a real, but otherwise arbitrary, potential that influences the scattered waves. The source $q(\mathbf{r})$ of the scattered waves is due to the action of H' (but not of V) on ψ_0. When H' is localized in space, $H'\psi_0$ acts as point source with a time dependence given by ψ_0. We next derive an expression for the probability density current of the scattered waves by considering $\psi_S^*(\mathrm{E13}) - \psi_S(\mathrm{E13})^*$, where the asterisk denotes complex conjugation and (E13) stands for Eq. (13). Integrating the result over a volume V, with boundary ∂V, and applying Green's theorem, gives

$$-\frac{mi}{2\pi\hbar}\Phi = \int_V \langle q^*\psi_S - q\psi_S^*\rangle dV, \tag{14}$$

where Φ is the average probability density flux of scattered waves $\Phi = (\hbar/2mi)\langle\oint_{\partial V}(\psi_S^*\nabla\psi_S - \psi_S\nabla\psi_S^*)\cdot d\mathrm{S}\rangle$. This quantity, which is also used in the definition of the scattering cross section, accounts for the probability per unit time that scattered particles traverse the surface.[18] This probability density flux takes the place of the energy flux of the acoustic waves in Sec. 2. For charged particles, Φ is proportional to the mean scattered electrical current through ∂V,[19,20] which reduces measurement of Φ to a measurement of perturbations in electrical current.

The same three experiments shown in Fig. 1 can be applied to this quantum mechanical system, and using the reasoning that led to Eq. (9) for identical sources ($q_A = q_B = W$) gives for the quantum case

$$G_S(\mathbf{r}_A,\mathbf{r}_B) - G_S^*(\mathbf{r}_A,\mathbf{r}_B) = \frac{mi}{4\pi\hbar|W|^2}(\Phi_A + \Phi_B - \Phi_{AB}). \tag{15}$$

This means that for quantum mechanics the Green's function G_S that accounts for wave propagation of scattered waves between $\mathbf{r}_A$ and $\mathbf{r}_B$ can be constructed by measuring the probability density fluxes through ∂V for the three source configurations of Fig. 1. Just as in acoustics the full Green's function, including the phase, can be found by measuring fluxes for three different experiments.

5. Discussion

The theory presented here provides a method to obtain the Green's function for acoustic waves from measurements of the integrated energy flux through a closed surface surrounding two sources. Measurement of the acoustic energy flux[21] can be carried out by using two nearby microphones,[22–24] or using devices that contain a microphone and an accelerometer.[25,26] The proposed method for Green's function extraction can be applied to such measurements. The theory holds for an arbitrary real density $\rho(\mathbf{r})$ and compressibility $\kappa(\mathbf{r})$. The derivation breaks down when the imaginary parts of these quantities are nonzero, which means that the system may not be attenuating. Physically, this makes sense, because in the presence of attenuation the energy flux changes as the surface ∂V is taken further

away from the sources. As shown in Eq. (11), the used sources need not be in phase, but their phase difference must remain constant during measurement. The two sources need not have the same amplitude, as long as the average product of the amplitudes is known. Both a nonzero phase difference and amplitude variations of the source change the amplitude of the estimated Green's function, when not properly accounted for, but not its phase. We show in Sec. 4 that the theory can be extended to the Schrödinger equation for an arbitrary real potential $V(\mathbf{r})$, and that in this case the integrated intensity flux is to be replaced by the integrated flux of scattered particles.

The method for Green's function extraction is based on a subtraction of quantities that are quadratic in the field variables. For example, for the term $p^*\mathbf{v}$ in Eq. (4), the method relies on the identity

$$(p_A^* + p_B^*)(\mathbf{v}_A + \mathbf{v}_B) - p_A^*\mathbf{v}_A - p_B^*\mathbf{v}_B = p_A^*\mathbf{v}_B + p_B^*\mathbf{v}_A. \tag{16}$$

Note that the right hand side only contains cross-terms of the two states A and B, hence the subtraction selects *interference terms* between the two states. In quantum mechanics this concept can be generalized to the following subtraction of the expectation values[18] of any operator O

$$\langle \psi_A + \psi_B | O | \psi_A + \psi_B \rangle - \langle \psi_A | O | \psi_A \rangle - \langle \psi_B | O | \psi_B \rangle = \langle \psi_A | O | \psi_B \rangle + \langle \psi_B | O | \psi_A \rangle, \tag{17}$$

which follows from the bilinear properties of the expectation value.

We next establish the connection between Green's function retrieval from measurements of the total energy flux presented here and earlier formulations. Inserting expressions (16) and (9) into Eq. (4), using the equation of motion (1) to eliminate $\mathbf{v}$, and substituting the Green's function $G(\mathbf{r}, \mathbf{r}_{A,B})$ for $p_{A,B}$

$$G_{AB} - G_{AB}^* = \frac{1}{2\omega^2} \oint_{\partial V} \frac{1}{\rho} \Big(G(\mathbf{r}, \mathbf{r}_A) \nabla G^*(\mathbf{r}, \mathbf{r}_B) - G^*(\mathbf{r}, \mathbf{r}_B) \nabla G(\mathbf{r}, \mathbf{r}_A) \Big) \cdot d\mathrm{S}, \tag{18}$$

where $G_{AB} = G(\mathbf{r}_A, \mathbf{r}_B)$. This integral has the same form as the integral used in Green's function extraction for acoustic waves [e.g., expression (9) of Wapenaar *et al.*],[27] except that in Eq. (18) the Green's function has arguments $G(\mathbf{r}, \mathbf{r}_{A,B})$, whereas in traditional Green's function extraction the integrand depends on $G(\mathbf{r}_{A,B}, \mathbf{r})$.[27]

The Green's function extraction of expression (18) relies on field fluctuations that are generated by pressure sources and forces[27] (monopole and dipole sources). When the surface is far away and spherical, the forces can be replaced by pressure sources,[27] but when the surface does not meet these criteria, pressure sources do not suffice. In contrast, the theory presented here is exact, and measurements of the energy current suffice for a surface of arbitrary shape, even when it is in the near field of the sources.

The integral (18) corresponds to the situation shown in the left panel of Fig. 2 where the energy flux of the field excited by sources at $\mathbf{r}_A$ and $\mathbf{r}_B$ is measured at locations $\mathbf{r}$ at ∂V. In Green's function extraction, shown in the right panel of Fig. 2, one cross correlates the fields measured at locations $\mathbf{r}_A$ and $\mathbf{r}_B$ that are excited by uncorrelated sources at locations $\mathbf{r}$ on the surface.[27] Because of reciprocity ($G(\mathbf{r}, \mathbf{r}_{A,B}) = G(\mathbf{r}_{A,B}, \mathbf{r})$) these two cases are identical. It was recognized earlier[8] that cross-correlation methods can yield the waves that propagate between sources. In this work we generalize this principle to obtain the Green's function from measurements of the total energy flux.

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