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Finding sets of acceptable solutions with a genetic algorithm with application to surface wave group dispersion in Europe

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Abstract. We discuss the use of a genetic algorithm (GA) to invert data for many acceptable solutions, in contrast to inversion for a single, "optimum" solution. The GA is a directed search method which does not need linearization of the forward problem or a starting model, and it can be applied with a very large model-space. Consequently, fewer assumptions are required and a greater range of solutions is examined than with many other inversion methods.

We apply the GA to fundamental Rayleigh group dispersion estimates for paths across Central Europe and across the East European Platform to determine "average", layered S velocity models separately for each region. The use of the GA allows an identical model parameterization and broad parameter search range to be used for both regions. The scatter of acceptable solutions shows velocity-depth trade-offs around the Moho, indicates the depth resolution of the inversion, and shows the uncertainty in upper mantle, S velocity estimates. The results indicate that a thicker crust and up to 0.3 km/sec (7%) higher S wave velocities in the upper 100 km of the mantle under the older East European Platform than under Central Europe explain most of the differences in the data sets.

Introduction

A goal of geophysical inversion is to find all earth models which, when operated upon by some forward method, produce synthetic data that gives an acceptable agreement with observed data [*Keilis-Borok and Yanovskaya*, 1967; *Aki and Richards*, 1980]. However, for practical reasons, most inversions include various assumptions, simplifications and restrictions which limit the allowable solutions to a small region of the physically plausible model-space. For example, direct inversions can find solutions only in the neighbourhood of some reference model, and many techniques require a simple parameterization of the model where important elements, such as crustal velocity and thickness, are fixed. There is, however, the possibility that the limited model-space excludes some reasonable solutions for a given data set and forward method. In addition, many methods produce a single solution, although geophysical problems are often non-unique, and many solutions, perhaps within different local minima, will produce an acceptable fit to the data. For many problems, as a complement to these more limited inversions, it is useful to explore as large a model space as possible, retaining large numbers of acceptable solutions.

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In general this goal cannot be achieved with direct, calculus based inversions. These methods require knowledge of an adequate starting solution, and that perturbations of the model are linearly related to changes in the data [*Aki and Richards*, 1980]. They operate through a single or an iterative perturbation of the solution using locally determined gradients of the misfit surface; this requires a smooth and differentiable misfit function. Only a single final solution is identified, and, in general, this solution will be in the neighbourhood of and strongly dependent on the starting solution [*Goldberg*, 1989].

Search techniques such as Monte Carlo and hedgehog allow a large model space to be explored and produce multiple solutions [*Keilis-Borok and Yanovskaya*, 1967]. However these techniques become inefficient or impractical in very large model spaces. The genetic algorithm (GA) method [*Goldberg*, 1989] is one of a number of newer techniques that give a more efficient sampling of a large model space; it has been used recently in geophysics [e.g., *Stoffa and Sen*, 1991; *Sen and Stoffa*, 1992; *Sambridge and Drijkoningen*, 1992; *Jin and Madariaga*, 1993; *King*, 1993].

The GA method is an iterative, non-local, controlled search, which operates with populations of trial solutions to find new solutions with lower "misfit", where the misfit is given by a comparison of predictions from the solutions with data. Beginning with a random initial population of solutions and corresponding misfits, succeeding populations are created by three stochastic processes: 1) *selection* - saving those solutions with smaller misfit, 2) *crossover* - combining parts of two solutions to form new trial solutions, and 3) *mutation* - making small changes to the solutions. New populations are created until a convergence criteria is reached. The GA search produces a large set of solutions which give an estimate of the misfit surface in the model space.

Genetic Algorithms for Acceptable Solutions

Unfortunately, many inverse problems in geophysics are non-linear and poorly constrained. Such problems may have multiple, broad or topologically complex regions of minimum misfit in the solution space. Our experience and other work [e.g., *Goldberg*, 1989; *Stoffa and Sen*, 1991] indicate that with this type of problem, each run of a GA tends to converge to a single, local minima, and in different runs different minima may be found depending on the parameters controlling the GA.

Many GA applications converge to a local minima because they are configured for rapid convergence with the goal of finding an "optimum" solution . Here, the GA is configured to attempt to find many acceptable solutions - solutions representing all regions of the model space that give a misfit with the data below some level (Figure 1). We begin with a GA similar to that described by *Sambridge and Drijkoningen* [1992], but set the rates of crossover and mutation relatively low so that many solutions pass unchanged to the following generations, we flip many bits in each mutated string, and the best solution of each generation is not explicitly saved. In addition, we define a minimum misfit value and reset lower misfits to this value; this reduces the stalling of the GA in deep minima that are much lower than the acceptable level. These adjustments tend to produce a smaller but more stable and diverse set of acceptable models relative to a GA configured for rapid convergence.

