NOTES AND CORRESPONDENCE

The Flywheel Effect in the Middle Atmosphere

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ABSTRACT

Because of the requirement of geostrophic balance, mechanical inertia can affect the thermal response of the atmosphere to transient heating. We examine some very simple linear models of this "flywheel effect," and discuss their possible reference to the Antarctic ozone hole.

1. Introduction

In the wake of the discovery of the late-spring antarctic ozone minimum has come a renewed interest in the response of zonally symmetric circulations to transient radiative forcing (Tung et. al. 1986; Mahlman and Fels 1986; Pyle 1986; Garcia 1987; Dunkerton 1988). The symmetric nature of the southern polar stratospheric vortex, as well as the spring occurrence of the ozone dip, led all of the above authors to speculate that the onset of solar heating might result in strong vertical motion in high latitudes, and hence to a decrease in the polar ozone column. This offers a dynamical rather than photochemical mechanism for the ozone hole.

The calculations to be discussed in what follows bear directly on this issue, but were undertaken for a rather different reason—to provide a context in which the results of seasonal GCM simulations of the middle atmosphere could be interpreted. It is in this more general context that we shall present them.

A natural starting point for the analysis of the middle atmosphere, or a model of it, is the zonally symmetric temperature field which results from the time-marched thermodynamic equation in which all dynamical processes are suppressed:

$$\frac{\partial T}{\partial t} = J_{\text{solar}}(\theta, z, t) - Q_{\text{IR}} \tag{1}$$

where J_{solar} is the solar heating, and Q_{IR} the infrared cooling. This is a simple extension of the idea of radiative equilibrium to allow for the existence of thermal inertia.

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At this point, we recognize that since U, the zonal wind, is largely determined by thermal wind balance, a change in T entails a change in U, which will be resisted by mechanical inertia. In reality, therefore, we expect that the existence of this "flywheel effect"[1] may alter the response of the system to a change in the radiative forcing. By virtue of this, the balance represented by Eq. (1) will be destroyed, and there must therefore exist zonally symmetric dynamical heating, signifying the presence of a mean meridional circulation induced by the transient heating. (Indeed, it is this circulation which has lead to the renewed interest in this sort of problem in the context of the "ozone hole.")

These ideas are anything but new—they are implicit in the seminal work of Leovy (1964), which we believe lay out many of the issues still of importance for the symmetric circulation in the middle atmosphere. In that paper, however, the phenomena related to the transient aspects of the radiative forcing were not central. In what follows, it is precisely these which will occupy our attention. Our paper is frankly tutorial in style—with a few exceptions, our models can be fully worked out analytically. They do, however, provide some insight into the magnitudes of the effects to be expected in the real world.

The paper is arranged as follows. Section 2 introduces the simple f-plane model which clearly brings out the basic issues involved in the adjustment problem; sections 3 and 4 extend this analysis to the sphere. In section 5, a harmonic analysis of a realistic model for the seasonally marched solar heating is given, and in section 6, solutions of the spherical model driven by realistic radiative heating and cooling are presented.

2. Radiative and dynamical control on the f-plane

As a first step it is instructive to consider the circulation on a f-plane using the Boussinesq approxima-

[1] We are indebted to M. E. McIntyre for this term.

tion. This example is simple enough so that analytical solutions can be found. The system is assumed to be independent of the x-coordinate, and is enclosed in a rectangular box with width L in the y-direction, and a depth $D \ll H$. At the boundaries no-normal-flow conditions are imposed, and the radiative forcing is assumed to be periodic in time. In that case the linearized model equations in log pressure coordinates are:

$$u_t = fv - \tau_m^{-1}u \tag{2}$$

$$fu_z = -\phi_{yz} \tag{3}$$

$$\phi_{zt} + N^2 w = \frac{R}{H}J - \tau_r^{-1}\phi_z \tag{4}$$

$$v_y + w_z = 0. \tag{5}$$

Note that the inertia terms in the zonal momentum equation and the thermodynamic equation are taken into account. It may seem inconsistent that there is no inertia term in equation (3), which is just the z-derivative of the meridional momentum equation. This can be justified for the midlatitude middle atmosphere with a simple scaling argument: If realistic values for the middle atmosphere are used, the ratio of the inertia term to the Coriolis term in the zonal momentum equation for the yearly cycle at midlatitudes is

$$u_t/fv \approx 2.0 \times 10^{-1}.$$

For the meridional momentum equation this ratio is

$$v_t/fv \approx 2.0 \times 10^{-5}$$

so that the inertia term in the meridional momentum equation can be neglected. Newtonian cooling and Rayleigh friction are imposed, but the damping time for these two processes may be different.

