1. This one is simple and straightforward, but I think the results are best absorbed if you just work out little examples of the results I gave you on determinants. Show the following:

   a) A common factor in each row or column may be factored out, hence show that \( \det A = \det \begin{pmatrix} 6a & 2b \\ 3c & d \end{pmatrix} = 6 \det B \) where \( B = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \).
   b) If any row or column is zero then \( \det A = 0 \). Calculate \( \det A \) where \( A = \begin{pmatrix} 0 & b \\ 0 & d \end{pmatrix} \) and when \( A = \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \).
   c) Interchanging two rows or columns changes the sign of the determinant. Calculate \( \det B \) when \( B = \begin{pmatrix} c & d \\ a & b \end{pmatrix} \) and \( \det C \) when \( C = \begin{pmatrix} b & a \\ d & c \end{pmatrix} \) as well as \( \det D \) when \( D = \begin{pmatrix} d & c \\ b & a \end{pmatrix} \) and compare to \( \det A \) for \( A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \).
   d) If any two rows or columns are equal, then \( \det A = 0 \). So evaluate \( \det A \) for \( A = \begin{pmatrix} a & b \\ a & b \end{pmatrix} \) and \( \det B \) for \( B = \begin{pmatrix} a & b \\ c & c \end{pmatrix} \).
   e) Show explicitly that for \( A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \) and \( B = \begin{pmatrix} e & f \\ g & h \end{pmatrix} \) then \( \det(AB) = \det(A) \det(B) \).
   f) A scalar multiple of a row or column may be added to another row or column without changing the determinant. Hence show that \( \det A = \det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \det \begin{pmatrix} a+2c & b+2d \\ c+3d & d \end{pmatrix} \).
   g) If the row or column "vectors" of a matrix are linearly dependent, then \( \det A = 0 \). So evaluate \( \det A \) when \( A = \begin{pmatrix} a & b \\ -3a & -3b \end{pmatrix} \).

2. Now things get a bit harder. These are proofs, not examples.

   a) Use any of the earlier results (a-e) to prove the next to last result (f).
   b) Use any of the earlier results (a-f) to prove the last result (g).

3. Evaluate the classical adjoint of \( M = \begin{pmatrix} a & b & c \\ d & e & f \\ h & i & j \end{pmatrix} \).

4. Supposing that \( M^{-1} \) in the previous exists, calculate it.

5. Consider a linear transformation which takes vectors in \( \mathbb{R}^3 \) and projects them onto a plane defined by the axis through \( x = y = z = 1 \) and the origin.

   a) Write down the matrix version of this linear transformation. And give an example of acting upon a vector with this operator to create the projected version of the vector. Don't use the trivial cases, i.e. a vector along the axis or a vector already in the plane.
b) Does this linear transformation have an inverse?

c) Find any eigenvalues and the associated eigenvectors associated with this linear transformation.

Now, imagine a change of basis which takes the axis defining the plane and aligns it with the \( x \)-axis.

d) Start by finding the operator matrix which takes vector components into their new form.

e) Now find the transformed version of the projection operator.

f) Verify that the transformed version of this story matches the pre-transformed version.