Equilibrium of Rigid Bodies – 2D

- For a rigid body in static equilibrium, the external forces and moments are balanced and will impart no translational or rotational motion to the body.

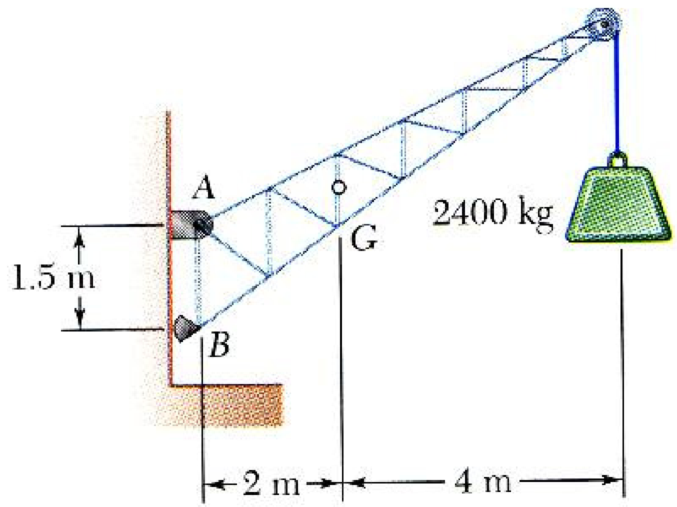
- The necessary and sufficient condition for the static equilibrium of a body are that the resultant force and couple from all external forces form a system equivalent to zero,

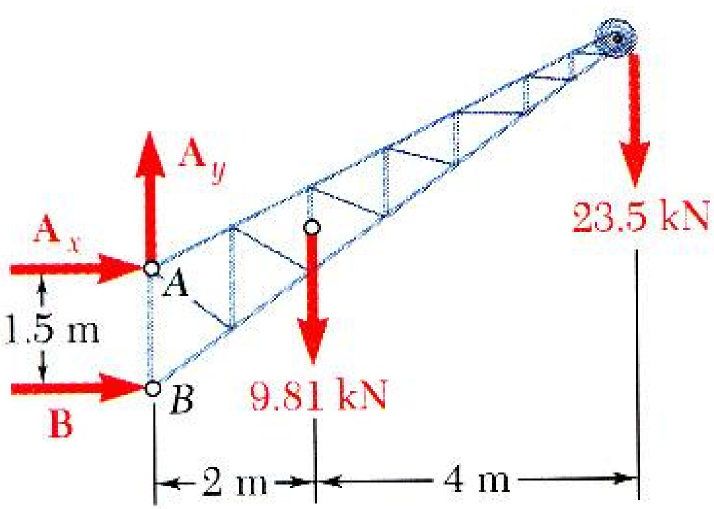
$$\sum \vec{F} = 0 \quad \sum \vec{M}_O = \sum \left(\vec{r} \times \vec{F} \right) = 0$$

- Resolving each force and moment into its rectangular components leads to 6 scalar equations which also express the conditions for static equilibrium,