From the continuity equation a streamfunction can be defined so that

$$v = -\psi_z, \quad w = \psi_y \tag{6}$$

For a periodic forcing the following elliptic equation for the streamfunction can be derived:

$$N^2\psi_{yy} + \left(\frac{\tau_r^{-1} - i\omega}{\tau_m^{-1} - i\omega}\right)f^2\psi_{zz} = \frac{R}{H}J_y. \tag{7}$$

The zonal momentum equation becomes

$$u = -f\psi_z/(\tau_m^{-1} - i\omega). \tag{8}$$

Let us now consider the thermodynamic equation in some more detail. It contains four terms, but it will be shown in this paper that in the middle atmosphere, various balances occur, so that two of these terms will approximately cancel each other. Figure 1 shows several examples of pairs of terms which may dominate the thermodynamic equation. If the tendency term and the adiabatic heating are small the longwave cooling will balance the shortwave heating. As noted previously, this is the radiative equilibrium case since the

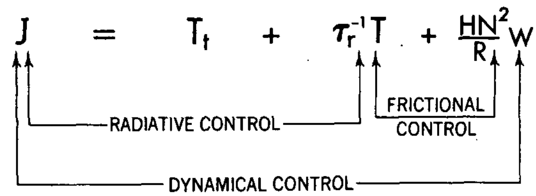


FIG. 1. Balances of terms in the thermodynamic equation leading to various types of control in the stratosphere.

temperature is determined by radiative terms only; we refer to it as "radiative control." If the radiative heating and the vertical motion balance, we speak of "dynamical control." In this case the meridional circulation is driven by the gradient of the shortwave heating, so that the absorbed shortwave radiation is redistributed by the meridional circulation.

A third type of balance occurs if both the shortwave heating and the tendency term are negligible. This is called "frictional control." This may appear to be a very unnatural situation because it is ultimately the shortwave heating that drives the atmosphere. However, it follows from (7) that the streamfunction arises from the shortwave heating through an elliptic operator, so that the streamfunction response is nonlocal. It is therefore conceivable that frictional control will exist in certain regions of the middle atmosphere. The tendency in the thermodynamic equation term will usually be of little importance, since the cooling time in the middle atmosphere is small compared to the frequency of the seasonal cycle, i.e., $\omega\tau_r \ll 1$.

Now suppose that the radiative forcing excites only one mode of the system, that is:

$$J = J_0 \cos\left(\frac{n\pi y}{L}\right) \sin\left(\frac{m\pi z}{D}\right) \tag{9}$$

For convenience the time dependence $\exp(-i\omega t)$ is not made explicit in (9) and the following expressions. In that case the zonal wind is

$$u = \frac{u_{eq}}{(1 - i\omega\tau_r)} \cdot \frac{1}{1 + \left(\frac{\tau_m^{-1} - i\omega}{\tau_r^{-1} - i\omega}\right)\left(\frac{\Delta D}{\Delta L}\right)^2\left(\frac{N}{f}\right)^2} \tag{10a}$$

where

$$u_{eq} = \frac{RT_0\tau_r}{H} \cdot \frac{N^2}{f}\left(\frac{\Delta D}{\Delta L}\right) \sin\left(\frac{n\pi y}{L}\right) \cos\left(\frac{m\pi z}{D}\right) \tag{10b}$$

and

$$\Delta L = L/n\pi, \quad \Delta D = D/m\pi \tag{10c}$$

The u_{eq} is the equilibrium zonal wind in the absence of mechanical dissipation, since u_{eq} follows from (10a) in the limit $\omega = \tau_m^{-1} = 0$. The term $(1 - i\omega\tau_r)$ describes the inertial effects due to the tendency term in the thermodynamic equation. In general this term will be close to 1 since $\omega\tau_r \ll 1$. The last term on the rhs of (10a) describes the effect of the meridional circulation on the evolution of the zonal wind. It is this term we shall be concerned with, since this term determines how transience and mechanical dissipation affect the zonal wind and the temperature.

It is instructive to see what fraction of the radiative forcing is used to change the temperature field, and how much of the radiative forcing is used to drive the meridional circulation. This is particularly important in the context of the Antarctic ozone hole. The dynamical hypothesis of Tung et al. (1986) requires that there exist vertical motion of about 100 m day^{-1} during September over the south polar region. In assessing the correctness of this mechanism, we require more than a knowledge of the solar heating—we must also know how it is partitioned between the temperature tendency and vertical motion terms.

The ratio of the adiabatic heating to the radiative forcing is given by

$$\frac{\frac{HN^2}{R}w}{J} = \frac{1}{1 + \left(\frac{\tau_r^{-1} - i\omega}{\tau_m^{-1} - i\omega}\right)\left(\frac{\Delta L}{\Delta D}\right)^2\left(\frac{f}{N}\right)^2} \tag{11}$$

Now consider two special cases: 1) the radiative forcing is tall and narrow:

$$\Delta L \ll \left|\frac{\tau_m^{-1} - i\omega}{\tau_r^{-1} - i\omega}\right|^{1/2}\frac{N}{f}(\Delta D) \tag{12}$$

Here, adiabatic heating cancels the radiative forcing because the rhs of (11) is equal to unity in this limit. The system is under dynamical control, because the radiative forcing does not alter the temperature since all the heating is used to drive the meridional circulation. It is in this limit that there may be a time lag between the radiative forcing and the zonal wind, because in this limit the zonal wind is given by

$$u = u_{eq}\left(\frac{\tau_r^{-1}}{\tau_m^{-1} - i\omega}\right)\left(\frac{\Delta L}{\Delta D}\right)^2\left(\frac{f}{N}\right)^2. \tag{13}$$

For a large Rayleigh friction time, one can speak of a flywheel effect because of the role played by the inertia of the zonal flow. The response of the system is slow even if the Newtonian cooling time is short because changes in the temperature field are opposed by the strong meridional circulation. It is through this mechanism that thermal wind balance is maintained.