Application to Group Velocity Dispersion

The older crust of the Precambrian East European Platform (EEP) adjoins the younger crust of Palaeozoic Central Europe (CE) along the geologically distinct Tornquist-Teisseyre Zone (TTZ, Figure 2). Body and surface wave studies have indicated a contrast of up to 10% in S wave velocity at the top of the mantle across the TTZ, with higher velocities under the EEP [*Snieder*, 1988; *Zielhuis and Nolet*, 1994]. Body wave studies also indicate increased P wave velocity under the EEP [*Hurtig et al.*, 1979; *Spakman et al.*, 1993].

In this study, the GA is used to determine two sets of layered velocity-models with group velocity dispersion curves that match group velocity estimates for wave paths on each side of the TTZ. The observations are digital seismograms from events located primarily to the south-east of Europe recorded at distances of about 10-30° at European NARS and GDSN stations (Figure 2).

Group dispersion for the fundamental Rayleigh mode in the period range of 7-300 sec is estimated from the observed waveforms using multiple-filter analysis (MFA) [*Dziewonski et al.*, 1969]. Peaks on the envelopes of the narrow-band seismograms form group-velocity - period data points for the GA inversion (Figure 3). Smooth dispersion curves are not estimated because the fitting of synthetic dispersion curves to the scattered peak values in the inversion produces an effective smoothing of the data, and because the scatter gives a frequency-dependent weighting to the curve fits.

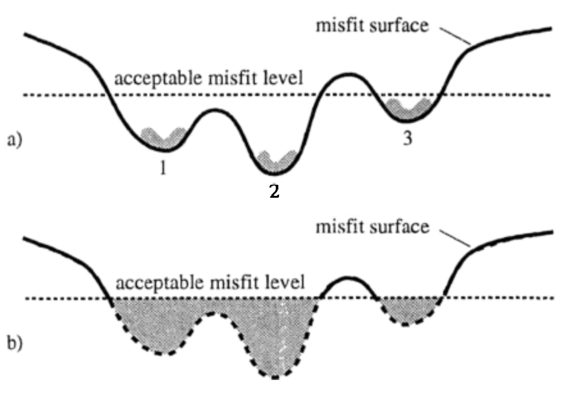

Figure 1. a) A standard GA may find solutions located near only one of the minima, 1, 2 or 3 in the data misfit surface. But a goal of geophysical inversion is to find all solutions with misfit below some acceptance level (e.g. data variance). b) The identification of a representative sample of all acceptable solutions is a practical way to achieve this goal of geophysical inversion.

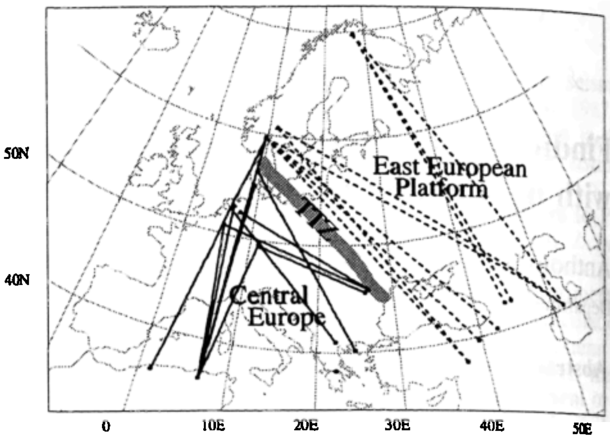

Figure 2. Map showing Tornquist-Teisseyre Zone (TTZ) and great circle paths between sources and receivers used in this study for CE (solid lines) and the EEP (dashed lines).

When divided by region, the group-velocity dispersion estimates form two groups which, despite their overlap and scatter, have distinctly different character (Figure 3). This grouping indicates that inversion of this data may resolve significant differences in "average" crustal and upper mantle structure between the EEP and CE.

Model Parameterization

The GA technique permits any model parameterization that is compatible with the employed forward algorithm. Since the GA technique can efficiently search a very large number of models, this parameterization can be liberally defined, with few assumptions and restrictions.

