1. (10pts) In a 1+2D spacetime (based on SO(1,2) invariance) you have a dual vector with components $V_{\mu} = (1 \ 1 \ 1)$ and a (1,1)-tensor with components $M_{\mu}^{\nu} = \begin{pmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{pmatrix}$.

Determine the components of $M_{\mu}^{\nu} V^{\mu}$. 

The first thing to do is find $V^{\mu}$.

$$V^{\mu} = V_{\nu} g^{\nu\mu} \text{ where } g^{\nu\mu} = (g_{\nu\mu})^{-1} = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$ needed to get $V$'s adjacent

Then: $M_{\mu}^{\nu} V^{\mu} = M^{T} V = \begin{pmatrix} -1 & 1 & 1 \\ -1 & -1 & 1 \\ -1 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix}$
2. (10pts) Consider the scattering event $A + A \rightarrow B + B$ where in the lab frame the incoming particles are moving at $90^\circ$ to each other and have equal mass $m_A$ and equal energies $E_A$. Afterwards the two outgoing particles have equal mass $m_B$ where $m_B > m_A$. Find the minimum energy of the incoming particles $E_A$ such that this process can occur. Show your work.

\[ p_A^0 + p_A' = p_B^0 + p_B' \]

\[ (p_A^0 + p_A')(p_A' + p_A') = (p_B^0 + p_B')(p_B' + p_B') \]

\[ p_A^{0,1} + p_A^{0,1} + 2 p_A^{1,1} = p_B^{0,1} + p_B^{0,1} + 2 p_B^{1,1} \]

\[ -2m_A c^2 + 2 p_A^{0,1} = -2m_B c^2 + 2 p_B^{0,1} \]

\[ \left( \frac{E_A}{c^2}, \frac{p_A}{c} \right) \left( \frac{-E_A}{c^2}, \frac{p_A'}{c} \right) \]

\[ \left( \frac{E_B}{c^2}, \frac{p_B}{c} \right) \left( \frac{-E_B}{c^2}, \frac{p_B'}{c} \right) \]

Evaluate each of these in the rest frame of the respective particle and use that $m_A = m_A'$, $m_B = m_B'$.

\[ -2m_A c^2 + 2 p_A^{0,1} = -2m_B c^2 + 2 p_B^{0,1} \]

Minimum energy final configuration in c.o.h. frame

\[ -2m_A c^2 - 2 \frac{E_A}{c^2} = -U \]

\[ m_B c^2 = E_A = c^2 \sqrt{2m_B^2 - m_A^2} \]