Mathematical Methods in Physics HW5

1. For the matrix \( M = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \), show that the standard construction of eigenvalues and eigenvectors agrees with the results of the extremizing \( I = (x, Mx) \) subject to \((x, x) = 1\).

2. Consider the matrix \( M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \). Find the set of vectors \( y \) such that \([M - \lambda I]x = y\), where \( \lambda \) is an eigenvalue of \( M \), has a solution.

3. Consider the matrix \( A_0 = \begin{pmatrix} 3 & 0 & i\sqrt{8} \\ 0 & 2 & 0 \\ -i\sqrt{8} & 0 & 1 \end{pmatrix} \). Now consider a perturbation given by \( A = \begin{pmatrix} 3 & 0 & i\sqrt{8} \\ 0 & 2 & 0 \\ -i\sqrt{8} & 0 & 1 \end{pmatrix} + \epsilon \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \). For the new matrix, determine the eigenvalues up to third order in \( \epsilon \) and the eigenvectors to first order in \( \epsilon \) using nondegenerate perturbation theory.

4. Check that your results from problem (3) for the eigenvalues agree with what you would obtain by directly determining the eigenvalues as an expansion in \( \epsilon \).