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

Free-Body Diagram

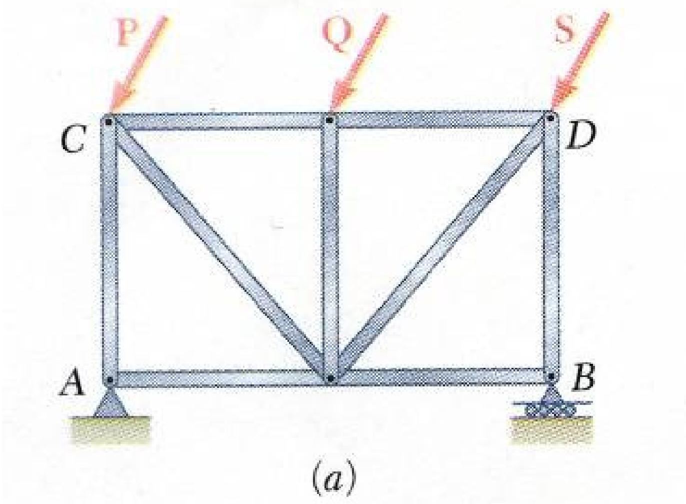




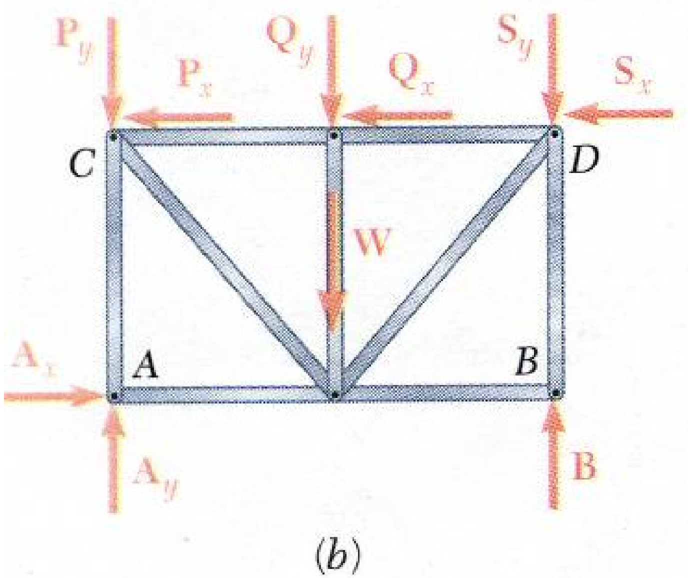
First step in the static equilibrium analysis of a rigid body is identification of all forces acting on the body with a *free-body* diagram.

- Select the extent of the free-body and detach it from the ground and all other bodies.
- Indicate point of application, magnitude, and direction of external forces, including the rigid body weight.
- Indicate point of application and assumed direction of unknown applied forces. These usually consist of reactions through which the ground and other bodies oppose the possible motion of the rigid body.
- Include the dimensions necessary to compute the moments of the forces.

Equilibrium of a Rigid Body in Two Dimensions



(a)



(b)

- For all forces and moments acting on a two-dimensional structure,

$$F_z = 0 \quad M_x = M_y = 0 \quad M_z = M_O$$

- Equations of equilibrium become

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0$$

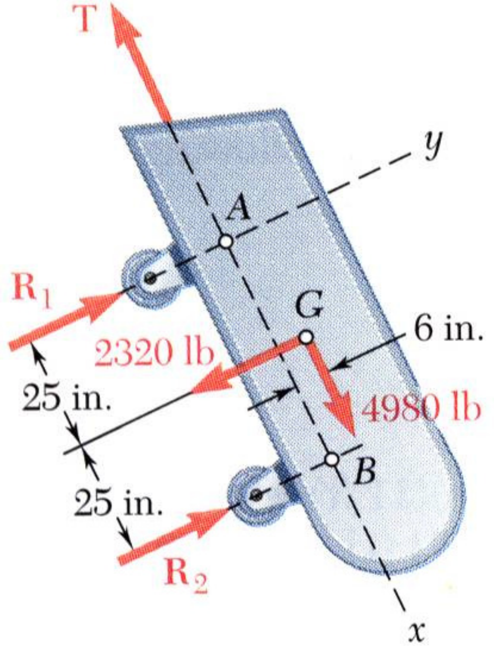
where A is any point in the plane of the structure.

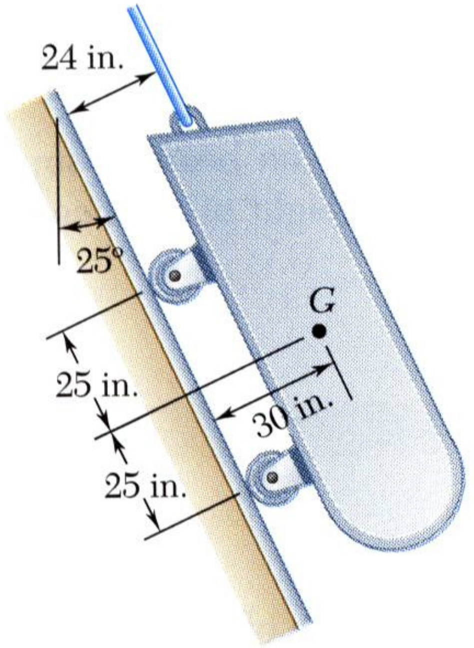
- The 3 equations can be solved for no more than 3 unknowns.

