Question 1 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . (22 points)

Consider a continuous signal $x_c(t)$ with the following amplitude spectrum:

The spikes in the spectrum at $\pm 60$ Hz are caused by power line noise. Now assume that this signal is sampled uniformly with interval $T = 0.01$ s to obtain a sequence $x[n]$.

(a) [2 points] What is the sampling frequency $F_s$, in Hz?

(b) [2 points] What is the Nyquist frequency $F_N$, in Hz?

(c) [4 points] Sampling in time causes replication in frequency. Sketch the amplitude spectrum implied by sampling. (Label any frequencies that are important in your sketch.)
(d) [2 points] Consider frequencies $F$ only in the interval $|F| < F_N$. After sampling, the noise appears to be at what frequencies (in Hz)?

(e) [2 points] Convert the noise frequencies $F$, in Hz, to frequencies $f$ in cycles per sample.

(f) [4 points] Determine and sketch the locations of two poles and two zeros for a simple digital filter that would eliminate the noise, while having little effect on other frequencies in the signal.

(g) [2 points] Write a difference equation \textit{with real coefficients} for your two-pole, two-zero filter that relates input $x[n]$ to output $y[n]$.

(h) [4 points] Given the noise-free sequence $y[n]$, how would you best implement the transformation $z_c(t) = y_c(\sqrt{t})$ in a digital system? That is, write an expression for a sequence $z[n] \equiv z_c(nT)$ in terms of the sequence $y[n]$.
Question 2. Find $z$-transforms $X(z)$, including the regions of convergence, of the following sequences:

(a) [3 points] $x[n] = \delta[n - 3]$

(b) [3 points] $x[n] = \left(\frac{1}{3}\right)^n u[n]$

(c) [3 points] $x[n] = \left(\frac{1}{3}\right)^{n+2} u[n + 2]$

(d) [3 points] $x[n] = \left(\frac{1}{3}\right)^n u[n + 2]$

(e) [3 points] $x[n] = 3^n u[-n - 1] + \left(\frac{1}{3}\right)^n u[n]$
Question 3 ................................................................. (18 points)

Consider a system with z-transform

\[ H(z) = 1 - z^{-3}, \quad |z| > 0. \]

(a) [2 points] How many zeros are in this system? How many poles?

(b) [4 points] Plot the poles and zeros in a sketch of the complex z-plane.

(c) [3 points] Sketch the amplitude spectrum of this system for frequencies \(-\pi < \omega < \pi\).

(d) [2 points] Write a difference equation for this system.

(e) [2 points] Is this system stable? Why or why not?

(f) [2 points] Sketch the impulse response of this system.

(g) [3 points] Sketch the output sequence \(y[n]\) of this system for the input sequence \(x[n] = u[n]\). (In other words, sketch the step response of this system.)