Feynman Rules for QED

1. Draw diagram and label matter lines w/ arrows to distinguish particles/anti: e.g., electron, positron

\[ \begin{array}{c}
\text{e}^- \rightarrow \text{e}^+ \\
\text{e}^+ \rightarrow \text{e}^-
\end{array} \]

2. Each external line gets a factor according to:

\[ \beta_i \]

Number factors w/ indices, e.g., \( \beta_i \rightarrow \beta_i(1) \)

Note: Each factor is a 4spins or 4vector.

3. Each vertex gets a factor: \( ig_\mu \delta_{\lambda\mu} \) (\( g_\mu = c \sqrt{\frac{\hbar}{2m^2}} \)) Note: Overall spin matrix.

4. Each internal line gets a factor: Electrons/Positrons \( \frac{i(\hbar g_{\mu\nu} + m c^2)}{\hbar c} \) Overall spin matrix.

\[ \begin{array}{c}
\text{Photons} \\
\frac{-i\gamma_{\mu\nu}}{\hbar c} \quad \text{Overall tensor}
\end{array} \]

(\( \gamma_{\mu\nu} \) match vertex indices)

5. Conserve momentum at each vertex w/ \( (2\pi)^3 \delta^4 \left( p_i + q_i - p_{out} - q_{out} \right) \)

6. Integrate over internal momenta w/ \( \int \frac{d^4k}{(2\pi)^4} \) for each \( q \).

7. Cancel overall \( (2\pi)^3 \delta^4 \left( p_i - p_{out} \right) \) and \( \gamma_{\mu\nu} \) to get \( \mathcal{M} \).

8. Anti-symmetrize between diagrams related by switching: 2 incoming electrons/positrons; 2 outgoing electrons/positrons or one incoming electron/positron with one outgoing positron/electron.

9. To get the order right for spinor elements (since we suppress indices) make “spinor sandwiches” from matter lines” by starting w/ an outgoing matter particle and tracing it back along pure matter lines inserting polarization and vertex factors as needed. The photon factors are less tricky since index notation gets them right.
Consider $c^4 \to e^4 e^c$:

1. $e^4 \to e^c$

\[ h = h_1 \pm h_2 \]

2. $e^c \to e^4$

\[ h = h_3 + h_2 \]

\[ M_1 \]

\[ M_2 \]

Switch one outgoing $e$ with one incoming $e$.

This is the same as in case 1.

So $h = h_1 - h_2$.
Consider $e^a \mapsto e^a$.

\[
H = \alpha(3) \gamma^2 \gamma^3 \alpha(4) \gamma^2 \gamma^4 \alpha(1) \left( \frac{-i \gamma^5}{(\lambda - \mu)^2} \right) i
\]

Once we have $H$, we can evaluate it given the following:

\[
\alpha(i) = \alpha(i) \gamma^3 \gamma^4 \gamma^1 \gamma^2
\]

However, in most experiments we average over $\alpha_i, \beta_i, \gamma_i, \delta_i$ and sum over $\alpha, \beta, \gamma, \delta$.

To do the average, we first note that $H$ contains terms of the form:

\[
H = \sum \alpha \gamma^a \gamma^b \gamma^c \gamma^d ...
\]

some combination of spin matrices

\[
\text{label to distinguish from other cases}
\]

\[
\text{spin sandwich}
\]

Then:

\[
\sum_{\alpha \gamma^a \gamma^b \gamma^c \gamma^d} \alpha(\gamma^a) \gamma^b \gamma^c \gamma^d = \alpha(\gamma^a) \gamma^b \gamma^c \gamma^d \left[ \alpha(\gamma^a) \gamma^b \gamma^c \gamma^d \right]^{*} ...
\]

This being a number in spin space as we can freely transpose it to form $+$. \n
\[
= \alpha(\gamma^a) \gamma^b \gamma^c \gamma^d \left[ \alpha(\gamma^a) \gamma^b \gamma^c \gamma^d \right]^{*}
\]

\[
\alpha(\gamma^a) \gamma^b \gamma^c \gamma^d = \alpha(\gamma^a) \gamma^b \gamma^c \gamma^d \left( \alpha(\gamma^a) \gamma^b \gamma^c \gamma^d \right)^{*}
\]

\[
= \alpha(\gamma^a) \gamma^b \gamma^c \gamma^d \left( \alpha(\gamma^a) \gamma^b \gamma^c \gamma^d \right)^{*}
\]

\[
\text{Now to average the first step will be to sum so we can use } \sum_{\alpha \gamma^a \gamma^b \gamma^c \gamma^d} = \gamma^a \gamma^b \gamma^c \gamma^d = \gamma^a \gamma^b \gamma^c \gamma^d
\]

\[
\sum_{\alpha \gamma^a \gamma^b \gamma^c \gamma^d} \alpha(\gamma^a) \gamma^b \gamma^c \gamma^d \left[ \alpha(\gamma^a) \gamma^b \gamma^c \gamma^d \right]^{*} ...
\]

Now we can use that $\gamma^a \gamma^b \gamma^c \gamma^d = \gamma^a \gamma^b \gamma^c \gamma^d$

\[
\sum_{\alpha \gamma^a \gamma^b \gamma^c \gamma^d} \alpha(\gamma^a) \gamma^b \gamma^c \gamma^d \left[ \alpha(\gamma^a) \gamma^b \gamma^c \gamma^d \right]^{*} ...
\]

Now sum over $\alpha$ to get:

\[
\sum_{\alpha \gamma^a \gamma^b \gamma^c \gamma^d} \alpha(\gamma^a) \gamma^b \gamma^c \gamma^d \left[ \alpha(\gamma^a) \gamma^b \gamma^c \gamma^d \right]^{*} ...
\]
If at some point we had summed over \( \bar{w} \rightarrow \psi - nc \).

Note: There are no spinors left in the expression! Only \( \bar{w} \)'s and \( \bar{\psi} \) 's.

\[
\text{To impose sum over } \bar{w}, \bar{\psi} \text{ replace } \bar{w}(a) \Gamma_{\nu} u(b) \left[ \bar{u}(a) \Gamma_{\rho} u(b) \right]^* \rightarrow \text{Tr} \left[ \gamma_{\nu} (\nu_c + h_c) \gamma_{\rho} (\rho_c + h_c) \right]
\]

Let's put this result to work:

\[
e + n \rightarrow e + n
\]

\[
\begin{align*}
H &= -\frac{q^2}{(\hbar - \beta)^2} \bar{u}(3) Y^\lambda u(1) \bar{u}(4) Y^\mu u(2) g_{\lambda \mu} \\
1 &= \left( \frac{q^2}{(\hbar - \beta)^2} \bar{u}(3) Y^\lambda u(1) \bar{u}(4) Y^\mu u(2) g_{\lambda \mu} \left[ \bar{u}(3) Y^\lambda u(1) \bar{u}(4) Y^\mu u(2) g_{\lambda \mu} \right]^* \right)
\end{align*}
\]

\[
\text{Tr} \left[ \gamma_{\nu} (\nu_c + h_c) \gamma_{\rho} (\rho_c + h_c) \right]
\]

\[
\langle \psi^{(1)} \rangle = \frac{q^2}{(\hbar - \beta)^2} \text{Tr} \left[ \gamma_{\nu} (\nu_c + h_c) \gamma_{\rho} (\rho_c + h_c) \right] \text{Tr} \left[ \gamma_{\nu} (\nu_c + h_c) \gamma_{\rho} (\rho_c + h_c) \right] g_{\lambda \mu} g_{\nu \rho}
\]