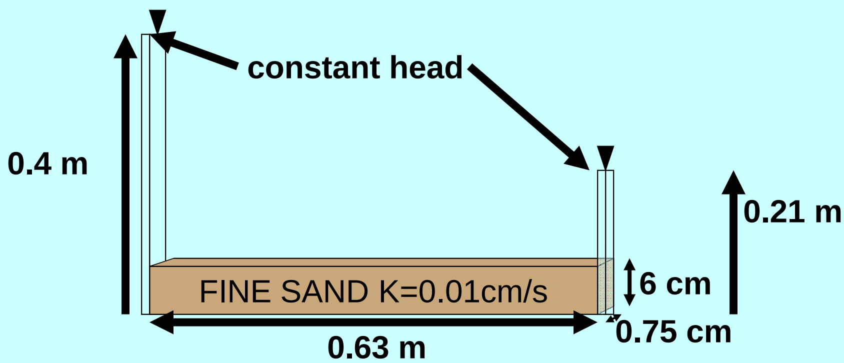
RECALL FIRST CLASS:

$$Q = K \, \frac{\text{Head Difference}}{\text{Distance between Heads}} \, \text{Area}$$

$$Q = 0.01 \, \frac{\text{cm}}{\text{sec}} \; \frac{0.19 \text{ m}}{0.63 \text{ m}} \; 6\text{cm} \; 0.75\text{cm} \; \frac{1 \text{ liter}}{1000\text{cm}^3} \; \frac{86400\text{sec}}{\text{day}} = 1.17 \, \frac{\text{liter}}{\text{day}} \sim \frac{1 \text{ liter}}{\text{day}}$$



For Water to Move
a driving force is needed

but it doesn't necessarily flow downhill
nor from high Pressure to low Pressure

Consider a siphon
What drives the flow in that case?

In a isothermal system of uniform electrochemical composition:

Flow Proceeds from High to Low Hydraulic Head i.e. from locations of high to low mechanical energy

Total mechanical energy depends on Fluid Pressure, Gravity, and Motion

$$E_{total} = P + \rho g z + \frac{1}{2}\rho v^2$$

Divide by density to get energy per unit mass

$$E_{unit\ mass} = \frac{P}{\rho} + gz + \frac{v^2}{2}$$

The Bernoulli Equation from Fluid Mechanics

States that for steady, laminar, flow of frictionless, incompressible fluid energy per unit mass $[L/T]^2$ is a constant
(in practice these assumptions can be relaxed)

$$E_{unit\ mass} = \frac{P}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

Steady Flow?
Not changing with time

Laminar Flow?
Reynolds number reflects flow regime
R < 100 laminar
R > 1000 turbulent
Between 100 to 1000 transitional

$$R = \frac{Velocity * diameter\ particle}{kinematic\ viscosity} = \frac{Vd}{\upsilon} \qquad \upsilon = \frac{dynamic\ viscosity}{fluid\ density} = \frac{\mu}{\rho}$$

Viscosity - resistance of a fluid to flow
More on this later

Frictionless Flow?
Real fluids are viscous (they require energy to overcome friction loss)

Incompressible Fluid?
Real fluids are compressible (their density changes with pressure)

Bernoulli's Equation is useful for comparing components of mechanical energy

$$\frac{P}{\rho} + gz + \frac{v^2}{2} = \text{constant} \xrightarrow{\text{units}} \frac{\left(\frac{M\frac{L}{T^2}}{L^2}\right)}{\frac{M}{L^3}} + \frac{L}{T^2}L + \frac{L^2}{T^2}$$

