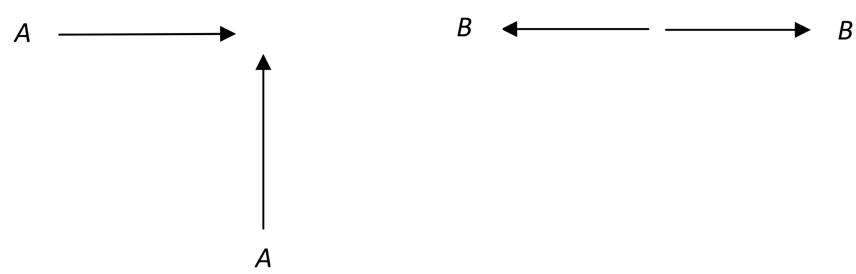
Particle Physics HW 3 Quiz Name___KEY___

You can try both problems below, but you will only receive credit for the most correct solution.

1. ($10pts$) Consider the scattering event $A + A \to B + B$ where in the **lab** frame the incoming particles are moving at 90° with respect to each other and have equal mass m_A and equal energies E_A. Afterwards the two outgoing particles have equal mass m_B and equal energies E_B. Determine the energies of the outgoing particles E_B in the **center of momentum** frame in terms of m_A and E_A. **Lab** **COM**



In the lab frame: $p^\mu_A = \begin{pmatrix} E_A/c \\ \vec{p}_A \end{pmatrix}$ $p^\mu_{A'} = \begin{pmatrix} E_A/c \\ \vec{p}_{A'} \end{pmatrix}$ where $\vec{p}_A \cdot \vec{p}_{A'} = 0$

In the c.o.m. frame: $p^\mu_B = \begin{pmatrix} E_B/c \\ \vec{p}_B \end{pmatrix}$ $p^\mu_{B'} = \begin{pmatrix} E_B/c \\ -\vec{p}_B \end{pmatrix}$ so $p^\mu_B + p^\mu_{B'} = \begin{pmatrix} 2E_B/c \\ \vec{0} \end{pmatrix}$

Using that $p^\mu p_\mu$ is invariant, we can relate the lab to com frame:

$$(p_A^\mu + p_{A'}^\mu)(p_{A\mu} + p_{A'\mu})_{lab} = (p_B^\mu + p_{B'}^\mu)(p_{B\mu} + p_{B'\mu})_{com}$$

$$p_A^\mu p_{A\mu} + p_{A'}^\mu p_{A'\mu} + 2 p_A^\mu p_{A'\mu} = -4 E_B^2/c^2$$

$$-m_A^2 c^2 - m_A^2 c^2 - 2\frac{E_A^2}{c^2} + 2\cancelto{0}{\vec{p}_A \cdot \vec{p}_{A'}} = -4\frac{E_B^2}{c^2}$$

Then: $E_B = \sqrt{\frac{1}{2} m_A^2 c^4 + \frac{1}{2} E_A^2}$

Turn over for second problem!!

2) (*10pts*) For the SU(3) algebra $[g_i, g_j] = i f^{ijk} g_k$ where

$$f^{123} = 1, \quad f^{147} = f^{165} = f^{246} = f^{257} = f^{345} = f^{376} = \frac{1}{2}, \quad f^{458} = f^{678} = \frac{\sqrt{3}}{2}$$

and the f^{ijk} are totally antisymmetric in the three indices, i.e. $f^{ijk} = -f^{jik}$ (if a particular index combination doesn't appear in this list (or from cylic or anticyclic permutations) it is 0), one solution for the generators is given by:

$g_i = \frac{\lambda_i}{2}$ and:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Explicitly compute the left **and** right hand sides of the algebra for $i = 7,\ j = 6$.

Left hand side: $[g_7, g_6] = \left[\frac{\lambda_7}{2}, \frac{\lambda_6}{2}\right] = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i/2 \\ 0 & i/2 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1/2 \\ 0 & 1/2 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1/2 \\ 0 & 1/2 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i/2 \\ 0 & i/2 & 0 \end{pmatrix}$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & -i/4 & 0 \\ 0 & 0 & i/4 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & i/4 & 0 \\ 0 & 0 & -i/4 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & -i/2 & 0 \\ 0 & 0 & i/2 \end{pmatrix}$$

Right hand side: $i f^{76k} g_k = \cancel{i f^{761}} g_1 + \cancel{i f^{762}} g_2 + i f^{763} g_3 + \cancel{i f^{764}} g_4 + \cancel{i f^{765}} g_5$

$$+ \cancel{i f^{766}} g_6 + \cancel{i f^{767}} g_7 + i f^{768} g_8$$

$$f^{678} = \frac{\sqrt{3}}{2} \Rightarrow f^{768} = -\frac{\sqrt{3}}{2}$$

$$f^{376} = \frac{1}{2} \Rightarrow f^{763} = \frac{1}{2}$$

$$i f^{76k} g_k = \frac{i}{2}\frac{\lambda_3}{2} - \frac{i\sqrt{3}}{2}\frac{\lambda_8}{2}$$

$$= \frac{i}{4}\begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} - \frac{i}{4}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & & \\ & -i/2 & \\ & & i/2 \end{pmatrix}$$ BAM!!