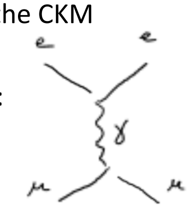
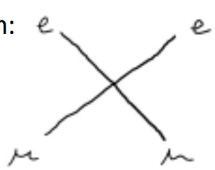
Particle Physics HW10

1. For two quarks, show that the color factor for any of the triplet states is $f_{\text{triplet}} = -\frac{2}{3}$, while for the sextet states it is $f_{\text{sextet}} = \frac{1}{3}$. Be aware that the factor of $\frac{1}{4}$ is pulled out of the color factor in this case, unlike in the meson case. So if you are off by that much, its likely that you included it.
2. For each of the gluons in the $p + p \rightarrow p + p$ process for which I drew the Feynman diagram in class, assign a gluon "direction", i.e. put an arrow on the gluon line, and then determine the color-anticolor combination that the gluon carries. Verify that color is conserved at each vertex in the diagram.
3. The CKM is a 3x3 complex matrix that is unitary.
   a) Determine the number of free parameters for the CKM matrix.
   b) The CKM matrix connects three quark states to three quark states. For each of these quark states we are free to adjust the phase, which means there are three degrees of freedom that we can "absorb" from the CKM matrix (or undo) by redefining quark phases. Also, the overall quark phase (of all quarks) is physically irrelevant and so increases this to four parameters. That means that the true number of physically relevant parameters in the CKM matrix is the answer of (a)-4. Using this, argue that you cannot render the CKM matrix to be purely real.
4. Repeat problem (3) for the 2x2 case, but this time argue that you can in fact make the CKM matrix in this case purely real.
5. Consider the QED process of $e + \mu \rightarrow e + \mu$ and the lowest order diagram given by:

   Now consider an effective field theory wherein this diagram is collapsed into the form:

   Is such a description even possible? If so, at what point will we need to go back to the fundamental QED description?
6. Let's see why the string theory result is finite. Usually, the infinities in quantum field theory arise from internal loops of virtual particles over which we must integrate their momenta, i.e. $\int_0^\infty dp$. Another way to view this is that the circular path of the virtual particles is going to zero in the limit of $p \rightarrow \infty$.
   If we consider a closed string that forms a loop, then we are actually considering a torus. We can coordinatize this torus by a complex number $\tau = \tau_1 + i\tau_2$ where we can take τ_1 to be the direction around the strings circumference, and τ_2 to be the direction of the circular path along which the string is moving.If we calculate the amplitude for such a diagram in string theory, then the answer is $\int_F \frac{d\tau d\bar{\tau}}{4\tau_2}$. In this expression F is the fundamental domain of values that τ_1 and τ_2 can take. Now what is important is that there are coordinate symmetries (which are just a topological version of Poincare transformations) that take the form $\tau \rightarrow \tau + 1$ and $\tau \rightarrow -\frac{1}{\tau}$. Argue that these will preclude the possibility of $\tau_2 \rightarrow 0$.