Question 1. Consider the digital sequence \( x[n] = \cos(\pi n/2) \).

(a) [2 points] Sketch this sequence. (Label axes.)

(b) [2 points] Suppose that we obtained this sequence \( x[n] \) by sampling a continuous-time signal \( x_c(t) \), with sampling interval \( T = 0.005 \text{ seconds} \). For this sampling interval, what is the Nyquist frequency, in Hz?

(c) [3 points] Suppose that our continuous-time signal \( x_c(t) \) is bandlimited to the frequency range \( 0 \leq |\Omega| < \Omega_N \), where \( \Omega_N \) is the Nyquist frequency in radians/second. Given the sampling interval \( T = 0.005 \text{ seconds} \), what is the continuous-time signal \( x_c(t) \)?

(d) [3 points] Now suppose that our continuous-time signal \( x_c(t) \) is bandlimited to the frequency range \( 400\pi \leq |\Omega| < 600\pi \text{ radians/second} \), and that our digital signal \( x[n] = \cos(\pi n/2) \) and sampling interval \( T = 0.005 \text{ seconds} \) are the same. What is the continuous-time signal \( x_c(t) \)?
Consider the filter $h[n] = \frac{1}{2}(\delta[n + 1] - \delta[n - 1])$.

(a) [2 points] Sketch this filter. (Label axes.)

(b) [3 points] What is the Z-transform $H(z)$ of this filter?

(c) [3 points] Where in the complex z-plane are the two poles and two zeros?

(d) [3 points] What is the frequency response $H(\omega)$ of this filter?

(e) [3 points] Sketch the amplitude response $A(\omega)$ of this filter. (Label axes.)

(f) [3 points] Sketch the phase response $\phi(\omega)$ of this filter. (Label axes.)
Consider the causal, continuous-time smoothing filter defined by:

\[ h_c(t) = \begin{cases} 
  e^{-\Omega_0 t} & ; \ t \geq 0 \\
  0 & ; \ t < 0 
\end{cases} \]

where \( \Omega_0 = \pi/2 \) radians/second, and \( t \) denotes time in seconds.

(a) [3 points] What is the frequency response (continuous-time Fourier transform) \( H_c(\Omega) \) of the continuous-time filter \( h_c(t) \)?

(b) [3 points] What is the amplitude response \( A_c(\Omega) = |H_c(\Omega)| \)?

(c) [3 points] Sketch the amplitude response \( A_c(\Omega) \), for frequencies \(-\pi \leq \Omega \leq \pi\). In your sketch, label the amplitudes at frequencies \( \Omega = 0 \) and \( \Omega = \Omega_0 = \pi/2 \) radians/second.
(d) [2 points] Create a digital filter $h[n]$ by sampling the continuous-time filter $h_c(t)$, with sampling interval $T = 1$ second. What is $h[n]$?

(e) [3 points] What is the frequency response (discrete-time Fourier transform) $H(\omega)$ of the digital filter?

(f) [3 points] What is the amplitude response $A(\omega) = |H(\omega)|$ of the digital filter?

(g) [3 points] Sketch the amplitude response $A(\omega)$, for frequencies $-\pi \leq \omega \leq \pi$. In your sketch, label the amplitudes at frequencies $\omega = 0$ and $\omega = \pi/2$ radians/sample.

(h) [3 points] For those who have not studied digital signal processing, sampling can yield surprises. In the example above, $h[n] = h_c(t = nT)$, exactly. Yet, the frequency responses $H(\omega)$ and $H_c(\Omega = \omega/T)$ are not equal, for any frequency $\omega$. [Compare, for example, $A(\omega = 0)$ with $A_c(\Omega = 0)$ in your sketches above.] Explain.