Question 1 …………………………………………………………………………………. (8 points)

(a) Label the time axis, with units of seconds.

(b) What is the Nyquist frequency, in Hz (cycles per second)?

(c) In the amplitude spectrum, the minimum frequency plotted is zero. The maximum frequency at the far right of the frequency axis is not the Nyquist frequency. Label the frequency axis, with units of Hz.

(d) At what frequency (in Hz) is the high-frequency noise apparent in the amplitude spectrum?
Question 2 ................................................................. (8 points)

Consider frequency-domain filtering to attenuate the high-frequency noise in the sequence $x[n]$ of Figure 1. Assume that we will use length $N = 1008 = 16 \times 9 \times 7$ for the FFT.

(a) Why not instead use the prime number $N = 1009$ for the FFT?

(b) Why not instead use the number $N = 504 = 9 \times 8 \times 7$ for the FFT?

(c) The sequence $x[n]$ is given for only $n = 0, 1, \ldots, 500$. Before FFT, what values should we put in the array for indices $n = 501, 502, \ldots, 1007$?

(d) Assume that we have performed the FFT to obtain complex numbers $X[k]$, where $k$ is the frequency sampling index. What value of $k$ most nearly corresponds to the frequency of the high-frequency noise?

Question 3 ................................................................. (6 points)

Consider the two-sided exponential smoothing filter with symmetric impulse response $h[n] = (\frac{1}{3})^{|n|}$. For this filter, specify the corresponding system response $H(z)$, with region of convergence, and the locations of all poles and zeros.
Question 4 ................................................................. (20 points)

Consider a linear time-invariant system with response \( H(z) = 1 - z^{-2} \).

(a) Sketch the impulse response for this system.

(b) Is this system causal? Is it stable? For both questions, explain your answers.

(c) What is the frequency response \( H(\omega) \) for this system?

(d) Sketch the amplitude and phase responses \( A(\omega) \) and \( \phi(\omega) \) for this system for \(-\pi < \omega < \pi\). Label axes carefully.

(e) Where are the poles and zeros for this system?

(f) What is the region of convergence for the system response \( H(z) \)?
(g) If input $x[n] = 3$ (is constant for all $n$), what is the output $y[n]$?

(h) Write a computer program fragment that for this system computes an output array of $N_t$ samples $y[n]$ from an input array of $N_t$ samples $x[n]$, for $n = 0, 1, 2, \ldots, N_t - 1$.

Question 5 ................................................................. (8 points)
   Consider the system $y[n] = x[4n]$ and its application to the sequence $x[n]$ of Figure 1.
   (a) Prove that this system is linear.

   (b) Prove that this system is not time-invariant.

   (c) For the output sequence $y[n]$, what is the Nyquist frequency (in Hz)?

   (d) In the output sequence $y[n]$, at what low frequency (less than 30 Hz) will the aliased high-frequency noise be apparent?