Now consider the case where: 2) the radiative forcing is shallow and wide:

$$\Delta L \gg \left|\frac{\tau_m^{-1} - i\omega}{\tau_r^{-1} - i\omega}\right|^{1/2}\frac{N}{f}\Delta D. \tag{14}$$

In that case the ratio of the adiabatic heating to the radiative forcing is given by

$$\frac{\frac{HN^2}{R}w}{J} \approx \left(\frac{N}{f}\right)^2\left|\frac{\tau_m^{-1} - i\omega}{\tau_r^{-1} - i\omega}\right|\left(\frac{\Delta D}{\Delta L}\right)^2 \ll 1$$

This means that almost all the radiative forcing is used to change the temperature, and only a small fraction of the absorbed radiation drives the meridional circulation. It is for this reason that one can say that there is radiative control in this situation. The zonal wind in this limit is

$$u = u_{eq}/(1 - i\omega\tau_r), \tag{15}$$

and only the thermal inertia influences the temporal evolution of the zonal flow. Usually the Newtonian cooling time is much smaller than the period for the radiative forcing, so that the zonal wind is almost in phase with the radiative forcing, and the adjustment of the system is very quick. Note that the zonal wind is independent of the mechanical damping time in this limit. This is a consequence of radiative control, because the temperature is largely determined by the radiative forcing and the cooling. By thermal wind balance the same applies to the zonal wind.

This behavior is completely analogous to the adjustment of the shallow water system to an external forcing. A Rossby radius of deformation can be defined by the rhs of (12) or (14), which separates the regime of radiative control from the regime of dynamical control. If there is radiative control the temperature adjusts itself quickly and the final state is determined by the details of the cooling scheme, so that the wind field adjusts itself to the temperature field. In the case of dynamical control the final state is determined by the mechanical dissipation, which means that the temperature is strongly constrained by the zonal wind.

The only difference with the shallow water system is that the Rossby radius of deformation is now dependent on the damping rates of the system, and the frequency of the radiative forcing. In Table 1 the Rossby radius of deformation is shown for values which are not too unrealistic for the middle atmosphere. Note that the Rossby radii of deformation are quite small compared to the global scale of the circulation in the middle atmosphere, suggesting that there is radiative control in the middle atmosphere. However, it will turn

TABLE 1. Rossby radii of deformation (km) for the following parameter setting: τ_m = 90 days, ΔD = 14 km, $N = 10^{-2}$ s^{-1}, $f = 10^{-4}$ s^{-1}. Data in the two left columns refer to the midlatitude case; these figures follow from Eqs. (12) or (31). Data in the right-hand columns are for the equatorial case; they follow from Eq. (35).

	Midlatitude L_R		Equatorial L_R	
Frequency	τ_r = 6 days	τ_r = 20 days	τ_r = 6 days	τ_r = 20 days
0	361	659	1257	1698
2π/360 days	492	875	1467	1957
2π/180 days	649	1085	1685	2178

out that the semiannual component of the radiative forcing is so narrow that there is dynamical control for this component.

3. System equations on the sphere

The treatment of the last paragraph was too simplified to give accurate results for the middle atmosphere. The circulation in the middle atmosphere is global, so the effects of sphericity and the variation of the Coriolis parameter have to be taken into account. Furthermore, the density should be height dependent. In order to do this the relevant equations on the sphere are rescaled in the same way as in Leovy (1964). The only difference is that in this study nondimensional time is defined by t = dimensional time/1 day. The nondimensional system equations in log-pressure coordinates are

$$u_t = yv - \tau_m^{-1}u \qquad (16)$$

$$yu_z + (1 - y^2)T_y = 0 \qquad (17)$$

$$T_t + \Gamma w = Q \qquad (18)$$

$$v_y + (w_z - w) = 0 \qquad (19)$$

In (18) the net radiative forcing is denoted by Q; this function describes both the shortwave heating and the longwave cooling; Γ is a stability parameter:

$$\Gamma = \left(\frac{H}{2\pi a}\right)^2 N^2 \qquad (20)$$

and y is defined by

$$y = \sin(\text{latitude}). \qquad (21)$$

Note that even though these look like beta-plane equations, they do take the full effects of sphericity into account. The reason for the simple form of the equations is that both the zonal wind and the meridional velocity have been rescaled with a factor 1/cos(latitude). Just as in section 2, the inertial term in the meridional momentum equation has been ignored. The scaling argument given in section 2 breaks down near the equator, but poleward of 20 deg latitude it is justified to omit the v_{zt} term. This is numerically quite convenient because leaving out the v_{zt} term filters out short-period inertial oscillations.

