Question 1. ................................................................. (18 points)
Find $z$-transforms $X(z)$, including the regions of convergence, of the following sequences:

(a) [3 points] $x[n] = \delta[n - 2]$

(b) [3 points] $x[n] = 2\delta[n + 2] - 3\delta[n - 2]$

(c) [3 points] $x[n] = \left(\frac{1}{4}\right)^n u[n]$

(d) [3 points] $x[n] = \left(\frac{1}{4}\right)^{n+1} u[n + 1]$

(e) [3 points] $x[n] = \left(\frac{1}{4}\right)^n u[n + 1]$

(f) [3 points] $x[n] = 4^n u[-n - 1]$
Consider a system with $z$-transform

$$H(z) = 1 + z^{-2}, \quad |z| > 0.$$ 

(a) [2 points] How many zeros are in this system? How many poles?

(b) [4 points] Plot the poles and zeros in a sketch of the complex $z$-plane.

(c) [3 points] Sketch the amplitude spectrum of this system for frequencies $-\pi < \omega < \pi$.

(d) [2 points] Write a difference equation for this system.

(e) [2 points] Is this system stable? Why or why not?

(f) [2 points] Sketch the impulse response of this system.

(g) [3 points] If applied to a sequence $x[n] = x_c(nT)$ with sampling interval $T = 4$ ms, what frequency $F$ (in Hz) is most attenuated by this filter?
Question 3 .......................................................... (16 points)
Assume that you are given a sampled sequence \( x[n] = x_c(nT) \), where the sampling interval \( T = 4 \) ms.

(a) [2 points] What is the sampling frequency \( F_s \)?

(b) [2 points] What is the Nyquist frequency \( F_n \)?

(c) [4 points] Sketch the amplitude spectrum \( A(F) \) of a non-zero continuous (not yet sampled) signal \( x_c(t) \) for which the corresponding sampled sequence \( x[n] \) is not aliased. In your sketch, (1) label the frequency axis \( F \) with units of Hz, (2) include both negative and positive frequencies, and (3) indicate both the sampling frequency \( F_s \) and Nyquist frequency \( F_n \).

(d) [2 points] Make a similar sketch for which the sequence \( x[n] \) is aliased.

(e) [2 points] Assume that the sequence \( x[n] \) is not aliased, and sketch the amplitude spectrum \( A(\omega) \) of this sequence for frequencies \(-2\pi < \omega < 2\pi\). In this sketch, the units of frequency \( \omega \) are radians/sample.
(f) [4 points] Assume that the sequence \( x[n] = x_c(nT) \) is not aliased, and write
a computer program fragment with two loops (one nested inside the other) that will compute a new sequence \( y[n] = x_c(t_0 + nT) \), where \( t_0 = 1 \text{ ms} \).

```java
private static float sinc(float x) {
    // assume this method exists
}
...
int nt = sx.getCount();
float dt = (float)sx.getDelta();
float t0 = 0.001f;
float pi = (float)Math.PI;
// TODO: good (but slow) implementation of sinc interpolation
```

Question 4 ........................................................................................................ (3 points)

After his most recent haircut, how much did Dr. Hale tip his barber?