Weight

Divide each term by g to use units of energy per unit weight : dimensions of length

$$\frac{P}{g\rho} + z + \frac{v^2}{2g} = \text{hydraulic head} \longrightarrow \frac{\left(\frac{M\frac{L}{T^2}}{L^2}\right)}{\frac{L}{T^2}\frac{M}{L^3}} + \frac{\frac{L}{T^2}L}{\frac{L}{T^2}} + \frac{\frac{L^2}{T^2}}{\frac{L}{T^2}}$$

units of length

Ground water velocity is generally so low that the kinetic term can be ignored

$$\frac{P}{g\rho} + z = \text{hydraulic head}$$

$$\text{hydraulic head} = \frac{P}{\rho g} + z$$

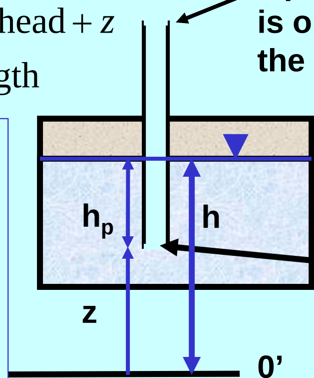
$$P = \rho g h_p \qquad h_p = \frac{P}{\rho g}$$

$$\text{hydraulic head} = h_p + z$$

$$\text{hydraulic head} = \text{pressure head} + z$$

all can be expressed in length

a pipe that is open at the ends



Hydraulic head, h, at location x,y,z is

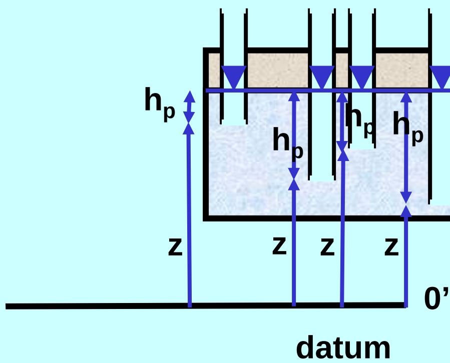
$h = h_p + z$

datum (often sea level)

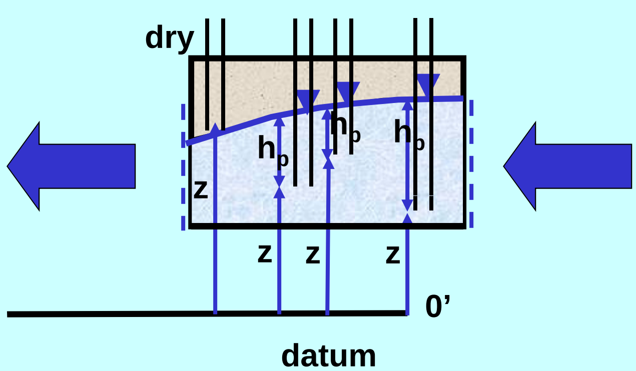
When comparing heads if the density of water in column h_p differs, normalize to the same density	**If the density were 10% greater at one well, would h_p increase or decrease to reflect an equivalent hydraulic head?**

Head exists at every location in a body of water
If no forces are exerted on it and we allow it to
"come to rest" head will be the same everywhere

We call this condition hydrostatic



If we force water through the system
Head will vary with space



Bernoulli's equation for total mechanical energy per unit mass
(here, omitting the velocity term)
defines the Force Potential

$$\text{force potential} = \Phi = \frac{P}{\rho} + gz$$

$$\text{and } P = \rho g h_p$$

$$\text{so} \quad \Phi = \frac{\rho g h_p}{\rho} + gz = g h_p + gz = g(h_p + z)$$

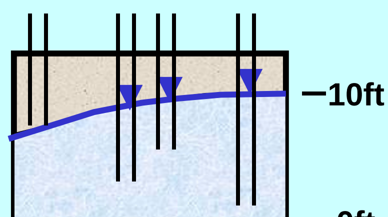
$$\text{and total hydraulic head} \quad h = h_p + z$$

$$\text{so} \quad \Phi = gh$$

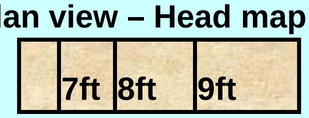
thus Φ and h are related by a constant

Connecting lines of equal head (potential)