As in section 2 a streamfunction can be defined which now must be interpreted as a mass streamfunction:

$$v = \rho^{-1}\frac{\partial}{\partial z}(\rho v) = \psi - \psi_z \qquad (22)$$

$$w = \psi_y. \qquad (23)$$

The streamfunction satisfies the following elliptic equation:

$$(1 - y^2)\Gamma\psi_{yy} + y^2(\psi_{zz} - \psi_z) = (1 - y^2)(Q_y + \tau_m^{-1}T_y). \qquad (24)$$

The imposed boundary conditions are that there is no normal flow at the top (100 kilometers) and at the bottom, and that $(1 - y^2)v \to 0$ at the poles.

The equations (16), (18) and (24) can be integrated in time and give the response of the atmosphere to a prescribed radiative forcing. Note that there is a feedback from the temperature to the streamfunction through the rhs of (24). The temperature appears explicitly through the mechanical damping term, and implicitly through the cooling term.

Further analytical progress is impossible without making any assumptions about the longwave cooling. Consider now as a first approximation Newtonian cooling with a constant damping time:

$$Q = J - \tau_r^{-1}(T - 240\text{ K}). \qquad (25)$$

In this expression J is the shortwave radiative heating. If the radiative forcing is assumed to be periodic, then it is possible to take the effect of the temperature on the streamfunction into account. In that case the relevant equations are:

$$u = y(\psi - \psi_z)/(\tau_m^{-1} - i\omega) \qquad (26)$$

$$T = (J - \Gamma\psi_y)/(\tau_r^{-1} - i\omega) \qquad (27)$$

$$(1 - y^2)\Gamma\psi_{yy} + \left(\frac{\tau_r^{-1} - i\omega}{\tau_m^{-1} - i\omega}\right)y^2 \times (\psi_{zz} - \psi_z) = (1 - y^2)J_y. \qquad (28)$$

The substitution $\psi = \phi\exp(z/2)$ removes the z-derivative in Eq. (28), transforming it into

$$(1 - y^2)\Gamma\phi_{yy} + \left(\frac{\tau_r^{-1} - i\omega}{\tau_m^{-1} - i\omega}\right)y^2\left(\phi_{zz} - \frac{1}{4}\phi\right) = (1 - y^2)e^{-z/2}J_y. \qquad (29)$$

4. An analytical solution for a simple radiative forcing

As in section 2, some analytical results can be obtained by exciting one mode of the streamfunction equation. This can be done by forcing the system with a zonally symmetric Hough function. Consider therefore a radiative forcing:

$$J = J_0H_n(y)e^{z/2}\sin(m\pi z/D) \qquad (30)$$

(The time dependence $\exp(-i\omega t)$ is again not shown.)

The function H_n is a zonally symmetric Hough function; the H_n are shown in Plumb (1982). The H_n is analogous to the $\cos(n\pi y/L)$ term in (9). The only difference with the cosine is that H_n varies most rapidly near the pole, but flattens out near the equator. The number of zeros of H increases with increasing n. For this particular heating function the zonal wind is

$$u = \frac{u_{eq}}{1 - i\omega\tau_r)}\cdot\frac{1}{\left[1 + \left(\frac{\tau_m^{-1} - i\omega}{\tau_r^{-1} - i\omega}\right)\frac{\Gamma\epsilon_n}{\mu_m}\right]} \qquad (31a)$$

where

$$u_{eq} = \frac{J_0 \tau_r \Gamma \epsilon_n}{\mu_m} y\sqrt{1 - y^2} B_n(y) e^{z/2}$$

$$\times \left[(m\pi z/D) \cos(m\pi z/D) - \frac{1}{2} \sin(m\pi z/D) \right] \quad (31b)$$

The function B_n is related to the derivative of the Hough function (see Plumb 1982), and is analogous to the $\sin(n\pi y/L)$ term in (7). The μ_m is the vertical separation constant, and ϵ_n the eigenvalue of the nth Hough function.

The important point is that these expressions have the same form as their counterparts (9a) and (9b) for the Boussinesq system on the f-plane. All the conclusions from section 2 can therefore be transferred to the system with variable density on the sphere. The only difference lies in the space dependence of the eigenfunctions, and in the values of the separation constants ϵ_n and μ_m.

