Introduction

- Kinematics of rigid bodies: relations between time and the positions, velocities, and accelerations of the particles forming a rigid body.

- Classification of rigid body motions:
  - translation:
    - rectilinear translation
    - curvilinear translation - Fig (a)
  - rotation about a fixed axis - Fig (b)
  - general plane motion
• Consider the motion of a rigid body in a plane perpendicular to the axis of rotation.

• Velocity of any point \( P \) of the slab,

\[
\vec{v} = \vec{\omega} \times \vec{r} = \omega \vec{k} \times \vec{r} = r \omega
\]

• Acceleration of any point \( P \) of the slab,

\[
\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{\omega} \times \vec{r} = \alpha \vec{k} \times \vec{r} - \omega^2 \vec{r}
\]

• Resolving the acceleration into tangential and normal components,

\[
\vec{a}_t = \alpha \vec{k} \times \vec{r} \quad \quad a_t = r \alpha
\]

\[
\vec{a}_n = -\omega^2 \vec{r} \quad \quad a_n = r \omega^2
\]
Motion of a rigid body rotating around a fixed axis is often specified by the type of angular acceleration.

Recall \( \omega = \frac{d\theta}{dt} \) or \( dt = \frac{d\theta}{\omega} \)

\( \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \omega \frac{d\omega}{d\theta} \)

**Uniform Rotation, \( \alpha = 0 \):**

\( \theta = \theta_0 + \omega t \)

**Uniformly Accelerated Rotation, \( \alpha = \text{constant} \):**

\( \omega = \omega_0 + \alpha t \)

\( \theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \)

\( \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \)
Two Rotating Bodies in Contact
Two Rotating Bodies in Contact - Same Velocities & Tangential Acceleration

\[ \text{v}_A, \text{v}_{A'} \]
Two Rotating Bodies in Contact – Different Normal Accelerations
General Plane Motion

- *General plane motion* is neither a translation nor a rotation.

- General plane motion can be considered as the *sum* of a translation and rotation.

- Displacement of particles $A$ and $B$ to $A_2$ and $B_2$ can be divided into two parts:
  - translation to $A_2$ and $B'_1$
  - rotation of $B'_1$ about $A_2$ to $B_2$
General Plane Motion

Plane motion

\[ = \]

Translation with \( A \) + Rotation about \( A \)

\[ (a) \]

Plane motion

\[ = \]

Translation with \( B \) + Rotation about \( B \)

\[ (b) \]
• Any plane motion can be replaced by a translation of an arbitrary reference point $A$ and a simultaneous rotation about $A$.

\[ \vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \]

\[ \vec{v}_{B/A} = \omega \vec{k} \times \vec{r}_{AB} \]

\[ \vec{v}_B = \vec{v}_A + \omega \vec{k} \times \vec{r}_{AB} \]
• Assuming that the velocity $v_A$ of end $A$ is known, wish to determine the velocity $v_B$ of end $B$ and the angular velocity $\omega$ in terms of $v_A$, $l$, and $\theta$.

• The direction of $v_B$ and $v_{B/A}$ are known. Complete the velocity diagram.
• Selecting point \( B \) as the reference point and solving for the velocity \( v_A \) of end \( A \) and the angular velocity \( \omega \) leads to an equivalent velocity triangle.

• \( v_{A/B} \) has the same magnitude but opposite sense of \( v_{B/A} \). The sense of the relative velocity is dependent on the choice of reference point.

• Angular velocity \( \omega \) of the rod in its rotation about \( B \) is the same as its rotation about \( A \). Angular velocity is not dependent on the choice of reference point.
• Plane motion of all particles in a slab can always be replaced by the translation of an arbitrary point \( A \) and a rotation about \( A \) with an angular velocity that is independent of the choice of \( A \).

• The same translational and rotational velocities at \( A \) are obtained by allowing the slab to rotate with the same angular velocity about the point \( C \) on a perpendicular to the velocity at \( A \).

• The velocity of all other particles in the slab are the same as originally defined since the angular velocity and translational velocity at \( A \) are equivalent.

• As far as the velocities are concerned, the slab seems to rotate about the *instantaneous center of rotation* \( C \).
Instantaneous Center of Rotation in Plane Motion

- If the velocity at two points $A$ and $B$ are known, the instantaneous center of rotation lies at the intersection of the perpendiculars to the velocity vectors through $A$ and $B$.

- If the velocity vectors are parallel, the instantaneous center of rotation is at infinity and the angular velocity is zero.

- If the velocity vectors at $A$ and $B$ are perpendicular to the line $AB$, the instantaneous center of rotation lies at the intersection of the line $AB$ with the line joining the extremities of the velocity vectors at $A$ and $B$.

- If the velocity magnitudes are equal, the instantaneous center of rotation is at infinity and the angular velocity is zero.
Absolute and Relative Acceleration in Plane Motion

- Absolute acceleration of a particle of the slab,

\[ \ddot{a}_B = \ddot{a}_A + \ddot{a}_{B/A} \]

- Relative acceleration \( \ddot{a}_{B/A} \) associated with rotation about \( A \) includes tangential and normal components,

\[
\begin{align*}
(\ddot{a}_{B/A})_t &= \alpha \hat{k} \times \hat{r}_{AB} \\
(\ddot{a}_{B/A})_n &= -\omega^2 \hat{r}_{AB}
\end{align*}
\]

\[
\begin{align*}
(a_{B/A})_t &= r \alpha \\
(a_{B/A})_n &= r \omega^2
\end{align*}
\]