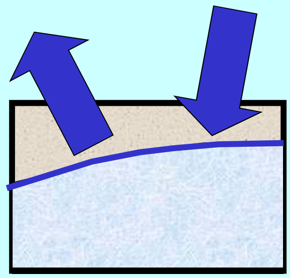
Yields head map / equipotential surface



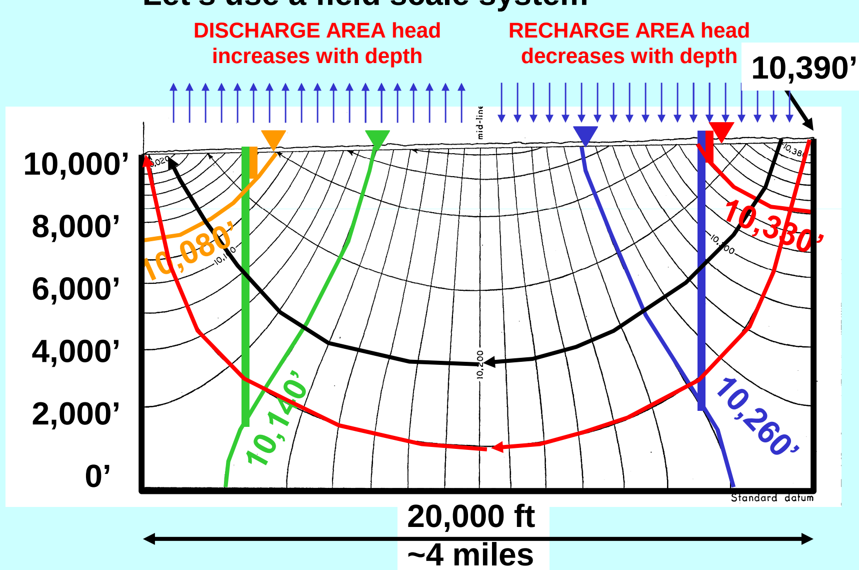
Plan view – Head map



It may be multi-dimensional



Let's use a field scale system



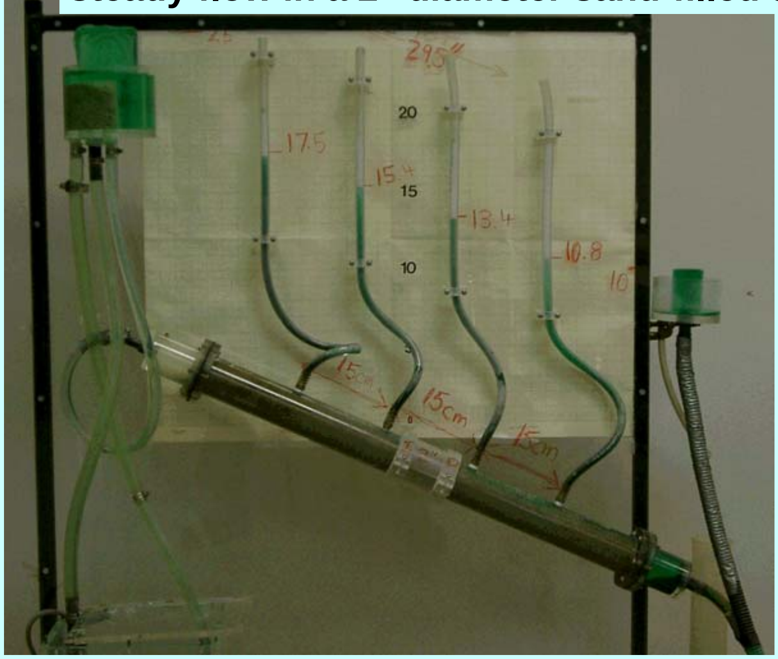
10,000 ft thickness
horizontal gradient
i = 390ft/20000ft ~0.02

