Question 1 ............................................. (10 points)

Find (you need not sketch) impulse responses $h[n]$ for linear time-invariant systems with the following system responses $H(z)$:

$$H(z) = \left(1 + 3z\right)\left(1 - 2z^{-1}\right) \quad ; \quad 0 < |z| < \infty$$

$$H(z) = \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{3}z^{-1}\right)} \quad ; \quad \frac{1}{2} < |z|$$
Question 2. ......................................................................................... (15 points)

For input and output sequences $x[n]$ and $y[n]$, consider the linear time-invariant system:

$$y[n] = \frac{1}{2} x[n + 1] + x[n] + \frac{1}{2} x[n - 1]$$

(a) Sketch the impulse response $h[n]$ of this system. (Label axes.)

(b) Sketch the frequency response $H(\omega)$ for $-\pi \leq \omega \leq \pi$. (Label axes.)

(c) What is the system response $H(z)$ for this system? (Include the ROC.)

(d) Sketch locations of pole(s) and zero(s) in the complex $z$-plane.

(e) Is this system causal? Why or why not?

(f) Is this system stable? Why or why not?
Question 3 ................................................................. (13 points)

Assume that the maximum frequency in a continuous signal \( x_c(t) \) is 8 Hz, and that we have sampled this signal to obtain a sequence \( x[n] \) with sampling interval \( T_x = 0.05 \) s.

(a) For the sequence \( x[n] \), what is the
   • sampling frequency \( F_S \) (in Hz)?
   • Nyquist frequency \( F_N \) (in Hz)?

(b) Is the sequence \( x[n] \) aliased? Why or why not?

(c) For frequencies \( F \) in the interval \([-F_N, F_N]\), sketch a possible Fourier transform \( X(F) \) for the sequence \( x[n] \). Include in your sketch some non-zero amplitude at the maximum frequency 8 Hz.

(d) Suppose we resample \( x[n] \) to create a new sequence \( y[n] \) with sampling interval \( T_y = 0.08 \) s. Write an equation that shows how you would compute the sequence \( y[n] \) from the sequence \( x[n] \).

(e) For the new sequence \( y[n] \), what is the
   • sampling frequency \( F_S \) (in Hz)?
   • Nyquist frequency \( F_N \) (in Hz)?

(f) Is the new sequence \( y[n] \) aliased? Why or why not?

(g) Sketch the Fourier transform \( Y(F) \) for the new sequence \( y[n] \).
Question 4 .................................................... (12 points)
Assume that we want to eliminate noise at frequency 25 Hz from a sequence $x[n]$ for which the time sampling interval is 0.01 s. We will design a notch filter with two zeros and two poles and the following linear constant-coefficient difference equation:

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] - a_1 y[n-1] - a_2 y[n-2]$$

(a) What is the Nyquist frequency (in Hz) for the input and output sequences?

(b) Sketch the locations of poles and zeros in the complex $z$-plane.

(c) Derive coefficients $b_0$, $b_1$, $b_2$, $a_1$ and $a_2$ for the difference equation to attenuate the noise at 25 Hz, while not altering any signal at 0 Hz.

(d) Suppose that we want to preserve only the noise at 25 Hz, while attenuating all other frequencies. How should we modify the coefficients of our difference equation to do this?