Why you [are] matter.

When discussing what “matter” fills our universe, for practical purposes it is sufficient to count the number of baryons, i.e. 3-quark bound states. These are largely in the form of protons and neutrons. Of course, the total matter in the universe is largely neutral so we know that for every proton there must exist a negatively charged particle, but these are usually electrons whose tiny mass relative to protons and neutrons makes them relatively inconsequential. BTW, we only need to be concerned with electrons, protons and neutrons because these are long lived – well so are neutrinos, but they are even less inconsequential then electrons. This also means that we need not worry about any mesons.

Okay, so let’s compare: # baryons in universe = \(10^8\) vs. # antibaryons in universe \(\approx 0\)

Baryon Asymmetry of the Universe

Why is this a problem?

The first important observation is that despite this being a very big problem, it is actually a very tiny problem, which makes it a big-tiny problem kinda like a black hole... just kidding.

In all seriousness, even though the difference seems huge, if we trace back through the history of the universe we find that at early times (~ a billionth of a second after the Big Bang) that the difference was actually only one part per billion! That is, at this time, for every billion +1 baryons, there were a billion antibaryons. Since then, the baryons have annihilated leaving us... the 0.0000001% behind!

So this is a tiny problem? No, it’s a big problem. It turns out that big numbers and 0 are often easy to explain. It’s the very tiny numbers that are tricky.

Okay, so perhaps the universe was born baryon-excess, you know a bit wobbly since its baryon leg was a bit longer than its antibaryon leg. Nope. It is highly doubtful that the universe was born with this asymmetry. Rather, the expectation is that it was born beautiful and symmetric, but through some later process became asymmetric.
So the challenge is to find a scenario where, starting from equal number of baryons and antibaryons, we generate an asymmetry.

In 1967, Sakharov enumerated a set of conditions that would have to be met for this to even be possible.

1) Baryon number violation
2) C and CP violation
3) Departure from thermal equilibrium

We will discuss each of these in detail and along the way provide suggested means by which they are achieved.

**Baryon number violation**

First, let’s define baryon number:

$\text{Each baryon } (B) \text{ gets } +1$

$\text{Each antibaryon } (\bar{B}) \text{ gets } -1$

For a collection of $B$ and $\bar{B}$, we add to get the total.

- Case: Symmetric universe $B_{\text{tot}} = 0$
  - Our current universe $B_{\text{tot}} \approx 10^{80}$

Baryon # conservation would imply that (taking the system to be the entire universe for which there is no where for things to come from or exit to) the total baryon # is constant.

But if the universe was born w/ $B_{\text{tot}} = 0$, then getting to $B_{\text{tot}} \approx 10^{80}$ requires non-conservation.

Now the tricky part is that if we take the SM Lagrange and all of the Feynman diagram vertices that it predicts, we discover that it is impossible to draw a diagram that connects a set of incoming states w/ $B_{\text{tot}} = 0$ to a set of outgoing states w/ $B_{\text{tot}} \neq 0$.

Hence, we say that the SM conserves baryon number... well at least perturbatively.

Recall that Feynman rules are a perturbative treatment of small fluctuations.

But we already know that non-perturbative effects play a role in the SM... I’m looking at you, Higgs!

So while Feynman diagrams happily describe:

they are not capable of describing:

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It turns out that in the 5th Lagrangian a non-perturbative effect that violates baryon number does arise. In fact:

\[ \mathcal{L}(t) \]

If it tunnels, this is an instanton process. Alternatively, if we jiggly things enough that they can get over the hump, then it is a sphaleron process.

Let's do jiggling \( \Rightarrow \) high energy density \( \Rightarrow \) early universe!

So we can violate baryon number conservation.

(Indeed it conserves \( B-L \) so if it creates 3 baryons, it also creates 3 anti-leptons)

These sphaleron processes are part of the electroweak sector of the 5th \( \mathcal{L} \).
One big problem with the story so far is that the arrows go both ways. That is, we can have $\pi^+ \rightarrow \pi^+ + \pi^+$ but also $\pi^- \rightarrow \pi^- + \pi^-$. Moreover, both processes have the same amplitude. So if we insist with $\Delta Q = 0$, then we would expect that for every $\pi^+ \rightarrow \pi^+$ process we would also have a $\pi^- \rightarrow \pi^-$ process canceling it out.

One way to relate the $\pi^+ \rightarrow \pi^+ + \pi^+$ process to the $\pi^- \rightarrow \pi^- + \pi^-$ is through charge conjugation ($C$). The transformation simply interchanges all particles with their antiparticle versions (and vice versa) for a process. Does reverse charge but also $\pi \rightarrow \bar{\pi}$.