along the red flow line
i = 390ft/24000ft ~0.016

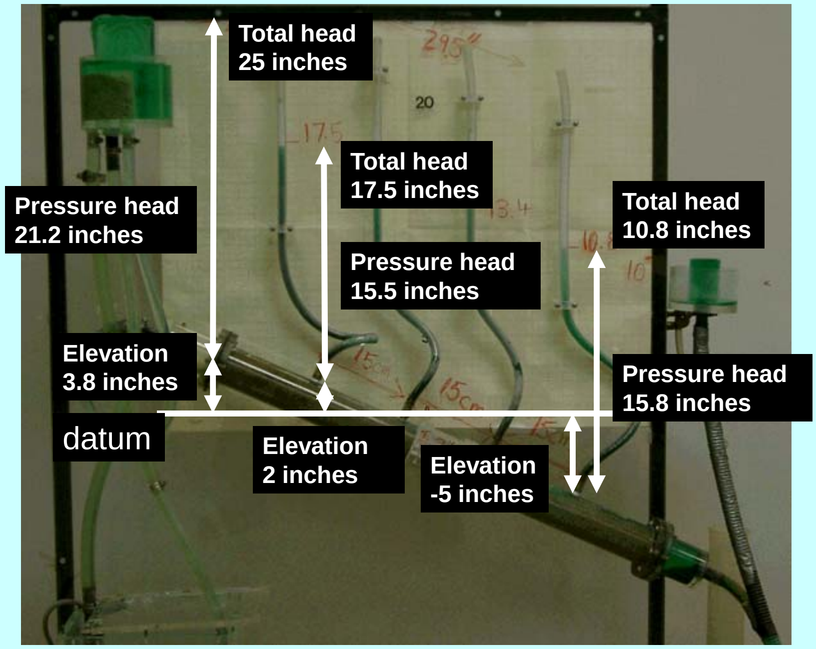
Flow in Porous Media

static conditions

steady flow in a 2" diameter sand-filled tube



What are the elevation and pressure heads?



Recall Darcy's Law

velocity of flow through the sand column is:

*** directly proportional to the**

head difference at the ends of the column

and

*** inversely proportional to the**

length of the column

$$V_{Darcy} = -K\left(\frac{h_2 - h_1}{l}\right) = -Ki$$

constant of proportionality (hydraulic conductivity if water) → $-K$

hydraulic gradient, i in direction of flow → $\left(\frac{h_2 - h_1}{l}\right)$

OTHER NAMES:

Darcy Velocity or Specific discharge LT^{-1}

(i.e. discharge per unit area, although sometimes multiplied by thickness to express as discharge per unit width as L^2T^{-1} **)**

Calculate K using data from the Apparatus

What measurements will you need?
What equation will you solve?

How will flow change if we rotate the sand column?
What if we bend the column?

Recall: Darcy velocity is a DISCHARGE per unit AREA

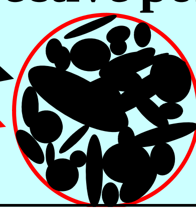
$$V_{Darcy} = \frac{Q}{A}$$

Average Linear Velocity

"as the crow flies" velocity through the pores

this governs rate of pollutant movement

$$V_{Interstitial} = \frac{Q}{A\phi_e} = \frac{V_D}{\phi_e} = \frac{V_D}{\text{effective porosity}}$$



Calculate Effective Porosity
using data from
Darcy Apparatus

What measurements will you need?

What equation will you solve?

Aquifer

permits appreciable amounts of groundwater to pass under normal field conditions (passes economic quantities of water)

Aquitard

Low hydraulic conductivity does not pass significant amounts of water but may store water

K ---- HYDRAULIC CONDUCTIVITY

when the fluid is water

The range of values spans many orders of magnitude:

Gravel ~1x10^{2}cm/sec

Unfractured Crystalline Rock ~ 1x10^{-11}cm/sec

k --- PERMEABILITY

the capacity of a porous medium to transmit fluid

MEASUREMENT OF K

FIELD TESTS - AQUIFER TESTS

LABORATORY - PERMEAMETERS

problems

not representative large rock mass

disturbed samples

orientation of sample

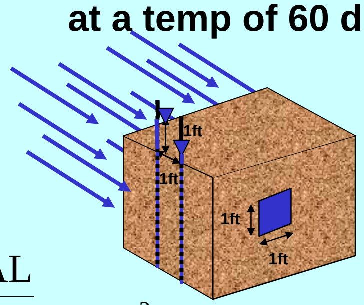
often knowing K to an order of magnitude is satisfactory and may be all that is obtainable within temporal and financial constraints