In this study the same model parameterization is used for the inversion of the data from each side of the TTZ to allow a relatively unbiased estimate of the differences in structure between the two regions. This model is defined by 4 crustal and 14 mantle velocity-depth nodes and a variable Moho depth between 15 and 70 km. The deepest crustal node is located at the Moho depth. The mantle nodes are spaced from the Moho to the bottom of the model at 2371 km depth, with the node spacing increasing approximately in proportion to depth. The location of two nodes at the Moho depth allows a step discontinuity between the crust and mantle. The S velocity at each

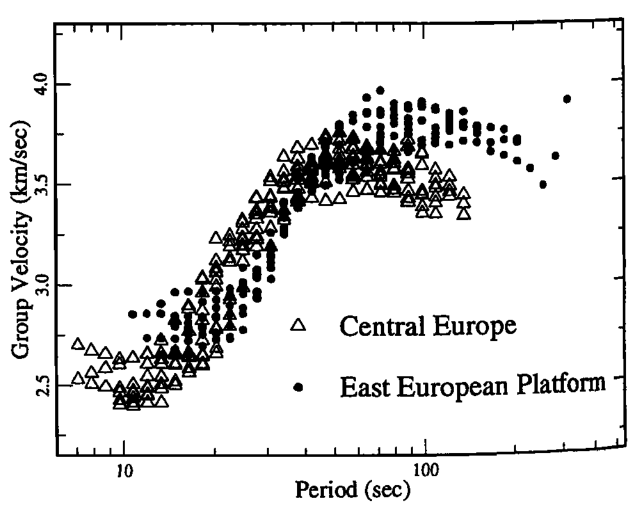

Figure 3. Fundamental mode, Rayleigh group-velocity estimates as a function of period from MFA analysis for wavepaths in CE and in the EEP.

node can vary within approximately ±1 km/sec (about ±20%) of the S velocity from the iasp91 model [*Kennett and Engdahl*, 1991] at the corresponding depth. The P velocity is determined from the S velocity using a Poisson's ratio of 0.25, the density profile is fixed separately for the crust and mantle and corresponds approximately to the values in iasp91. The Moho depth and S velocity parameters are discretized with a very small step size, giving an effectively continuous variation between the limiting values. This parameterization gives about 10^{45} possible models, though the number of significantly different models is the order of 10^{10}. (For comparison, in a similar study with phase velocities, Calcagnile and Panza [1978] use a "hedge-hog" grid-search for about $2x10^4$ possible crust/mantle models.)

A centrally-weighted, 5 point smoothing of velocity is applied separately to the crust and mantle sections of the node model to suppress rapid oscillation of the solution. This smoothed model is converted to a constant-velocity layered model to calculate synthetic dispersion curves. A starting population of models for the GA search is shown in Figure 4a; the search range is indicated in Figure 5.

Inversion Results

Figures 4b and 4c shows the results from the GA applied to the EEP data. The distribution of acceptable models (solid lines) and corresponding dispersion curves shows the mapping of uncertainty between the data space and the model space. Here, acceptable models are defined as those models giving predicted group velocity values with an r.m.s misfit in group velocity less than $0.9E_d$, where E_d is the r.m.s of the differences in group velocity between each of the data values and all the other data values at each period. This definition of acceptance level is chosen so that the scatter of acceptable dispersion curves falls within the average scatter of the data.

The scatter of acceptable models is lowest from just below the Moho to about 350 km depth. This indicates the depth range where the velocity is best constrained by the dispersion data. Below about 350 km the models begin to fan out and span the range of tested models (light lines). This increase in scatter shows the loss of constraint on the solutions at depth primarily due to the lack of dispersion data at greater than 300 sec period.

There is increased scatter on the dispersion curves at shortest periods; the corresponding scatter in the models occurs in the upper crust. This scatter indicates poor constraint on velocity at shallow depth due to the lack of shorter period data. There is also increased scatter and diversity in the models near the Moho discontinuity because a truncated set of low-order surface-wave modes cannot uniquely resolve a discontinuity. Some models have a strong velocity contrast across a deep Moho discontinuity and a distinct, high-velocity mantle lid, while other models have a weaker Moho discontinuity at shallower depth and no high-velocity lid.

Figure 5 shows the 1σ and 2σ distribution of acceptable models obtained with the modified GA for both CE and the EEP paths. (For completeness, the results of three GA runs for each region are shown, though a single run of the GA as configured here can indicate most of the significant variations in acceptable solutions.) The results for CE show a sharp increase in velocity across a Moho discontinuity at about 25 km depth, and a well defined upper mantle velocity structure to a depth of about 150 km. Below this depth the increasing spread indicates a lack of constraint on the solutions; the scatter in models for the EEP does not increase significantly until below 300 km. There is a difference in the maximum depth constrained in the two regions because the longest period data available for CE is only about 150 sec, while the data for the EEP extends to around 300 sec. Because of the lack of constraint, the depth extent of the low velocity zone under CE cannot be determined from this inversion.