$C: e + e \rightarrow \mu + \mu \Rightarrow e + e \rightarrow \mu + \mu$

It turns out that for many $\Delta Q$ processes, the $C$ version of any given process has the same amplitude as the original. You don't have to take my word for it! Indeed this is true for any electromagnetic or strong interaction.

However, the weak interactions give us $C$-violation, i.e. different rates for a process and its $C$-conjugated version.

Consider: $\pi^+ \rightarrow \mu^+ +\nu_{\mu}$
\[\text{Angular momentum:} \quad 0 \quad \text{left handed} \rightarrow \text{left handed}\]

Then: $\pi^- \rightarrow \mu^- +\bar{\nu}_{\mu}$
\[\text{Angular momentum:} \quad 0 \quad \text{left handed} \rightarrow \text{left handed}\]

We never see this!! $H = 0$.

This is a pure maximal violation of $C$!

Now if we imagine that this effect also carries over to the $\Delta Q$-violating sphaleron processes (which we electromagnetically remember) then we could imagine:

$\text{Beq = 0} \Rightarrow \text{Beq + 3} \Rightarrow H \neq 0$

Note: This is not violating charge conservation, but rather charge conjugation!
However this is not the end of the story.

While we do not observe the C-conjugated version of $\pi^-$ decay, we do observe the $CP$-conjugated version, where $P$ is parity, which (among other things) inverts handedness. 

\[
\begin{array}{ccc}
\pi^- & \rightarrow & \mu^- + \bar{\nu}_\mu \\
\text{left-handed} & \rightarrow & \text{right-handed}
\end{array}
\]

So we observe: 

\[
\pi^- \rightarrow \mu^- + \bar{\nu}_\mu \quad \text{and} \quad \pi^+ \rightarrow \mu^+ + \nu_\mu
\]

The problem now is that we could imagine: 

\[
\begin{array}{c}
B_{\text{tot}} = 0 \rightarrow \bar{B} + 3 \\
\text{L.H.} & \rightarrow \text{R.H.}
\end{array}
\]

The total baryon number does not distinguish L.H. from R.H. baryons, so again there could cancel.

So we need to go further and demand that not only $C$, but also $\bar{C}P$ be violated in the $SU(3)$ (beware the sphaleron processes). This is in fact the case as we will see when we study the weak interactions perturbatively. The difference is that while $C$ and $P$ violation is maximal in the weak interactions, $\bar{C}P$-violation is actually a very small effect. But hey... at least it's there.
3) Now the early universe is not a single interaction at a time, but rather a hot mess of particles colliding, appearing, disappearing, etc. This is where yet another issue arises.

Suppose we have a CP and B violating process \( B^{u} = 0 \rightarrow B^{u} + 3 \)

Well, if we start with \( B^{u} = 0 \) and start generating \( B^{u} \# 0 \), then suddenly the reverse process becomes important, i.e. \( B^{u} + 3 \rightarrow B^{u} = 0 \). In fact a state of equilibrium is such that sufficient quantities of everything is present that reactions happen on average in both directions at the same level.

So, it turns out that we need a departure from equilibrium which will allow a dominance of reaction rates in one direction over the other.

Fortunately, the B-violating process we encounter corresponds to a first-order phase transition which typically proceed via bubble nucleation; i.e. a localized region of the new phase (vacuum) is formed, i.e. the bubble, and then grows taking over the old vacuum regions or colliding with other bubbles. Think boiling!

Fortunately, a bubble wall corresponds to a highly non-equilibrium configuration and hence satisfies the 3rd and 4th Subbarao requirement.

While the pieces are there, the details are still to be completely worked. Non-perturbative calculations are hard. There are cosmological effects to take into account. This is still considered a not-completely solved problem in particle cosmology.
Here these be...

Often in math we can find arbitrarily complicated or large examples of structures we define. For example in group theory, when I consider $SO(n)$, I am free to take $n$ to be as large a positive integer as I like.

However suppose I gave you a construction indexed by some $N$, and then said that it only made sense for $N$ up to 12. That might seem weird, unless of course we had secretly limited the number 12 in the definitions. For example, if I asked what dimensions of spheres can be embedded in 13 dimensions. In that case I can embed $S^0, S^1, \ldots, S^{11}$ with corresponding rotation isometries $I, SO(2), SO(3), \ldots, SO(12)$. But this limit is intimately tied to the any having chosen the number of dimensions to be 13.

What is really weird is when our definitions do not rely on numbers that are in any way (or obvious way) related to maximal cases.

