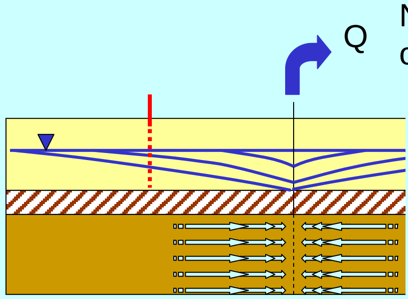
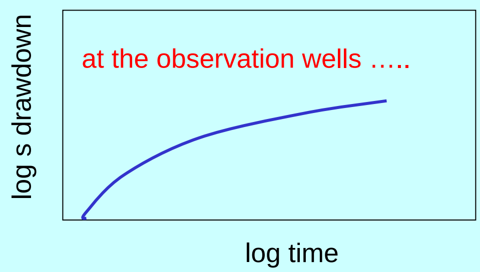
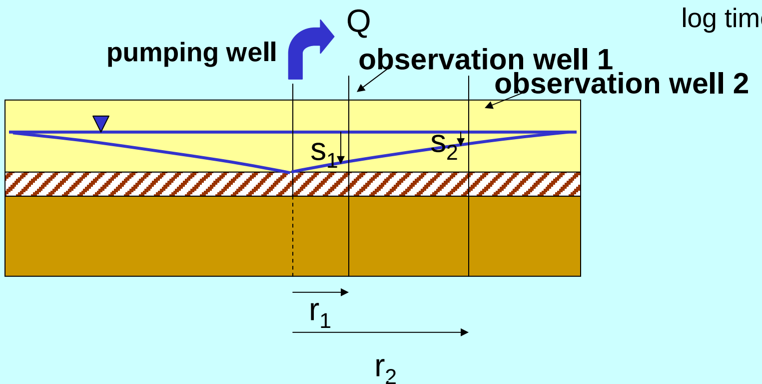
Well Hydraulics

- When a well is pumped water flows toward the well from storage, so the head declines forming a <u>cone of depression.</u>
  - The amount of decline is called <u>drawdown</u> so this is called the drawdown cone.
    - The time required to reach steady state depends on S(torativity) T(ransmissivity) BC(boundary conditions) and Q(pumping rate).
      - Monitoring the development and final form of this cone in observation wells around the pumping well allows us to determine aquifer properties (e.g. T and S).

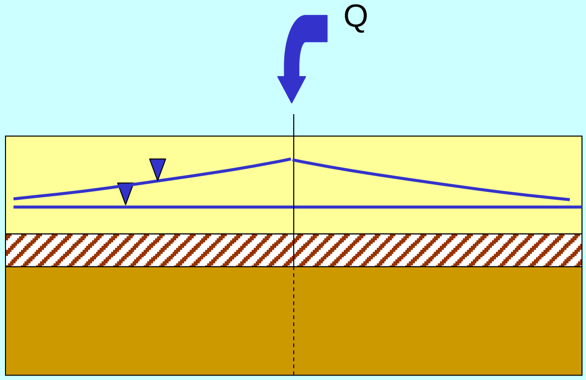
Note the confined character of the aquifer.







The same behavior will occur during injection but in reverse



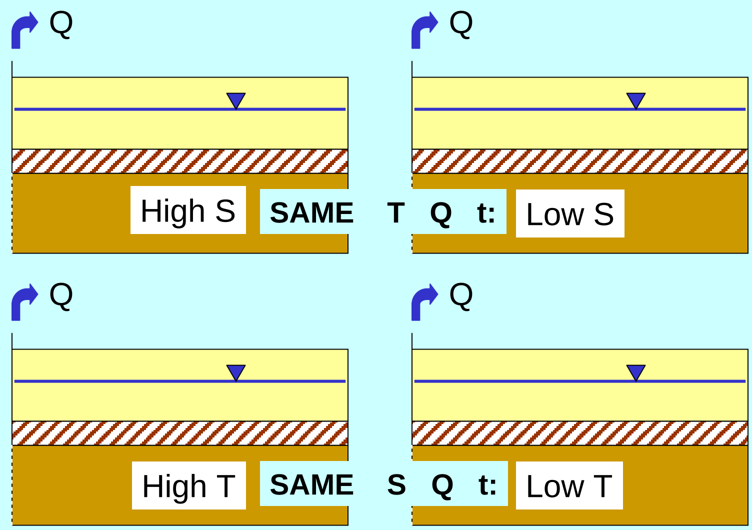
The Qualitative Viewpoint:

