Question 1 ................................................................. (2 points)
Sketch the impulse response of any digital filter with an exactly linear non-zero phase response.

Question 2 ................................................................. (3 points)
Sketch the impulse response of any digital filter that has an impulse response with exactly three non-zero samples and a zero-phase response.
Question 3 .......................................................... (18 points)
Consider the LTI digital filter defined by $H(z) = \frac{1}{2} + \frac{1}{2}z^{-2}$.

(a) [2 points] Sketch the impulse response $h[n]$ of this filter. (Label axes.)

(b) [3 points] Sketch the locations of the poles and zeros of $H(z)$ in the complex $z$-plane.

(c) [2 points] What is the region of convergence (ROC) for $H(z)$?

(d) [2 points] Is this system causal? Stable? Why or why not?

(e) [2 points] What is the frequency response $H(\omega)$ of this filter?

(f) [3 points] Sketch the amplitude response $A(\omega)$ for $-\pi \leq \omega \leq \pi$. (Label axes.)

(g) [2 points] Sketch the phase response $\phi(\omega)$ for $-\pi \leq \omega \leq \pi$. (Label axes.)

(h) [2 points] When applied to a digital signal with sampling interval $T = 0.5$ s, what frequency (in Hz) is most attenuated by this filter?
Question 4 ................................................................. (14 points)
Consider the causal LTI digital filter defined by \( H(z) = (1 + z^{-2})/(1 + 0.81z^{-2}) \).

(a) [3 points] Sketch the locations of the poles and zeros of \( H(z) \) in the complex \( z \)-plane.

(b) [2 points] What is the region of convergence for \( H(z) \)?

(c) [2 points] Is this system stable? Why or why not?

(d) [3 points] Sketch the amplitude response \( A(\omega) \) for \(-\pi \leq \omega \leq \pi\). (Label axes.)

(e) [2 points] Sketch the phase response \( \phi(\omega) \) for \(-\pi \leq \omega \leq \pi\). (Label axes.)

(f) [2 points] To implement this filter, what difference equation would you solve?
Question 5 ................................................................. (13 points)
Assume that \( x[n] = x_c(nT) \) is a non-aliased sequence obtained by uniformly sampling a continuous bandlimited signal \( x_c(t) \) with sampling interval \( T = 0.01 \text{ s} \). (The time of first sample is zero.) By “bandlimited”, we mean that the Fourier transform of \( x_c(t) \) is zero for frequencies greater than some maximum frequency \( F_m \), in Hz.

(a) [2 points] For the specified sampling interval \( T \), what is the Nyquist frequency (in Hz)?

(b) [2 points] What is an upper bound for the maximum frequency \( F_m \) (in Hz)?

(c) [2 points] If \( y_c(t) \equiv x_c(2t) \), how would you compute the corresponding sequence \( y[n] = y_c(nT) \) from \( x[n] \)? (Express your answer without a sinc function.)

(d) [2 points] For the specified sampling interval \( T \), what maximum frequency \( F_m \) for \( x_c(t) \) will ensure that \( y[n] \) is not aliased?

(e) [3 points] If \( z_c(t) \equiv x_c(t - T/3) \), how would you compute the corresponding sequence \( z[n] = z_c(nT) \) from \( x[n] \)? (Express your answer with a sinc function.)

(f) [2 points] For the specified sampling interval \( T \), what maximum frequency \( F_m \) for \( x_c(t) \) will ensure that \( z[n] \) is not aliased?