Scale analysis of this equation proceeds with the replacement of the nondimensional y derivative by $a/\Delta L$ and the nondimensional z-derivative by $H/\Delta z$, where ΔL is defined as:

$$\Delta L = a\Delta[\sin(\text{latitude})]. \quad (32)$$

It follows then that in the thermodynamic equation, the ratio of w_y-term to the v_z-term is determined by the width of the streamfunction. There is radiative control if

$$\Delta L \gg \left(\left|\frac{\tau_m^{-1} - i\omega}{\tau_r^{-1} - i\omega}\right|^{1/2}\left(\frac{N}{f}\right)\right)\Delta D \quad (33)$$

and there is dynamical control if

$$\Delta L \ll \left|\frac{\tau_m^{-1} - i\omega}{\tau_r^{-1} - i\omega}\right|\left(\frac{N}{f}\right)\Delta D. \quad (34)$$

In these expressions f is given by the usual definition $f = 2\Omega y$, and ΔD is determined by

$$\frac{1}{(\Delta D)^2} = \frac{1}{(\Delta Z)^2} + \frac{1}{(2H)^2} \quad (35)$$

where ΔZ is the dimensional height of the streamfunction. It follows from (35) that ΔD is always slightly smaller than $\min(\Delta Z, 2H)$. This means that usually:

$$\Delta D \approx 2H \quad (36)$$

so that twice the scale height is the appropriate height scale in the expression for the Rossby radius of deformation. Thus, the results for the f-plane can be taken over to the midlatitude case on the sphere. Apart from the effect of the variable density the only difference is that ΔL is now given by (32), so that only at low latitudes ΔL is close to the geometrical width of the circulation. (ΔL can be interpreted as the width of the circulation projected on the earth's axis.)

Again the same pattern emerges as in section 2. Modes with a large horizontal scale and a small vertical scale respond mostly radiatively, while modes with a small horizontal scale and a large vertical scale respond more dynamically.

The foregoing scale analysis assumes that f is approximately constant, an assumption which breaks down in the tropics. In that region standard beta-plane theory can be used (see Fels et al. 1980), but an equatorial Rossby radius of deformation can also be obtained by replacing f in the rhs of (33) by βL_R, and by replacing ΔL by L_R, in which L_R is the Rossby radius of deformation. This gives

$$L_R = \left|\frac{\tau_m^{-1} - i\omega}{\tau_r^{-1} - i\omega}\right|^{1/4}\left(\frac{N\Delta D}{\beta}\right)^{1/2}. \quad (37)$$

Table 1 shows the equatorial Rossby radius of deformation. Within one Rossby radius of deformation of the equator there is always dynamical control in the regions of shortwave absorption, regardless the width of the radiative forcing. At higher latitudes the width of the radiative forcing determines if there is radiative control or dynamical control in the regions of radiative forcing.

5. The shortwave heating and its spectral components

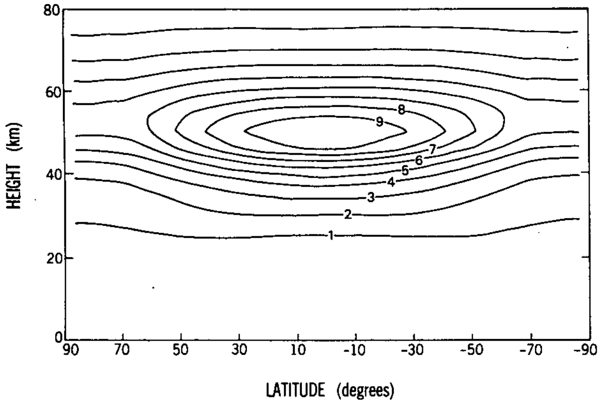
In the preceding sections criteria were developed to determine when there is radiative and when there is dynamic control. In order to see what type of control there is in the middle atmosphere, it is necessary to specify the shortwave heating. Since this quantity does not in general vary sinusoidally in time, a Fourier transform has to be applied in order to make the theory of the previous sections applicable.

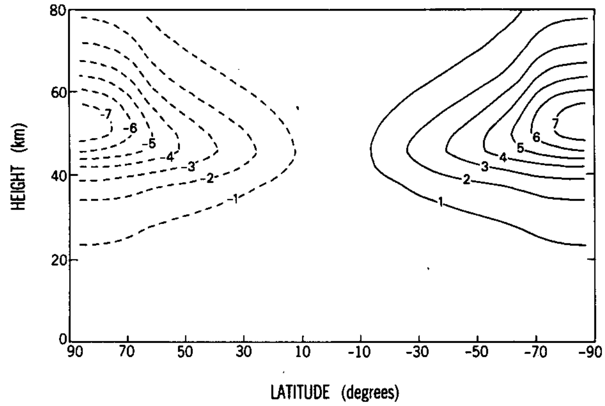
The shortwave heating is calculated with the parameterization of Lacis and Hansen (1974). The only absorber is a single ozone layer which is kept constant throughout the year. The tropopause is ignored in the calculations of the shortwave heating because absorption by water is ignored.

The spectral components of the shortwave heating are shown in Figs. 2a–c. Almost 90% of the shortwave heating can be explained by the annual mean, the annual cycle and the semiannual cycle of the shortwave heating. The remaining Fourier components have been taken into account in the numerical experiments, but they are not shown.

Note that in large regions of the middle atmosphere the annual mean of the shortwave heating is larger than the time varying components of the shortwave heating. This means that a large part of the shortwave heating is not subjected to any transient effects.

