1. We found in an example in class that a rotation in the xy-plane in $\mathbb{R}^3$ has three distinct eigenvalues and three distinct eigenvectors. Prove that the same is true for any rotation in $\mathbb{R}^3$.

$$M = \begin{pmatrix} 3 & 0 & 0 & -1 \\ -\frac{3}{\sqrt{2}} & 2 & 0 & -\frac{3}{\sqrt{2}} \\ 0 & 0 & 4 & 0 \\ -1 & 0 & 0 & 3 \end{pmatrix}$$

2. Is the following matrix diagonalizable via a similarity transformation? Explain your reasoning.

3. Consider the vector space of even polynomials up to 4th degree armed with the inner product defined by $(x, y) = \int_0^1 x(t)y(t) \, dt$. A natural basis would be $X = \{x_0, x_2, x_4\} = \{1, t^2, t^4\}$.
   a) Confirm that this basis is not orthogonal.
   b) Starting with the basis vector $x_0$, use the Graham-Schmidt process to determine an orthonormal basis.
   c) Starting this time with the basis vector $x_2$, use Graham-Schmidt to determine an orthonormal basis.

4. Consider the derivative operators $D = \frac{d}{dt}$ and $D^2 = \frac{d^2}{dt^2}$ acting on the vector space considered in question (3).
   a) Are both of these linear operators on this space? Explain.
   b) For any of these two that are linear operators, find their matrix representations with respect to the three bases discussed in problem (3). Verify the matrix forms by acting on the vector $x = a_0 + a_2 t^2 + a_4 t^4$ (rewritten in terms of each basis) with both the straight derivative(s) and via matrix multiplication of the components.
   c) Find the transformation that carries the operators between each of the three basis sets.

5. Find two 2x2 matrices that are both self-adjoint, but which do not commute with each other. Verify that their product is not self-adjoint.

6. Consider the following statement: For a Hermitian matrix that is fully degenerate, i.e. all of its eigenvalues are the same, then the matrix is necessarily diagonal. If this true, prove it. If not, find a counter example.