Infinite Aquifer, Initially hydrostatic

Water flows "more easily" in high T material vs low T,
Thus for the same Q
steeper gradients occur in low T material

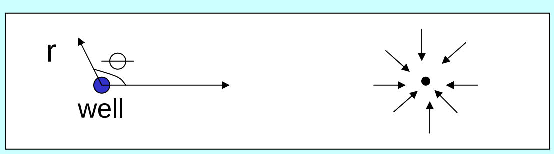
Initially water is removed from storage near the well bore
If S is high: we get more water
for the same drop in head
over the same area
compared with low S

Sketch the relative drawdown cones for the cases below
Pair up with a partner, compare your sketches to discuss differences and try to come to a consensus



When evaluating well hydraulics we use the flow equations in Polar Coordinates

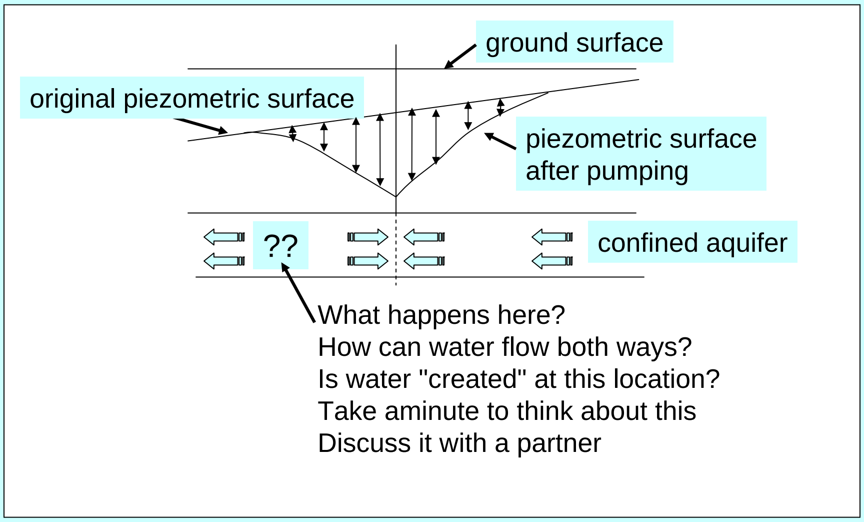
Plan View - Assume Radial Symmetry



$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}=\frac{S}{T}\frac{\partial h}{\partial t} \rightarrow \frac{\partial^2 h}{\partial r^2}+\frac{1}{r}\frac{\partial h}{\partial r}=\frac{S}{T}\frac{\partial h}{\partial t}$$

NOTE: **drawdown, not head, is uniform with theta**

Drawdown from pumping is superimposed on the initial flow field i.e.



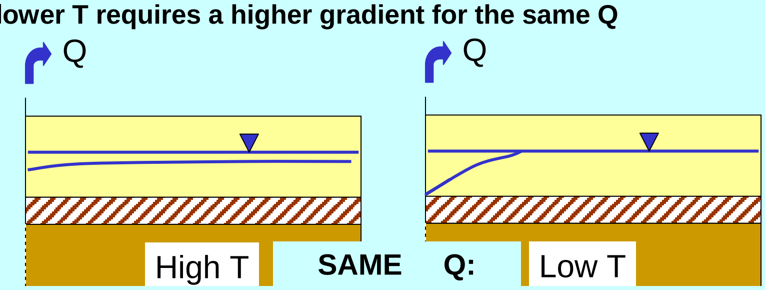
Simplest Situation

Steady State, Confined, inflow balances outflow

Only relationships between drawdown and distance need be considered

Only Transmissivity matters and can be determined ... Storage properties are irrelevant

The shape of the drawdown cone is controlled by pumping rate Q and Transmissivity,
lower T requires a higher gradient for the same Q



Darcy's Law Q=KiA is satisfied on every cylinder around the well, the gradient decreases linearly with distance from the well as the area increases linearly ($2\pi rh$)