The annual cycle has a seesaw structure, but the polar parts of the semiannual cycle are in phase with each other. The reason for this is that the time dependence of the shortwave heating is dominated by the variation of the length of the day. The length of the day near the





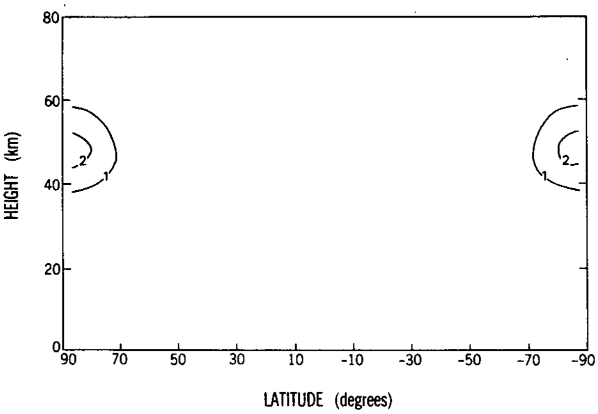


FIG. 2. (a) Annual mean component of stratospheric solar heating. (b) Annual component of stratospheric solar heating. (c) Semi-annual component of stratospheric solar heating. (°K day^{-1}).

poles has a boxcar-like shape, rather than a sinusoidal shape, since the transition between the polar night and the polar day occurs very rapidly. This means that the higher harmonics of the annual cycle give a contribution to the radiative forcing in the polar regions. The shortwave heating also has a semiannual component over the equator because the sun passes the equator twice a year. However, this effect is very weak because the length of the day does not vary much at the equator, and the absorption is only weakly dependent on the zenith angle for small zenith angles.

By comparing the results shown in Figs. 2b and 2c with the numbers in Table 1, we see clearly that the spatial scale ΔL of the annual cycle is always much larger than the quantity L_R defined by equations (33) and (37). It is therefore not surprising that effects of transience are completely negligible. For the semiannual component, however, ΔL is on the order L_R; for this reason, we anticipate that we may have dynamical control at high latitudes, and significant transient vertical motion.

6. A simulation of the seasonal cycle

To evaluate the roles of transience and of mechanical friction in inducing meridional circulation in the middle atmosphere, a number of simulations of the seasonal cycle have been carried out. The zonal momentum equation (16), and the thermodynamic equation (18) were integrated in time, and at each step the streamfunction was calculated using a spectral method to solve the elliptic equation (24). The effects of the tendency terms in (16) and (18) were evaluated by carrying out equilibrium calibrations with these terms omitted at various times. Table 2 shows which terms are taken into account in the time marched and equilibrium solutions. For completeness, we also evaluate the "radiative" solution discussed earlier.

For the purpose of the present study, an extremely simple ad hoc cooling scheme was employed. It goes slightly beyond the local Newtonian cooling parameterization by both including a diffusive term and a temperature dependent damping rate:

TABLE 2. Terms taken into account in the different models (+: yes, −: no).

Model	u_t in (16)	u in (17)	T_t in (18)	w in (18)
Radiative	−	−	+	−
Equilibrium	−	+	−	+
Time marched	+	+	+	+

$$Q = (J - C_1) + C_2(B(T) - B(240)) + C_3 \times \frac{\partial}{\partial z}\left((e^{-C_4 z}\frac{\partial}{\partial z}(B(T) - B(240))\right) \quad (38)$$

Here, J is the solar heating discussed earlier, and $B(T) = \exp(-960/T)$ is an approximation to the Planck function for the 15 μm CO_2 band. The temperature at 300 mb is fixed at its annual average climatological value, and a no-flux condition is imposed at the top. The parameters C_1–C_4 are tuned in such a way that the radiative temperature resembles that produced by the more detailed radiation scheme of Fels and Schwarzkopf (1981). The solstitial radiative temperature fields calculated from the approximate scheme shown in Fig. (3). The chief difference between the "exact" and approximate radiative temperature fields is in the winter polar lower stratosphere, where the parameterized version is about 15°K too warm at 20 km; in that this is precisely the region of interest for the Antarctic polar ozone hole, we shall return to this defect later on.

We note the extremely cold polar night region which is almost 90° K colder than its observed value. The discrepancy between this radiative temperature and the observed temperature has to be explained by the dynamics of the middle atmosphere. The nonlocality of the cooling makes the cooling rate scale dependent (see Fels 1982), but a good impression of the cooling rate can be obtained by considering the cooling rate of a temperature perturbation with a very large vertical scale. (This amounts to putting $C_3 = 0$). The effective cooling time is shown in Fig. 4 as a function of temperature. Observe that the cooling time is strongly dependent on temperature, especially for low temperatures. Through this temperature dependence, the radiative damping time depends implicitly on height. In this simple model, there is no additional explicit height dependence of τ, and for this reason the relaxation times in the lower stratosphere are too short by about a factor of two. As a result, the effects of transience there will be somewhat underestimated.