An interesting example happens in group theory. Consider finite (discrete) groups. A simple example is $G = \text{Tr}$: \[ I, R_{90}, R_{180}, \Gamma, \Gamma^{-1}, \{I, R_{90}, R_{180}, \Gamma, \Gamma^{-1}\} \]

We can readily identify some subgroups of $\text{Tr}$, e.g. $\{I, R_{90}, R_{180}\}, \{I, \Gamma, \Gamma^{-1}\}, \{I, \Gamma\}, \{I, \Gamma, \Gamma^{-1}\}$

It is often exceptionally useful to identify normal subgroups. A normal subgroup $H$ of $G$ is such that $g H g^{-1} = H$ for any $g \in G$. This does not mean $g H g^{-1} = H$ for all $h \in H$ but rather that $g H g^{-1}$ takes the whole set of $h \in H$ and returns the whole set again.

For $\text{Tr}$ we have a normal subgroup $\{I, \Gamma\}$ since $g \Gamma g^{-1} = \Gamma$ for any $g \in G$

e.g. $g = R_{180} \Rightarrow R_{180} \Gamma R_{180}^{-1} = \Gamma$

e.g. $g = R_{180} \Gamma R_{90} \Rightarrow \Gamma R_{90}$

e.g. $g = \Gamma R_{180} \Rightarrow \Gamma R_{90}$

Now for complicated reasons it is useful to define a simple group as a group $G$ for which the only normal subgroups are $H = I$ and $H = G$ itself.
Now an interesting question in mathematics was to try and classify all of the finite simple groups. An example is the cyclic group $\mathbb{Z}_n$ “generated” by $g$ s.t. $g^n = 1$. So the elements are $\{1, g, g^2, \ldots, g^{n-1}\}$. Clearly this can be generalized to arbitrarily large $n$! Notice the order is determined by $n$, i.e., $|\mathbb{Z}_n| = n$.

In classifying all finite simple groups it was determined that there are 18 families of these, each family consisting of similar definitions but with 1 or more discrete parameters to adjust. For example $\mathbb{Z}_n$ is a finite simple group w/ one parameter $n$. In all cases the order is determined by the parameter choices.

So far this is not that surprising. What are surprising are the so called “sporadic” groups. There are finite simple groups which do not belong to any of the 18 families, and which are “disconnected” in a parameter sense from each other.

The smallest (and one of the earliest understood) is $M_{11}$, i.e. the Hakeisen sharp 4-transitive group acting in 11 symbols. Its order is merely $|M_{11}| = 7920$.

What is particularly interesting is the largest of the sporadic groups, the so-called Monster group $M$. It is big $|M| = 8.08 \times 10^{53}$. The next largest is Baby-Monster w/ mere $10^{23}$ elements.

Why does such a large and isolated mathematical structure exist? This is still not completely understood.

But maybe some drinking will help...
The 3 smallest (complex) representations of \( V \) have dimensions 1, 196883, 21296876 respectively.

and you thought your matches were bad!

So what? Well consider something unrelated. Take the upper half complex plane \( \mathbb{H} \) and map it to the Riemann sphere \( \mathbb{C}^\infty/\mathbb{Z} \) using a function \( j(z) \).

There exists a function \( j(z) \) which is elliptic and \( \mathbb{Z}/\mathbb{Z} \)-modular whose Laurent expansion in terms of \( q = e^{2\pi iz} \) is given by:

\[
j(z) = \frac{1}{q} + 744 + 196884q + 212968760q^2 + \ldots
\]

Nothing anything weird? \[
196883 = \frac{196883}{1} = 196883
\]
\[
21296876 = 1 + 196883 + 21296876.
\]

Based on this observation, a conjecture was made that these two different constructions must somehow be related. But most considered the idea so far fetched it couldn’t be meaningful.

Here Monster Moonshine!

The final understanding of the connection was eventually understood through what I think is the most fascinating thing I have ever learned in physics.

The Monster Moonshine conjecture was eventually understood in part with the vertex operator algebras that appear in String Theory.

Now String Theory is cool... quantum gravity, extra dimensions, D-branes, dualities, etc. But what is most amazing to me are its \\*finite*.

For QFT (based on 0-dimensional particle like excitations) we can have consistency with any gauge groups \( SU(N), SO(N) \) w/ \( N \in 1, 5, 8 \ldots \). They make sense in any dimension.

But the moment we move to stringy (10-1) suddenly weird numbers appear, e.g. dimensions of 10, 11 and 25 become special. Gauge groups w/ 278 and 476 become preferred.