Particle Physics HW 1 Quiz

You can try both problems below, but you will only receive credit for the most correct solution.

1. (6pts) Consider the set of transformations in 3D on a rectangle which carries corners into corners based on the representation pictured. Draw and label the full set of configurations, select a set of basis vectors and construct the corresponding group transformations as matrices.

\[ V_1 = (1) \]
\[ V_2 = (-1) \]

\[ G = \{ 1, -1 \} \] Note: \[ 1 V_1 = V_1 \]
\[ -1 V_1 = V_2 \]
\[ 1 V_2 = V_2 \]
\[ -1 V_2 = V_1 \]

There is only one non-trivial transformation.

You could do this with larger vectors, but there is no need. For example, you could use \((1, 0)\), \((0, 1)\) then \(G = \{(1, 0), (0, 1)\}\).

(4pts) Construct the multiplication table for this group. Is it abelian or non-abelian?

\[
\begin{array}{c|cc}
 & 1 & -1 \\
\hline
1 & 1 & -1 \\
-1 & -1 & 1 \\
\end{array}
\]

Yes it is abelian!

Turn over for second problem!!
(6pts) Consider the set of transformations in 2D on an equilateral triangle which carries corners into corners based on the representation pictured. Draw and label the full set of configurations, select a set of basis vectors and construct the corresponding group transformations as matrices.

\[
\begin{align*}
\begin{pmatrix} a & b & c \\
 0 & d & e \\
 0 & g & h \\
\end{pmatrix} \begin{pmatrix} 1 \\
 0 \\
 0 \\
\end{pmatrix} &= \begin{pmatrix} 0 \\
 1 \\
 0 \\
\end{pmatrix} \Rightarrow a = 0 \\
\begin{pmatrix} a & d & e \\
 b & c & f \\
 g & h & i \\
\end{pmatrix} \begin{pmatrix} 0 \\
 1 \\
 0 \\
\end{pmatrix} &= \begin{pmatrix} 0 \\
 0 \\
 1 \\
\end{pmatrix} \Rightarrow b = 0 \\
\begin{pmatrix} a & b & c \\
 d & e & f \\
 g & h & i \\
\end{pmatrix} \begin{pmatrix} 0 \\
 0 \\
 1 \\
\end{pmatrix} &= \begin{pmatrix} 0 \\
 0 \\
 0 \\
\end{pmatrix} \Rightarrow c = 0
\end{align*}
\]

\[
\begin{align*}
R_{130} &= \begin{pmatrix} 0 & 0 & 1 \\
 0 & 1 & 0 \\
 1 & 0 & 0 \\
\end{pmatrix} \\
R_{230} &= \begin{pmatrix} 0 & 1 & 0 \\
 0 & 0 & 1 \\
 1 & 0 & 0 \\
\end{pmatrix} \\
R_{430} &= \begin{pmatrix} 1 & 0 & 0 \\
 0 & 1 & 0 \\
 0 & 0 & 1 \\
\end{pmatrix}
\end{align*}
\]

Note: You can obviously do this in 2D or even 1D complex rep, but use the homework for examples of that!

(4pts) Construct the multiplication table for this group. Is it abelian or non-abelian?

\[
\begin{array}{c|ccc}
\text{I} & R_{130} & R_{230} & R_{430} \\
R_{130} & R_{230} & R_{430} & I \\
R_{230} & R_{430} & I & R_{130} \\
R_{430} & I & R_{130} & R_{230}
\end{array}
\]

Yup, it's abelian!