---

Hydraulic Conductivity K velocity units L/T or LT^{-1}

FIELD COEFFICIENT OF HYDRAULIC CONDUCTIVITY

describes K in terms of the
rate of flow of water in gallons per day
through a cross sectional area of 1 square foot
under a hydraulic gradient of 1
at a temp of 60 degrees F



$$\frac{\frac{GAL}{day}}{ft^2}$$

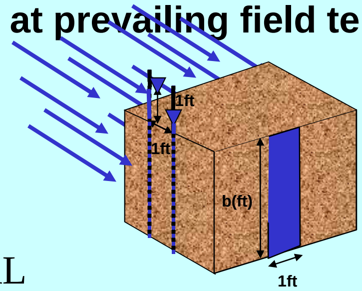
$$\frac{\frac{1\,GAL}{day}}{ft^2}\ \frac{1\,ft^3}{7.48GAL} \sim 0.13\frac{ft}{day} \sim 3.5x10^{-5}\frac{cm}{sec}$$

TRANSMISSIVITY T = K m, T units L²/T L²T

Theis, 1935

FIELD COEFFICIENT OF TRANSMISSIVITY

rate of flow of water in gal per day through a vertical strip of aquifer 1 ft wide, extending the full saturated height of the aquifer under a hydraulic gradient of 100% or 1 ft per ft at prevailing field temperature



$$\frac{\mathrm{GAL}}{\frac{\mathrm{day}}{\mathrm{ft}}}$$

$$\frac{\frac{1\,\mathrm{GAL}}{\mathrm{day}}}{\mathrm{ft}}\ \frac{1\,\mathrm{ft}^3}{7.48\mathrm{GAL}} \sim 0.13\frac{ft^2}{day} \sim 1x10^{-7}\ \frac{cm^2}{\sec}$$

Very important

T = Kb

Transmissivity =

Hydraulic Conductivity * Aquifer Thickness

K

hydraulic conductivity

applies to a material only for water passing through it, and at that,

water of particular temp & pressure (i.e. viscosity)

units of LT^{-1}

e.g. cm sec^{-1}

k

intrinsic permeability

a characteristic of the medium

independent of the fluid

units of L^2

e.g. cm^2

k

intrinsic permeability

$$k = \frac{K\mu}{\rho g} : \frac{\frac{L}{T}\frac{M}{LT}}{\frac{M}{L^3}\frac{L}{T^2}} : L^2$$

$$\mu = dynamic\ viscosity : \frac{M}{LT}$$

$$\rho = density : \frac{M}{L^3}$$

$$g = acceleration\ of\ gravity : \frac{L}{T^2}$$

k

<u>intrinsic permeability</u>

may be expressed in darcies

1 darcy is

the permeability that will lead to a specific discharge, q, of 1cm/sec for a fluid with a viscosity of 1 centipoise under a hydraulic gradient that makes the term:

$$\left(\frac{\rho g\ dh}{dl}\right) = \frac{1atm}{cm}$$

$$\mathbf{q} = \frac{\mathbf{k}}{\mu}\left(\frac{\rho \mathbf{g\ dh}}{\mathbf{dl}}\right)$$

Let's convert darcies to fundamental units of length squared

$$\mathbf{q} = \frac{\mathbf{k}}{\mu}\left(\frac{\rho \mathbf{g\ dh}}{\mathbf{dl}}\right)$$

Recall Viscosity?

Viscosity - resistance of a fluid to flow

(i.e. resistance to: pouring, deforming under shear stress, layers moving past each other)

dynamic viscosity is the tangential force per unit area required to move one horizontal plane with respect to the other at unit velocity when maintained a unit distance apart by the fluid)

dynamic viscosity units: N s m^{-2} or Pa s or $kgm^{-1}s^{-1}$ note:**1Pa-s=1N s m^{-2}=1kg m s^{-1}**
or: $gcm^{-1}s^{-1}$ or dyne s cm^{-2} or poise note: **1poise = $1gcm^{-1}s^{-1}$ = 1dyne s cm^{-2} = 0.1Pa-s**

dynamic viscosity of water at 68.4°F (20.2°C) is 1centipoise

$$kinematic\ viscosity = \upsilon = \frac{dynamic\ viscosity}{fluid\ density} = \frac{\mu}{\rho}$$