- The 3 equations can not be augmented with additional equations, but they can be replaced

$$\sum F_x = 0 \quad \sum M_A = 0 \quad \sum M_B = 0$$

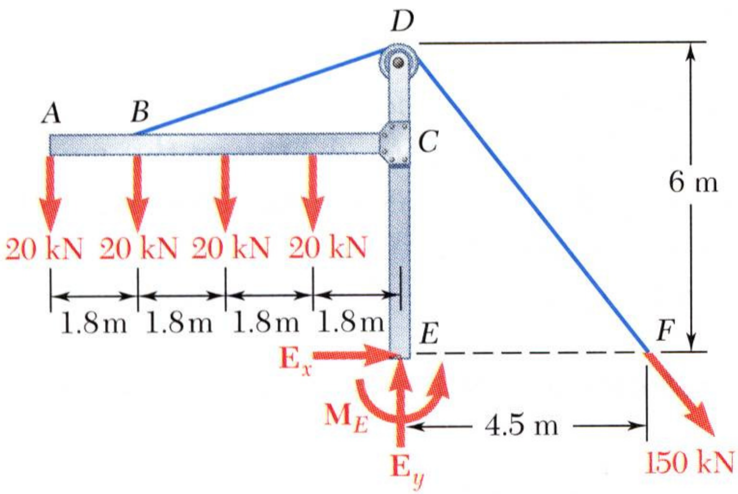
Free Body Diagram - 2D

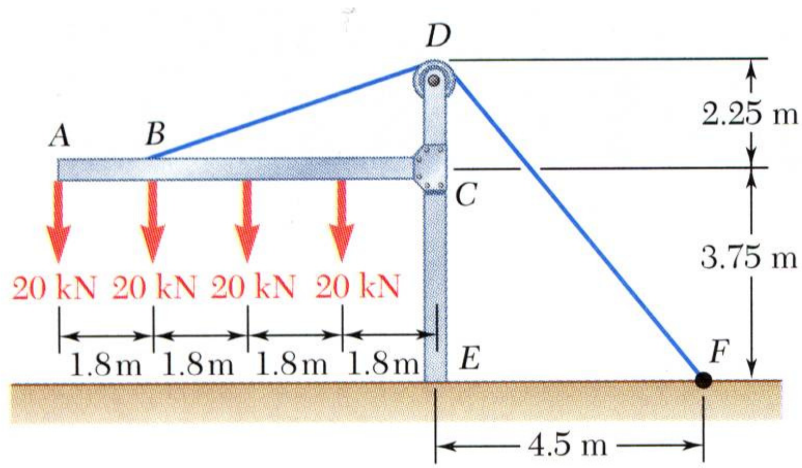




- Create a free-body diagram

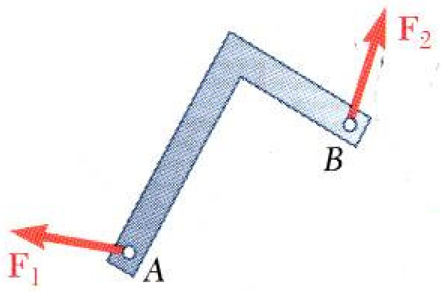
Free Body Diagram – 2D

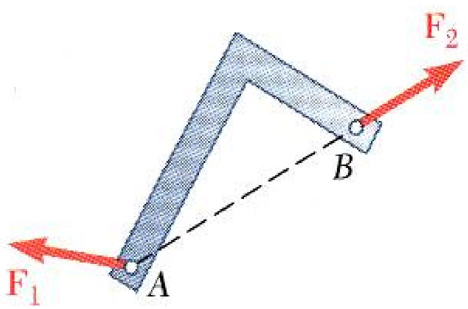


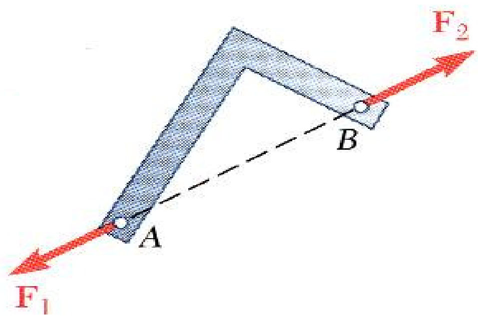


- Create a free-body diagram for the frame and cable.

