1. Consider the function \( f(x) = \begin{cases} 1 & \text{for irrational } x \\ 0 & \text{for rational } x \end{cases} \) on the closed domain \( x \in [0,1] \) with inner product \((f_1, f_2) = \int_0^1 f_1(x)f_2(x)dx\). Find \( \|f\|^2 \) using both the Riemannian measure as well as the Lebesgue measure. Do they agree?

2. a) For the example in class of a space which is not complete, i.e. continuous functions with norm defined by \( \|x\| = \int_0^1 |x(t)|dt \), show that for the given sequence \( x_n(t) = \begin{cases} 0 & 0 \leq t \leq \frac{1}{2} - \frac{1}{n} \\ 1 + nt - \frac{n}{2} & \frac{1}{2} - \frac{1}{n} \leq t \leq \frac{1}{2} \\ 1 & \frac{1}{2} \leq t \leq 1 \end{cases} \), \( \|x_n - x_m\| = \frac{1}{2}|\frac{1}{n} - \frac{1}{m}| \).

   b) Now imagine that the sequence was instead in a space which has as its norm \( \|x\| = \max_{t \in [0,1]} |x(t)| \). In this case, prove that the sequence is not Cauchy.

3. Show that the sequence \( g_n(x) = \frac{\cos(nx)}{\sqrt{n}} \) converges uniformly to \( g(x) = 0 \) for \( x \in \mathbb{R} \). Does it converge pointwise?

4. Show that the sequence \( f_n(x) = x^n \) on \( x \in [0,1] \), converges pointwise but not uniformly. Does it converge in the mean?