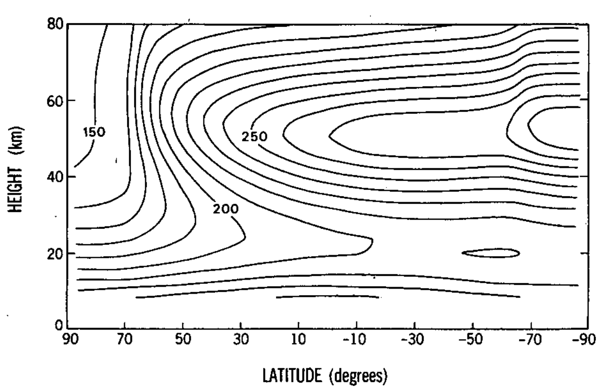


FIG. 3. Time-marched solstitial temperature using the simplified scheme of Eq. (36).

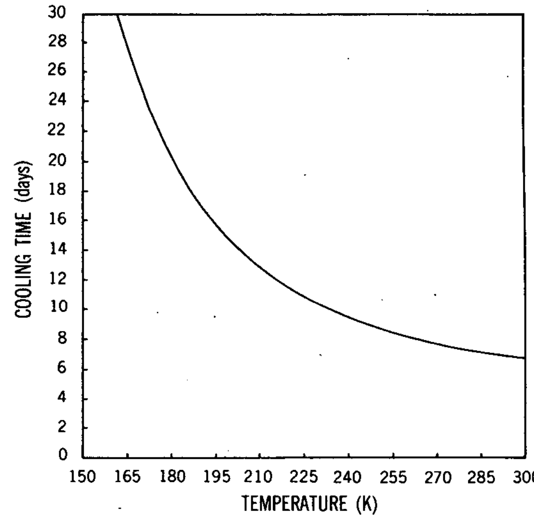


FIG. 4. Radiative relaxation time as a function of temperature.

Two different versions of Rayleigh friction were used. In the first and simplest case, we choose τ_m to have a constant value of 90 days. In this case, the model zonal winds reach the excessive value of 240 m s^{-1}, and the polar stratopause remains far too cold. The solstitial polar temperature for the time marched, equilibrium, and radiative solutions at 50 km are shown in Table 3. Although the time marched temperature is somewhat warmer than that of radiative case, it is clear that the flywheel effect is completely irrelevant to the problem of the warm winter pole.

The zonal wind can be reduced by using a Rayleigh friction time which becomes very small in the mesosphere, a fact first discussed by Holton and Wehrbien (1980), and later by Strobel and Schoeberl (1982). In this way, these authors obtained reasonable simulations of the zonal wind during solstice. Their simulations showed that the adiabatic heating was strong enough to reverse the temperature gradient in the mesosphere, so that the zonal wind jets were closed off. It is thus worthwhile to investigate what the role of transience is in the presence of a stronger mechanical dissipation. For this purpose, we shall employ the Rayleigh friction profile of Holton and Wehrbien (HW), for which the mechanical damping time is 90 days below 60 kilometers, and is 2 days above 80 kilometers. Between 60 and 80 kilometers there is a smooth transition region.

TABLE 3. Temperature at 50 km at the winter pole during solstice. The middle column is from the simulation with a mechanical damping time of 90 days; the right-hand column refers to the experiment with the HW friction.

Model	τ_m = 90 days	τ_m = HW
Radiative	150	150
Equilibrium	157	203
Time marched	165	206

The temperature and zonal wind during the solstice is shown in Figs. 5a and 5b. Note that the wintertime westerlies are stronger than the westerlies found by HW, though the summertime easterlies are very similar. This is because the cooling scheme of HW doesn't allow the polar night to become cold enough, as was already noted by Wehrbein and Leovy (1982).

The meridional circulation for both the equilibrium calculation and the time marched calculation at the solstices is very similar to the meridional circulation obtained by HW. The mesosphere temperature gradient is reversed and the polar night at the stratopause is 50 K warmer than in the case of weak Rayleigh friction. In the upper stratosphere and mesosphere, the short mechanical damping time makes the effect of transience very small. This is not true in the lower stratosphere, however, where the mechanical damping is very weak. In that region, the existence of meridional motion is importantly influenced by the transient character of the heating.

In light of the Antarctic ozone hole problem, it is of interest to examine the vertical velocity fields which

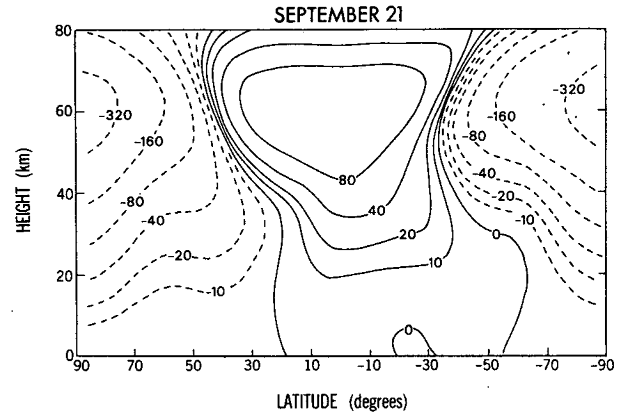


FIG. 6. Vertical velocity w (m day^{-1}) on 21 September in the radiative–dynamical model.