Two-Force Members







- Consider a plate subjected to two forces $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$
- For static equilibrium, the sum of moments about *A* must be zero. The moment of $\boldsymbol{F_2}$ must be zero. It follows that the line of action of $\boldsymbol{F_2}$ must pass through *A*.
- Similarly, the line of action of $\boldsymbol{F_1}$ must pass through *B* for the sum of moments about *B* to be zero.
- Requiring that the sum of forces in any direction be zero leads to the conclusion that $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ must have equal magnitude but opposite sense.

Equilibrium of a Rigid Body in Three Dimensions

- Six scalar equations are required to express the conditions for the equilibrium of a rigid body in the general three dimensional case.

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

- These equations can be solved for no more than 6 unknowns which generally represent reactions at supports or connections.

- The scalar equations are conveniently obtained by applying the vector forms of the conditions for equilibrium,

$$\sum \vec{F} = 0 \quad \sum \vec{M}_O = \sum \left(\vec{r} \times \vec{F} \right) = 0$$

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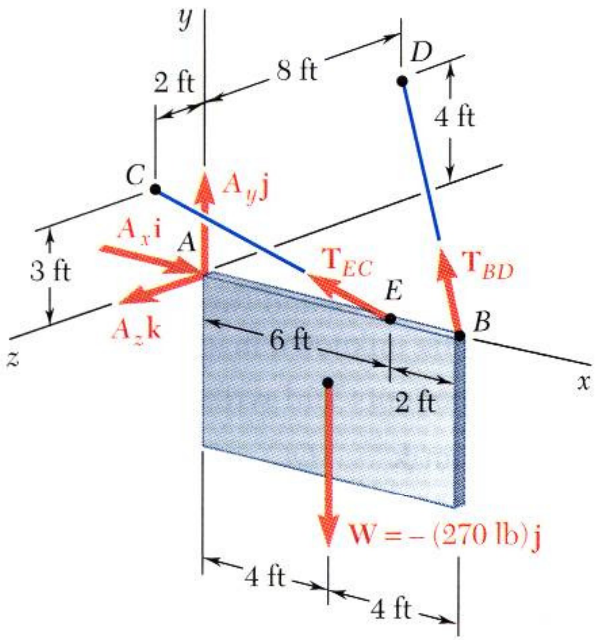
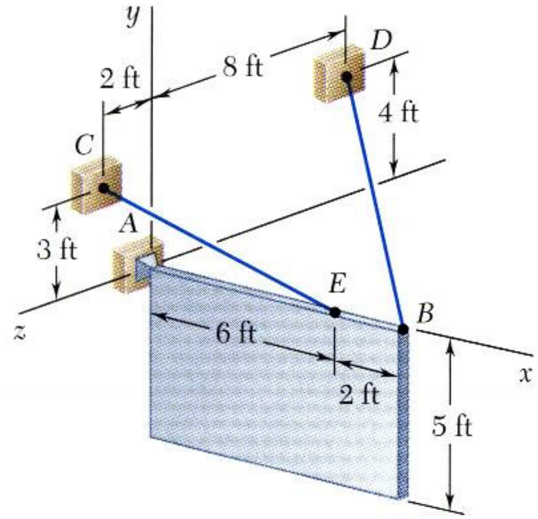
Free Body Diagram – 3D





- Create a free-body diagram for the sign.

  Since there are only 5 unknowns, the sign is partially constrain. It is free to rotate about the *x* axis. It is, however, in equilibrium for the given loading.