The results in Figure 5 indicate that significant changes in crustal thickness and in upper mantle S velocity across the TTZ can explain the main differences between the dispersion curves for the two regions. The velocity structures in the upper 100 km of the mantle are well constrained in both regions and show up to 7% higher average S wave velocities under the EEP than under CE. Taking in to account the differences in data sets, model parameterization and inversion methods, this result is in agreement with the contrast in upper mantle S velocity across the TTZ determined by *Zielhuis and Nolet* [1994].

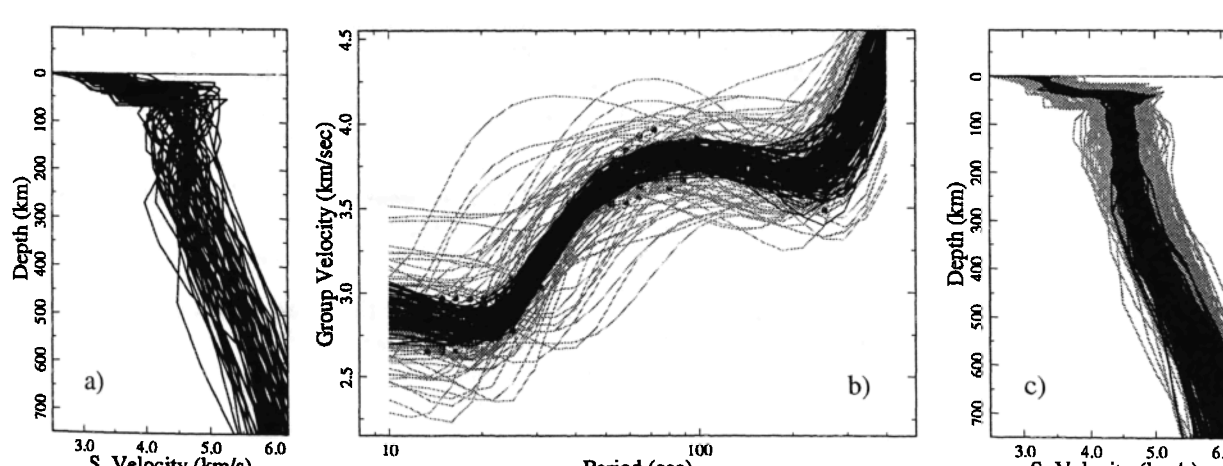

Figure 4. Results from one GA inversion for the EEP data. a) Initial population of layered earth models. b) Observed dispersion data (dots) and representative set of synthetic dispersion curves. c) Representative set of Earth models. Acceptable results are drawn with solid black lines, a sample of all tested results are plotted in grey. About 4500 models were sampled from a model space with about 10^{45} members using a population size of 60 and 200 generations.

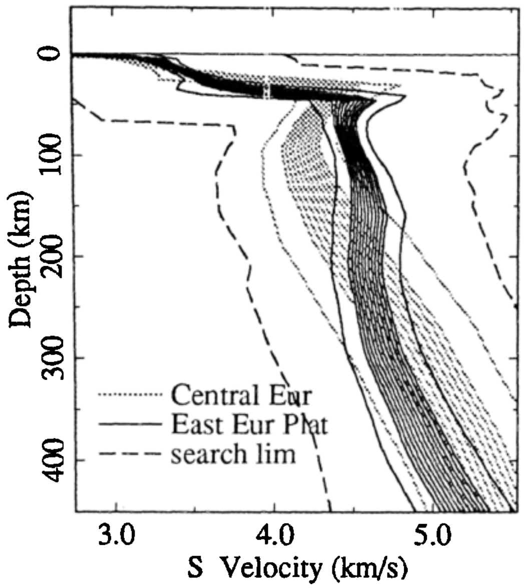

Figure 5. Upper 450 km of results from 3 GA runs for the EEP and CE showing the $\pm 1\sigma$ (closely spaced lines) and the $\pm 2\sigma$ (outer lines) spread in S velocity of acceptable models and the search limits (long dashed lines). The spread indicates little resolution below about 150 km for CE.

Discussion

Two families of dispersion curves from Eurasia were inverted with a GA configured to find large sets of acceptable solutions. The scatter in the set of acceptable models shows how the variance and the resolution of the dispersion data maps between data and the model spaces. This scatter shows trade-offs between Moho velocity contrast and depth, and between layer velocities and thicknesses, and indicates the maximum depth resolution of the inversion and the uncertainty in the conclusion that the upper mantle S velocities vary across the TTZ.

The GA presented here was configured to improve the stability of the inversion, producing more consistent images of the better-fitting regions of the model space. This stability is obtained at the expense of slower convergence and somewhat poorer-fitting best solutions than with a more "standard" GA and the results still show some dependence on the GA parameters. More significant modifications to the GA, or perhaps some other search method, may be required to adequately define the complex topology of the acceptable misfit region of the solution space for many geophysical problems.

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