Lecture 16 - The Higgs Mechanism and Spontaneous Symmetry Breaking Page 1

\[ V(\phi) = -\frac{1}{2} \lambda \phi^2 + \frac{1}{4!} \lambda_4 \phi^4 \]

\[ \mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi) = \frac{\partial^2}{\partial \phi^2} - \lambda \phi^2 - \lambda_4 \phi^4 = 0 \]

If we look for stable solutions, i.e., \( \partial^2 \phi^2 = 0 \), then we want \( \frac{\partial^2}{\partial \phi^2} = -\phi^2 + \phi^4 = 0 \) \( \phi = \begin{cases} 0 \text{ symmetric} \\ \pm 1 \text{ not symmetric} \end{cases} \)

This is even more explicit when we focus on small fluctuations (as we do when we study particle-like behavior of the underlying fields in the SM).

Consider \( \phi(x) = \phi_0 + \delta \phi(x) \), where \( \phi_0 \) is one of the static solutions above. To determine the equation of motion for the fluctuations \( \delta \phi(x) \), we simply substitute this back into the original Lagrangian:

\[ \mathcal{L}(\delta \phi) = \partial_\mu (\partial^\mu \delta \phi)^* (\partial^\mu \delta \phi) - (\partial_\mu \delta \phi)^* (\partial^\mu \delta \phi) \]

\[ = \partial_\mu \partial^\mu \delta \phi_0 + \partial_\mu \partial^\mu \delta \phi + 2 \partial_\mu \delta \phi \partial^\mu \delta \phi_0 + \partial_\mu \partial^\mu \delta \phi_0 - (\partial_\mu \delta \phi)^* (\partial^\mu \delta \phi) \]

\[ \mathcal{L}_0 = \partial_\mu \partial^\mu \delta \phi_0 - \delta \phi_0 + \delta \phi^3 \]

\[ \mathcal{L}_{\text{eff}} = \partial_\mu \partial^\mu \delta \phi - (\partial_\mu \delta \phi)^* (\partial^\mu \delta \phi) \]

\[ \mathcal{L}_0 \] for \( \phi_0 \) has symmetry breaking \( \mathcal{L}(\delta \phi) \neq \mathcal{L}(-\delta \phi) \)

It is important to realize that the full underlying potential in both cases is symmetric. It is just when we focus on fluctuations about a particular solution that the symmetry is not realized, i.e., it appears to be broken.
Recall that for the complex scalar Higgs we had:

\[ L(\phi, \phi^*, A_\mu) = \frac{1}{2} \left( \partial_\mu \phi \right)^* \partial^\mu \phi + \frac{1}{2} \lambda (\phi^* \phi)^2 + \frac{1}{16 \pi^2} F^\mu \nu F_{\mu \nu} \]

This has a symmetric solution: \( \phi = 0, A_\mu = 0 \) where the \( L(\phi, \phi^*, A_\mu) \) looks just like the expression above.

We also considered the more interesting solution: \( \phi = \frac{A}{\lambda} \) where the Lagrangian for fluctuations \( \phi^* \phi \) becomes:

\[ \phi^* \phi = \frac{1}{\lambda} \]

\[ A_\mu = 0 \]

\[ \lambda = \lambda \]

\[ \lambda = \lambda \]

\[ L(\phi, \phi^*, A_\mu) = \frac{1}{2} \left( \partial_\mu \phi \right)^* \partial^\mu \phi + \frac{1}{2} \lambda \left( \phi^* \phi \right)^2 + \frac{1}{16 \pi^2} F^\mu \nu F_{\mu \nu} + \frac{1}{\lambda} \left( \frac{\partial^\mu A_\mu}{\lambda} \right)^2 + \text{various interactions} \]

What we have done here is pretty much like what we just discussed for symmetry breaking with the difference that here we are breaking a continuous symmetry (i.e. \( U(1) \)), whereas before the symmetry was discrete (i.e. \( U(1) \)).

