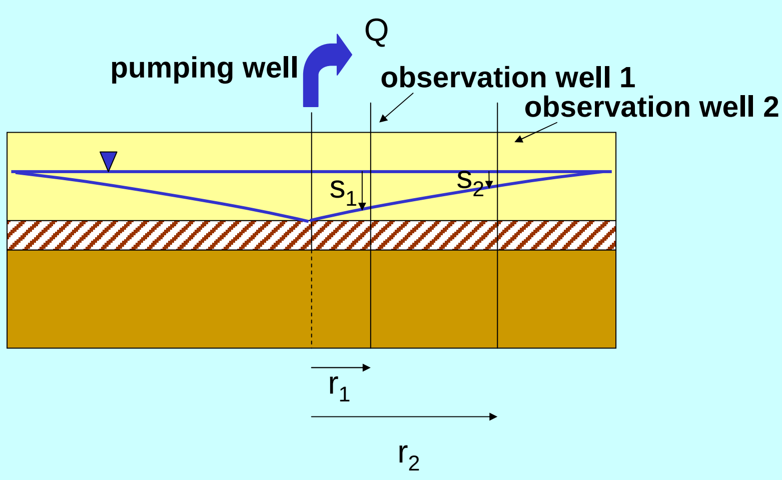
USING TRANSIENT RESPONSE OF SYSTEMS TO PUMPING

Benefits/Disadvantages

- can estimate storativity values
- when estimating aquifer properties, we get results @ early time
- easier to detect extraneous effects
- analysis is more complex

Assumptions

1. homogeneous, isotropic, infinite areal extent
2. pumping well fully penetrates and receives water from the entire thickness of the aquifer
3. Transmissivity is constant in space and time
4. well has infinitesimal diameter
5. water removed from storage is discharged instantaneously with decline in head