the model predicts for September and October; they are shown in Figs. 6 and 7. It is apparent that on 21 September, the vertical motion is *downward* over the entire Antarctic; dynamical explanations of the seasonal ozone dip of course require *upward* motion. The downward motion in the model is simply explained by the spatial structure of the transient heating. In late September, the sun is still very low in the polar sky, and hence the transient heating *decreases* in the poleward direction. This results in descending motion over the pole. It is true that by 11 October, weak upward motion has indeed set in over the South Pole, but its magnitude (8 m day^{-1} at 20 km) is far too small, and its onset much too late to explain the ozone dip. Although of no practical importance, it is still interesting to note that this weak upward motion is entirely due to transience; this is easily seen by examining the equilibrium vertical velocity (not shown) for the same time, which displays weak downward motion over the South Pole.

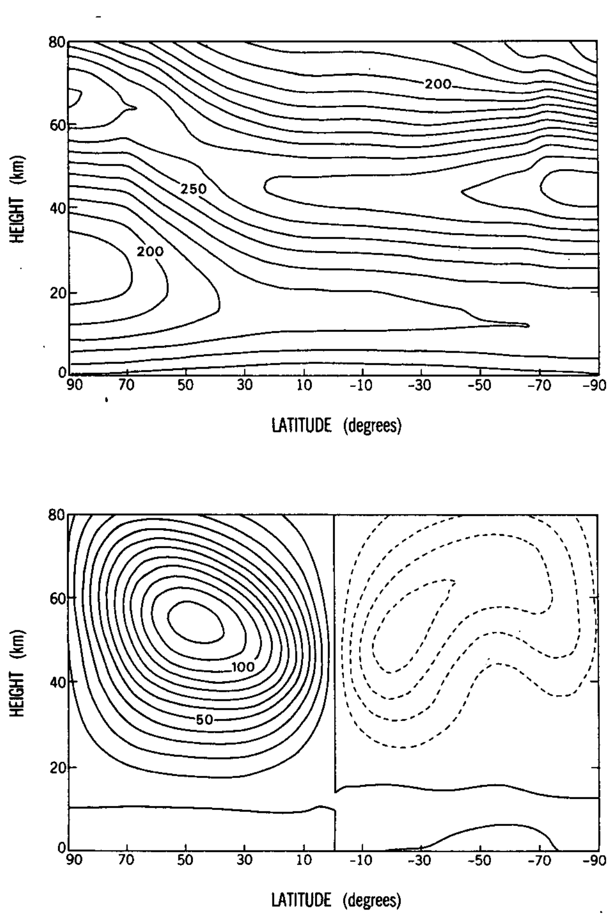


FIG. 5. Solstitial temperature (a) and zonal winds (b) from the radiative–dynamical model with HW friction.

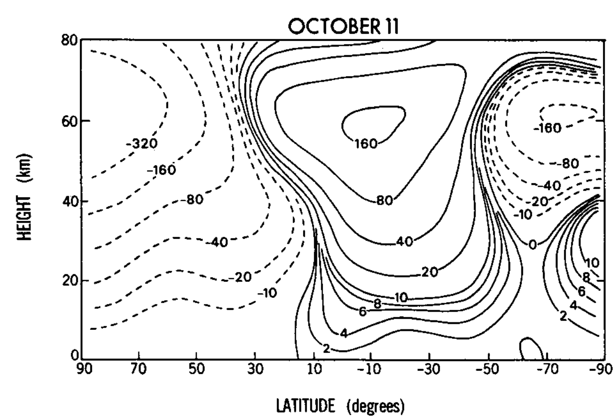


FIG. 7. As in Fig. 6 but on 11 October.

The numerical calculations described here include only that part of the transient heating due to the absorption of solar radiation by ozone, and it is legitimate to ask whether there might exist other sources of radiative heating which might lead to stronger upward motion. Simple arguments based on Eq. (11) show that heating functions which are deep and have large horizontal gradients are most effective in this respect. In addition, of course, it is necessary that the heating be largest at high latitudes. In Mahlman and Fels (1986), it was suggested that solar heating due to the presence of polar stratospheric clouds (PSCs) might produce the required upwelling. One may show, however, that heating rates on the order of 1° day^{-1} are required for the whole month of September. Since the PSCs disappear by the middle of the month, they are unable to provide the necessary heating.

Conclusion

Our emphasis has been on the character of the meridional circulation induced by time-varying insolation in the middle atmosphere. We have shown that in middle and high latitudes, even the semiannual cycle induces only small circulations, and that these are unable to produce the vertical motions required to explain the September decrease in Antarctic ozone.

This conclusion, also reached by Pyle (1986), does not by itself imply that dynamical mechanisms are irrelevant to the problem. Indeed, it is possible that eddy-forced symmetric circulations of the sort discussed by Dunkerton (1988), Garcia (1987), or Tung and Yang (1988) could play a role. It does, however, eliminate transient relative forcing as an important effect in this context.

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