You can try both problems below, but you will only receive credit for the most correct solution.

1. Consider the matrix $M = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ acting in $\mathbb{R}^3$ with basis $\{\hat{i} = (1,0,0), \hat{j} = (0,1,0), \hat{k} = (0,0,1)\}$.

   a) (8pts) Is this diagonalizable by a similarity transformation? If so find its diagonal form, if not explain how you determined so.

   b) (2pts) What would the eigenvalues be for a version of this operator that is similarity transformed by a rotation around the $\hat{i}$ basis vector by $39^\circ$ followed by a rotation about the $\hat{k}$ basis vector by $666^\circ$?
2. Consider the basis \( \{ x_0, x_1, x_3 \} = \{ 1, t, t^3 \} \) in the space of polynomials consisting of a constant plus linear and cubic terms.

   a) (7pts) Going in the order 0 \( \rightarrow \) 1 \( \rightarrow \) 3, Graham-Schmidt the shit out of it. No need to evaluate the normalization factor on the third vector.

   b) (3pts) Is any derivative operator a linear transformation on this space? If yes, which one(s)?