General Relativity HW4 Problems

1. Imagine a particle following a path through spacetime given by \( x^\mu(\tau) = \left( \tau^2 + \tau, \tau^2, \frac{4}{3} \tau^3, -10 \right) \).
   a) Compute the four-velocity of the particle as it passes through the point
      \( x^\mu = (20, 16, \frac{32}{3}, -10) \).
   b) For the function \( f(t, x, y, z) = -t^2 + x^2 + y^2 - yz \), calculate the rate of change of this
      function along the path from part (a), i.e. \( \frac{df}{d\tau} \), at the point \( x^\mu = (20, 16, \frac{32}{3}, -10) \).
      Hint: You will need to break up the derivative into two terms using \( \partial x^\mu \) in various places so
      that can use your result for the four-velocity.

2. Consider the example in class where I found the vector which defined the area of a two-sphere
   in 3D. In this case, I want you to do the same but this time for a cube of side length 1 which is
   centered on the origin with edges along the coordinates. You will need an equation which
   defines the surface, then proceed as I did in class. The answer should be obvious once you get it.

To be continued....