You know the drill!

1. (10pts) Prolate spheroidal coordinates are related to the usual Cartesian coordinates \( \{x, y, z\} \) of Euclidean three-space by

\[
x = \sinh \chi \sin \theta \cos \phi \\
y = \sinh \chi \sin \theta \sin \phi \\
z = \cosh \chi \cos \theta
\]

What does the invariant interval \( ds^2 \) look like in prolate spheroidal coordinates when \( \theta = \frac{\pi}{2} \)?
2. (10pts) Consider the open annulus which is the set of points in $\mathbb{R}^2$ such that $a < r < b$, when $\mathbb{R}^2$ is described in terms of polar coordinates $(r, \theta)$. Show that this space is a manifold that can be covered by a single chart. In your answer make sure you provide the explicit chart map.