Warning: Most of these will only be doable after Tuesday's lecture. However playing around with them in advance of Tuesday's lecture could open up your curiosity on how to work with them, which will probably make Tuesday's lecture more effective. This is important because Tuesday's lecture is a very technical one!!

1. Show that at least one of the nonzero momentum solutions presented in class actually solves the Dirac equation.

2. To find helicity eigenstates class we cheated and aligned our coordinates so that the z-axis pointed along the momentum. In this problem you will show that helicity eigenstates can be constructed for an arbitrary orientation of the coordinates. Explicitly construct the helicity projection operators $P_{\pm} = \frac{1}{2} \left( 1 \pm \frac{2}{\hbar} S_\theta \right)$. Evaluate $P_+ \psi^{(1)}$ where $\psi^{(1)}$ is the first nonzero momentum solution shown in class. Then explicitly show that the result of $P_+ \psi^{(1)}$ is an eigenstate of $S_\theta$ with eigenvalue $+ \frac{\hbar}{2}$. **Hint:** Once you have constructed the explicit form of $S_\theta$, the rest is plug and chug. Remember that each component of $S_\theta$ should be weighted by the nonzero momentum in that direction, e.g. the z-component should include a factor of $\frac{p_z}{p}$ where here $p$ is the magnitude of the spatial momentum. Also, remember that we are not dealing with chirality here, so $\gamma^5$ should not be part of your work!

3. Show that Dirac Lagrangian for a massive field, expressed in terms of Weyl spinors $\psi_+$ and $\psi_-$, takes the form shown in class.

4. Consider the Proca equation with zero mass. If we look for plane-wave solutions of the form $A^\nu = A e^{i \frac{P_\mu x^\mu}{\hbar}} \epsilon^\nu$ where $\epsilon^\nu$ is a polarization vector and $P^\mu$ is the four-momentum, show that the four –momentum and polarization are “orthogonal” in the 4D sense.