A picture will help...
The mass of a fluctuation can be associated with the coefficient of the quadratic term:
\[ V(\phi) = \frac{1}{2} \lambda \phi^2 + \frac{1}{4} \lambda' \phi^4 + \text{higher order terms} \]
\[ \Rightarrow \frac{\partial^2 V(\phi)}{\partial \phi^2} = 2 \lambda \phi \]

But the 2nd derivative is just expressing concavity!

\[ \text{Note:} \quad \frac{\partial^2 V(\phi)}{\partial \phi^2} > 0 \quad \Rightarrow \quad \phi = \phi_0 \quad \text{(The massless Higgs boson)} \\
\[ \frac{\partial^2 V(\phi)}{\partial \phi^2} = 0 \quad \Rightarrow \quad \phi \neq \phi_0 \quad \text{(The massive Higgs boson)} \]

What happened to original $\phi \rightarrow e^{i \theta_1} \phi$?

The original symmetry is now encoded by “shifts” in $\phi$, i.e., $\phi = \phi + \delta \phi$.

But this is a mathematical gauge symmetry, so we can set $\phi = 0$ leaving only $\pi \phi$, where $\lambda > 0$!

In words: The Higgs Mechanism” gives mass to the gauge fields of a “spontaneously broken” gauge symmetry through the coupling to an extra Higgs field $\phi$ (which of course has its own parity!).

The 2-polarization massless spin-1 gauge field “eats” the spin-0 Goldstone boson to get its 3rd polarization state, which is required when $\lambda > 0$.

This simple U(1) example can be generalized to the breaking of SU(2) x U(1), to U(1) explaining the masses of the W+/Z- bosons.

Furthermore, by coupling the fundamental matter fermions (taken to be massless for SU(2) invariant) we can generate effective masses for them as well.

This still leaves the question: “How did the Higgs field even get into the unstable solution in the first place?”
Suppose that the Higgs potential itself has "evolved" over the history of the universe.

Lower energy solution is symmetric \( SU(2) \times SU(2) \)

Still symmetric \( SU(2) \times SU(2) \)

Lowest energy solutions have broken symmetry \( U(1)_{em} \)

How does this happen? Recall that as the universe ages, it expands and cools; hence the average energy density is decreasing.

So we can consider:

One of the important things we will discover when we start doing calculations is that the "constant" coefficients in our Lagrangian actually change with the energy scale. But constants like \( m \) and \( \lambda \) are what determine the shape of the Higgs potential!

This might all sound strange, but you are probably already familiar with an example of this...
Consider a solid of magnetic dipoles at very high temperature. In this case the thermal motion is so extreme, that it overcomes the dipole-dipole interaction and everything looks random.

\[ \text{High } T : \]
\[
\begin{array}{cccc}
\bullet & \bullet & \bullet & \bullet \\
\downarrow & \downarrow & \downarrow & \downarrow \\
\end{array}
\]

As we cool this system, the thermal motion eventually slows and is overtaken by the dipoles tending to align.

\[ \text{Low } T : \]
\[
\begin{array}{cccc}
\bullet & \bullet & \bullet & \bullet \\
\end{array}
\]

What is perhaps counter-intuitive is that the high \( T \) state is actually more symmetric! It has \( SO(3) \) invariance.

As \( T \) gets low, there is a preferred axis in space leaving only \( SO(2) \) invariance.

So \( SO(3) \) is spontaneously broken to \( SO(2) \) in this system.

All of this can be modelled in terms of an “effective potential” for the dipole alignment in perfect analogy to the Higgs.

Note: The final preferred axis is completely undetermined in this case, hence the name spontaneous symmetry breaking.
The Higgs mechanism is an example of perturbative physics in the Standard Model. In this case, the entire Higgs field (not a localized excitation) does something interesting, but due to its coupling to other fields, their particle-like excitations get new behavior.

A redless analogy: If your quantum field was a lake, then particle-like excitations behave like small ripples on the surface. But the lake is coupled to another field" which is the ripples of the ground beneath it. In real situations, the ground is relatively flat, lake and ripples behave as usual.

But now let's give the ground a nontrivial structure:

Now we have a waterfall, and the behavior of ripples will definitely be altered.

In string theory gauge fields arise as open strings connecting Dp-branes. For a collection of N coincident Dp-branes, we get N² different string acting as the generators of \( U(N) \).

These are really coincident.

Since stretched strings get ripples, this breaks \( U(3) \rightarrow U(2) \times U(1) \)

This distance is the Higgs field!