Question 1 ......................................................... (10 points)
Consider each of the following sampled sequences. Determine whether or not the sequence is periodic. If so, specify the period $N$.

(a) $\sin(\pi n)$

(b) $\sin(\pi n/4)$

(c) $\sin(3\pi n)$

(d) $\sin(3\pi n/4)$

(e) $\sin(3n/4)$

Question 2 ......................................................... (10 points)
Consider the following systems $y[n] = T \{x[n]\}$. For each system, specify whether or not it is linear (L), time-invariant (TI), and/or stable (S).

(a) $y[n] = 2x[n]$

(b) $y[n] = 2x[n] - 2$

(c) $y[n] = x[2n]$

(d) $y[n] = x[2 - n]$

(e) $y[n] = x[n - 2]$
Question 3 .............................................................. (20 points)

You are given two arrays $x$ and $y$, both of length $nt$. The array $x$ contains an input sequence of values and the output array $y$ initially contains some arbitrary (not zero) random values that you will replace.

(a) Using any programming language, write a computer program fragment that implements an approximation to a derivative system $y(t) = x'(t)$.

(b) Specify the name of the programming language that you used and the type of finite-difference approximation that your system implements.

(c) For output samples near bounds of the array $y$, what values does your system assume for input samples outside bounds of the array $x$?

(d) Is your system linear? time-invariant? causal? stable? If stable, specify a bound $B_y$ on the output in terms of a corresponding bound $B_x$ on the input. If not stable, why not?

(e) Sketch the unit-impulse response of your system, the output $y[n] = h[n]$ for input $x[n] = \delta[n]$.

(f) Sketch the output $y[n]$ of your system for input $x[n] = u[n - 3]$, where $u[n]$ denotes the unit step sequence.

(g) Sketch the output $y[n]$ of your system for input $x[n] = \delta[n - 1] + u[n - 4]$. 
Question 4 ........................................................................................................ (5 points)

Let $x[n]$ and $y[n]$ denote system input and output, respectively, and consider two linear time-invariant systems with impulse responses $h_1[n] = u(n) - u(n - 4)$ and $h_2(n) = h_1(-n)$.

(a) Sketch the impulse responses of these two systems.

(b) Sketch the impulse response of a composite system $y[n] = h_2[n] * h_1[n] * x[n]$.

Question 5 ........................................................................................................ (5 points)

Consider the linear time-invariant system with impulse response $h[n] = (2/3)^n u(n)$.

(a) Write a linear constant-coefficient difference equation that relates the system output $y[n]$ to the input $x[n]$.

(b) Show that this system is stable, by specifying the bound $B_y$ on the output in terms of the bound $B_x$ on the input.