1. (10pts) Consider the set of transformations in 2D on an equilateral hexagon which carries corners into corners based on the representation pictured. Draw and label the full set of configurations, select a set of basis vectors and construct the corresponding group transformations as matrices. You can use any dimension of representation you like. You should think about this one a bit before hammering away!

We can rotate by 60° and then 120°, but beyond that we get back to the identity.

So the group of transformations must have 3 elements: \( G = \{ \mathbb{I}, g, g^2 \} \).

We can use any set of transformations with this structure, including \( R(120°), R(240°) \).

3D: \[
\begin{align*}
A' &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} B' = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} C' = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \\
\Rightarrow \quad \mathbb{I} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} R(120°) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} R(240°) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}
\end{align*}
\]

2D: \[
\begin{align*}
B' &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \mathbb{I} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad R(120°) &= \begin{pmatrix} \cos(120°) & -\sin(120°) \\ \sin(120°) & \cos(120°) \end{pmatrix} \quad R(240°) &= \begin{pmatrix} \cos(240°) & -\sin(240°) \\ \sin(240°) & \cos(240°) \end{pmatrix}
\end{align*}
\]

\[
\begin{align*}
A' &= R(120°) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos(120°) \\ \sin(120°) \end{pmatrix} \quad B' &= R(240°) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos(240°) \\ \sin(240°) \end{pmatrix}
\end{align*}
\]
2. (10pts) In your homework and in class we have (without naming them) been working with the idea of "events" which are simply points in spacetime (as opposed to "positions" which are locations in space). Two events are considered distinct if they are separated in either time or space. Show that if two distinct events occur simultaneously to one observer, then they can appear to happen at different times to another observer. That is, demonstrate the "relativity of simultaneity".

\[
\begin{pmatrix}
\Delta t \\
\Delta x \\
\Delta y \\
\Delta z
\end{pmatrix}_{S} \rightarrow \begin{pmatrix}
\Delta t' \\
\Delta x' \\
\Delta y' \\
\Delta z'
\end{pmatrix}_{S'} = \begin{pmatrix}
1 & -vs & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
\Delta t \\
\Delta x \\
\Delta y \\
\Delta z
\end{pmatrix}_{S} = \begin{pmatrix}
\Delta t - \gamma \Delta v \Delta x \\
\Delta x - \gamma \Delta v \Delta t \\
\Delta y \\
\Delta z
\end{pmatrix}_{S'}
\]

If \( \Delta t = 0 \) in \( S \) and the events are distinct, then we need at least one of \( \Delta x, \Delta y \) or \( \Delta z \) to be nonzero. If \( \Delta x = 0 \) then we see that \( \Delta t' = 0 \) as well, so we need to consider the case when \( \Delta x \neq 0 \).

Then: \( \Delta t' = -\gamma \Delta v \Delta x \Rightarrow \Delta t' = -\frac{\beta}{\gamma} \Delta x \neq 0 \)