Steady State, Confined

Assuming

- aquifer is **homogeneous, isotropic, areally infinite**
- pumping well **fully penetrates** and receives water from the entire thickness of the aquifer
- **Transmissivity is constant** in space and time
- pumping has continued at a **constant rate long enough** for steady state to prevail
- **Darcy's law is valid**

Plot s vs log r from a number of wells as follows

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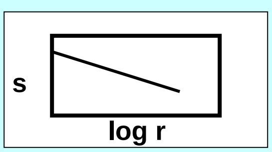
s vs. log r - is a straight line, if assumptions are met, drawdown decreases logarithmically with distance from the well because **gradient decreases linearly** with **increasing area (2π rh)**



$$Q = \frac{2\pi T(h_2 - h_1)}{\ln(r_2/r_1)}$$

Theim Eqtn

T = transmissivity [L^2/T]
Q = discharge from pumped well [L^3/T]
r = radial distance from the well [L]
h = head at r [L]

and rearranging to get T from field data:

$$T = \frac{Q}{2\pi(h_2 - h_1)} \ln\left(\frac{r_2}{r_1}\right)$$

Plot before applying equations. WHY?

In an unconfined aquifer, T is not constant

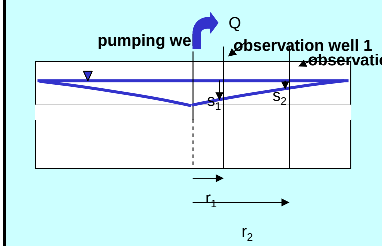
If drawdown is small relative to saturated thickness, confined equilibrium formulas can be applied with only minor errors

Otherwise call on Dupuit assumptions and use:

$$Q = \pi K \frac{(h_2^2 - h_1^2)}{\ln(r_2/r_1)}$$

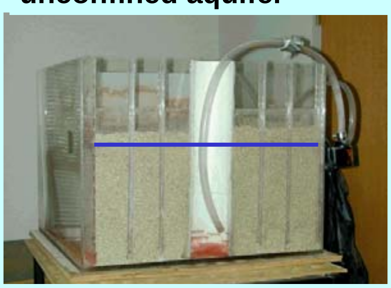
or, to determine K from field measurements of head:

$$K = \frac{Q \ln(\frac{r_2}{r_1})}{\pi(h_2^2 - h_1^2)}$$



Q = pumping rate [L^3/T]
K = permeability [L/T]
h_i = head @ a distance r_i from well [L]
using the aquifer base as datum

The sand tank simulates pumping in an unconfined aquifer



When I turn on the pump, it quickly reaches steady state

Collect data Left side: (r,h)
Right side:(r,h)
Q

With a partner, calculate K of the sand

$$K = \frac{Q \ln(\frac{r_2}{r_1})}{\pi(h_2^2 - h_1^2)}$$

Q = pumping rate [L^3/T]
K = permeability [L/T]
h_i = head @ a distance r_i from well [L]
using the aquifer base as datum

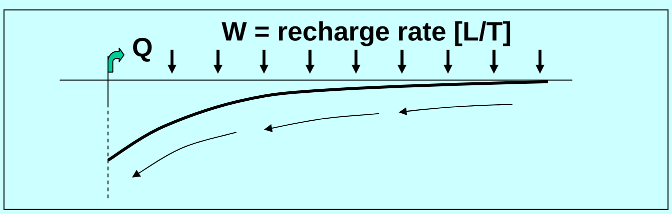
What are K? T?
Are they different on the left and right?
If you like, use:
wh1_theim_tank_class.xls

$$K = \frac{Q \ln(\frac{r_2}{r_1})}{\pi(h_2^2 - h_1^2)}$$

$$T = \frac{Q}{2\pi(h_2 - h_1)} \ln(\frac{r_2}{r_1})$$

Q = pumping rate [L^3/T]
T = transmissivity [L^2/T]
K = permeability [L/T]
h_i = head @ a r_i [L]

More likely, vertical leakage will satisfy Q
with w = recharge rate, then:



We can include recharge in the expression:

$$h_2^2 - h_1^2 = \frac{w}{2K}(r_1^2 - r_2^2) + \frac{Q}{\pi K} \ln(\frac{r_2}{r_1})$$

and solve for K:

$$K = \frac{\frac{w}{2}(r_1^2 - r_2^2) + \frac{Q}{\pi} \ln(\frac{r_2}{r_1})}{(h_2^2 - h_1^2)}$$