kinematic viscosity units: m^2s^{-1} or Stokes 1St=$10^{-4}m^2s^{-1}$
Since the specific gravity of water at 68.4°F (20.2°C) is almost 1g cm^{-3}, kinematic viscosity of water at 68.4°F is for all practical purposes 1centiStoke (cSt)

$$q = \frac{k}{\mu}\left(\frac{\rho g\, dh}{dl}\right)$$

k intrinsic permeability

To convert darcies to cm²:

$$by\ definition: \frac{1cm}{\sec} = \frac{1darcy}{1centipoise}\left(\frac{1atm}{cm}\right)$$

$$\frac{1cm}{\sec}\frac{1centipoise\ \ cm}{1atm} = 1darcy$$

$$1centipoise = \frac{0.01g}{cm\sec} \qquad 1atm = \frac{1.01325x10^{-6}g}{cm\sec^2}$$

$$darcy = \frac{1cm}{\sec}\frac{\frac{0.01g}{cm\ \sec}cm}{\frac{1.01325x10^{-6}g}{cm\sec^2}} = 9.869x10^{-9}\frac{cm\ g\ cm\ cm\sec^2}{cm\sec^2\ g} = 1x10^{-8}cm^2$$

To convert 1 darcy to K cm/sec for water:

$$K = \frac{k\rho g}{\mu} = \frac{9.869x10^{-9}cm^2\frac{1g}{cm^3}\frac{980cm}{\sec^2}}{\frac{1.01x10^{-2}g}{cm\sec}}$$

$$= 9.6x10^{-4}cm^2\frac{g}{cm^3}\frac{cm}{\sec^2}\frac{cm\sec}{g} = 1x10^{-3}\frac{cm}{\sec}$$

DARCY'S LAW - summary of basics details and assumptions

$$V_D = -K\frac{dh}{dl} \quad \text{or} \quad Q = KiA$$

NOT VALID FOR:

***very high velocities where turbulent conditions might prevail**

***extremely low velocities in fine grained materials where there may be a threshold for which no flow occurs**

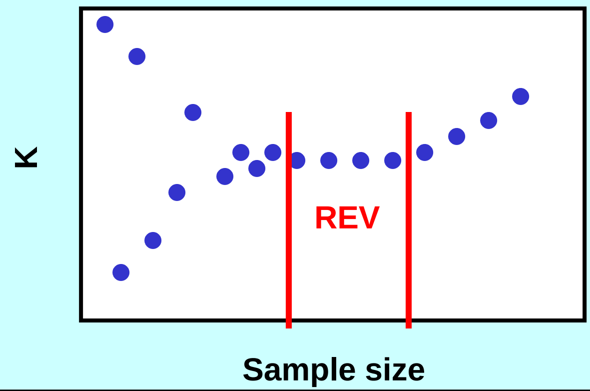
***compressible fluids**
(recall we divded total energy by a constant density to define hydraulic head as energy per unit mass for Bernoulli's equation)

***a discontinuum or variable properties (see next slide REV)**

*** pressure & elevation are not the only driving forces**
(see subsequent slide on other driving forces)

Darcy's is a macroscopic law
we assign uniform constant K to the entire porous medium mass

REPRESENTATIVE ELEMENTARY VOLUME
REV
a macro-continuum, below this volume there is no single value of a given parameter that can represent the material



***other phenomena contribute to the energy gradient that drives flow**

temperature
electric currents
chemical variations

hence we should really write:

$$V = -L_1 \frac{dh}{dl} - L_2 \frac{dT}{dl} - L_3 \frac{dC}{dl} - L_4 \frac{dV}{dl}$$

temperature & chemical gradients make water flow
but flowing water will transport heat & dissolved chemicals
this causes the
temperature & chemical gradients that drive water flow to change
which causes the rate of water flow to change
which changes the rate of heat and chemical transport by the water etc
this is **coupled flow**

SUMMARIZING ASSUMPTIONS OF DARCY'S LAW

***LAMINAR FLOW**

***ABOVE THRESHOLD V, IF IT EXISTS**

***THE FLUID IS INCOMPRESSIBLE**

***CONTINUUM & K IS CONSTANT (in space and time)**

***PRESSURE/ELEVATION HEAD IS ONLY DRIVING FORCE**