Question 1 ................................................................. (8 points)
Consider the sequence $x[n] = \sin(2\pi fn)$.

(a) [4 points] If $x[n]$ is periodic with period $N = 20$ samples, what is the frequency $f$ (in cycles per sample)?

(b) [2 points] Specify a second frequency $f$ that yields the same sequence $x[n]$.

(c) [2 points] Specify a frequency $f$ for which the sequence $x[n]$ is not periodic.

Question 2 ................................................................. (5 points)
Let $h[n] = 2\delta[n+1] - \delta[n] - \delta[n-1]$, where $\delta[n]$ denotes the unit-impulse sequence. Let $x[n] = u[n] - u[n-3]$, where $u[n]$ denotes the unit-step sequence. Sketch the sequences (a) $h[n]$, (b) $x[n]$, and (c) $y[n] = h[n] \ast x[n]$, where $\ast$ denotes convolution. (Label axes.)

Question 3 ................................................................. (5 points)
Given only the impulse response $h[n]$ of an LTI system, how can you

(a) [3 points] compute the output $y[n]$ for any input $x[n]$?

(b) [2 points] determine whether the system is stable?
Question 4 .............................................................. (6 points)
Given that the discrete-time Fourier transform (DTFT) of \( x[n] \) is \( X(\omega) \), prove that
(a) [3 points] the DTFT of \( x[n+3] \) is \( X(\omega)e^{j\omega 3} \).
(b) [3 points] the DTFT of \( x[-n] \) is \( X(-\omega) \).

Question 5 .............................................................. (8 points)
(a) [4 points] Describe in words (not a computer program) a non-linear system
\( y[n] = T\{x[n]\} \) that removes isolated noise spikes from any sequence \( x[n] \).
(b) [4 points] For inputs \( x_1[n] = \delta[n] \), \( x_2[n] = u[n-1] \), and \( x[n] = x_1[n] + x_2[n] \),
sketch the corresponding outputs \( y_1[n] \), \( y_2[n] \), and \( y[n] \) for your system, and thereby prove that your system is non-linear.
Consider a stable system described by the constant-coefficient difference equation
\[ y[n] - \frac{1}{4} y[n-1] = 2 x[n-2]. \]

(a) [3 points] Is this system linear? Time-invariant? Causal?

(b) [3 points] Sketch the impulse response \( h[n] \) for this system. (Label axes.)

(c) [3 points] What is the frequency response \( H(\omega) \) of this system?

(d) [3 points] How would you modify the scale factor 2 for \( x[n-2] \) in this system so that the DC response \( H(0) = 1 \).

(e) [3 points] Assume a bounded input sequence \( x[n] \) such that \( |x[n]| < 1 \) for all \( n \). For such an input sequence, and for the original unmodified system above, what is the bound on \( |y[n]| \) for the output sequence \( y[n] \)?

(f) [3 points] Write computer code to implement the original unmodified system. That is, write code to compute \( y[n] \) for \( n = 0, 1, 2, ..., N - 1 \), given input \( x[n] \) for \( n = 0, 1, 2, ..., N - 1 \).