Figure 1: A sequence $x[n]$ and its amplitude spectrum. The sequence $x[n]$ is not aliased and consists of 101 samples of a continuous signal $x_c(t)$.

Question 1. ...........................................................................................................(8 points)

(a) [2 points] What is the time sampling interval $T$, in seconds?

(b) [2 points] What is the Nyquist frequency, in Hz?

(c) [2 points] What is the frequency (in Hz) of the periodic fluctuations in $x[n]$.

(d) [2 points] Label the frequency axis below the amplitude spectrum.
Question 2. Consider the moving average filter with system function \( H(z) = \frac{1}{5}(z^2 + z + 1 + z^{-1} + z^{-2}) \).

(a) [2 points] Sketch the impulse response \( h[n] \) of this system.

(b) [2 points] Use a well-known formula to rewrite \( H(z) \) as the ratio of two polynomials in \( z \).

(c) [2 points] For what two frequencies \( \omega \) between 0 and \( \pi \) is the amplitude response \( A(\omega) \) of this filter zero?

(d) [3 points] Assume that the sequence displayed in Figure 1 is \( x[n] = cn + \sin(\omega_0 n) \), for some constant \( c \) and frequency \( \omega_0 \). (You should already have determined the frequency \( \omega_0 \).) For this input \( x[n] \), and ignoring samples near the ends, what is the output \( y[n] \) of this moving average filter? Hint: the input \( x[n] \) to this LTI system is the sum of two sequences.

(e) [2 points] Write the main loop of a computer program that implements this filter.
Question 3 ................................................................. (12 points)
Design a causal time-domain notch filter to attenuate the periodic fluctuations in $x[n]$ of Figure 1, as follows:
(a) [2 points] What is the frequency to be attenuated, in radians/sample?

(b) [2 points] Sketch the locations of filter poles and zeros in the complex z-plane.

(c) [2 points] What is the system function $H(z)$ for your filter? (Include the region of convergence.)

(d) [2 points] Modify your system function $H(z)$ so that your filter does nothing at frequency zero (DC).

(e) [2 points] Write a constant-coefficient difference equation relating filter output $y[n]$ to input $x[n]$.

(f) [2 points] Write the main loop of a computer program that implements your filter.
Question 4 ................................................................. (10 points)
Assume that you are given a discrete Fourier transform $X[k]$ of the sequence $x[n]$ of Figure 1. Also assume that a fast Fourier transform (FFT) was used, and that the number of samples after padding $x[n]$ with zeros was $N = 128$.

(a) [2 points] Why typically are only positive frequencies $\omega$ sampled in the array $X[k]$?

(b) [2 points] Assuming that only positive frequencies $\omega$ are sampled, how many values are provided in the array $X[k]$?

(c) [2 points] What is the frequency sampling interval $\Delta \omega$, in radians/sample?

(d) [2 points] What is the index $k_0$ of the sample corresponding to the frequency nearest that of the periodic fluctuations in the sequence $x[n]$?

(e) [2 points] Describe in words how you would use this index $k_0$ to implement a linear time-invariant filter that attenuates the periodic fluctuations in the sequence $x[n]$?
Question 5 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . (5 points)
Given the sequence \( x[n] = x_c(nT) \) from Figure 1, ...

(a) [3 points] Implement the transformation \( y_c(t) = x_c(\sqrt{t}) \). That is, write an expression that exactly defines a new output sequence \( y[n] = y_c(nT) \) in terms of the samples of the input sequence \( x[n] \).

(b) [2 points] In practice, we often sacrifice precision for efficiency in such transformations. How would you modify your computation of \( y[n] \) to make it faster, while still approximately correct?

Question 6 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . (4 points)
Give an example of \( H(z) \) (system function with ROC) for a linear time-invariant system that is ...

(a) [2 points] Stable but not causal.

(b) [2 points] Causal but not stable.