Theis Equation

Drawdown given Q T S r t

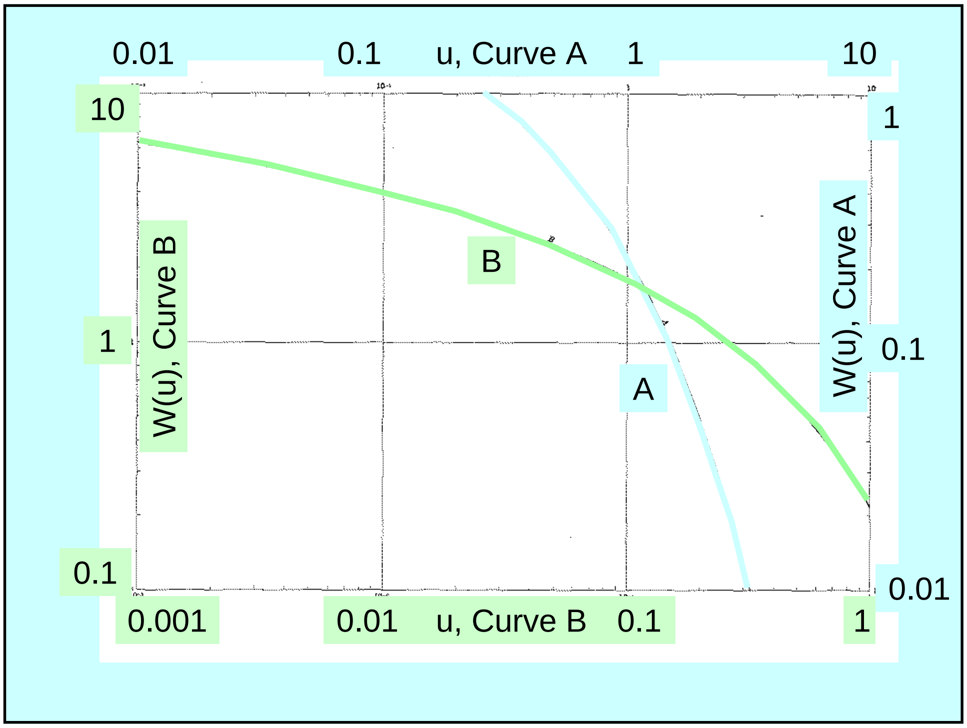
$$s = h_o - h = \frac{Q}{4\pi T} W(u)$$

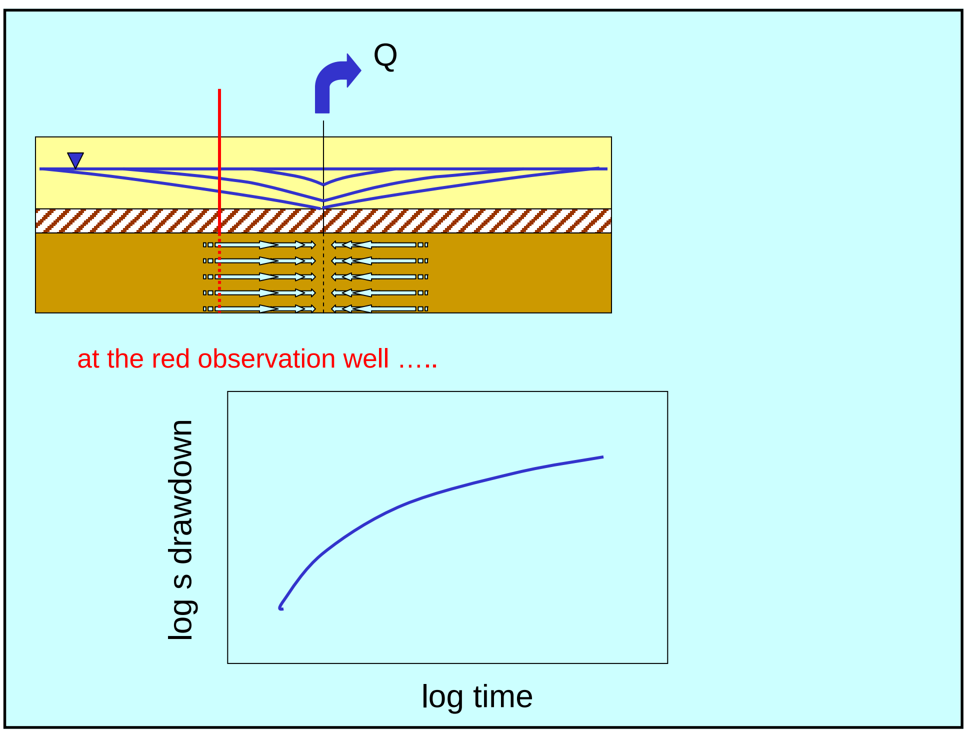
$$u = \frac{r^2 S}{4Tt} \quad or \quad \frac{r^2}{t} = \frac{4T}{S} u$$

s = drawdown [L]
h_o = initial head @ r [L]
h = head at r at time t [L]
t = time since pumping began [T]
r = distance from pumping well [L]
Q = discharge rate [L^3/T]
T = transmissivity [L^2/T]
S = Storativity [ ]

$$W(u) = \int_u^\infty \frac{e^{-u}}{u} du = \left[-0.5772 - \ln u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} + ...\right]$$

u	$W(u)$	u	$W(u)$	u	$W(u)$	u	$W(u)$
1×10^{-10}	22.45	7×10^{-8}	15.90	4×10^{-5}	9.55	1×10^{-2}	4.04
2	21.76	8	15.76	5	9.33	2	3.35
3	21.35	9	15.65	6	9.14	3	2.96
4	21.06	1×10^{-7}	15.54	7	8.99	4	2.68
5	20.84	2	14.85	8	8.86	5	2.47
6	20.66	3	14.44	9	8.74	6	2.30
7	20.50	4	14.15	1×10^{-4}	8.63	7	2.15
8	20.37	5	13.93	2	7.94	8	2.03
9	20.25	6	13.75	3	7.53	9	1.92
1×10^{-9}	20.15	7	13.60	4	7.25	1×10^{-1}	1.823
2	19.45	8	13.46	5	7.02	2	1.223
3	19.05	9	13.34	6	6.84	3	0.906
4	18.76	1×10^{-6}	13.24	7	6.69	4	0.702
5	18.54	2	12.55	8	6.55	5	0.560
6	18.35	3	12.14	9	6.44	6	0.454
7	18.20	4	11.85	1×10^{-3}	6.33	7	0.374
8	18.07	5	11.63	2	5.64	8	0.311
9	17.95	6	11.45	3	5.23	9	0.260
1×10^{-8}	17.84	7	11.29	4	4.95	1×10^{0}	0.219
2	17.15	8	11.16	5	4.73	2	0.049
3	16.74	9	11.04	6	4.54	3	0.013
4	16.46	1×10^{-5}	10.94	7	4.39	4	0.004
5	16.23	2	10.24	8	4.26	5	0.001
6	16.05	3	9.84	9	4.14		


0.01
0.1
u, Curve A
1
10
10
1
W(u), Curve B
W(u), Curve A
B
1
0.1
A
0.1
0.01
0.001
0.01
u, Curve B
0.1
1


Q
at the red observation well …..
log s drawdown
log time

Predict Drawdown Using Theis Equation

Class picks: T S r t Q

Take 3 minutes to calculate: s

$$s = h_o - h = \frac{Q}{4\pi T} W(u) \qquad u = \frac{r^2 S}{4Tt}$$

$$W(u) = \int_u^\infty \frac{e^{-u}}{u} du = [-0.5772 - \ln u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} + ...]$$

What value did you get for s?

Theis Formula cannot be solved directly for T and S from observations of drawdown (why?)

$$s = h_o - h = \frac{Q}{4\pi T} W(u) \qquad u = \frac{r^2 S}{4Tt}$$

$$W(u) = \int_u^\infty \frac{e^{-u}}{u} du = [-0.5772 - \ln u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} + ...]$$

Consequently, we use curve matching techniques

Type Curve is W(u) vs u OR W(u) vs 1/u

Plot s vs 1/t or s vs r^2/t for field data OR s vs t

Type curve & field data must be plotted on same log-log paper

Field curve is overlaid on Type Curve

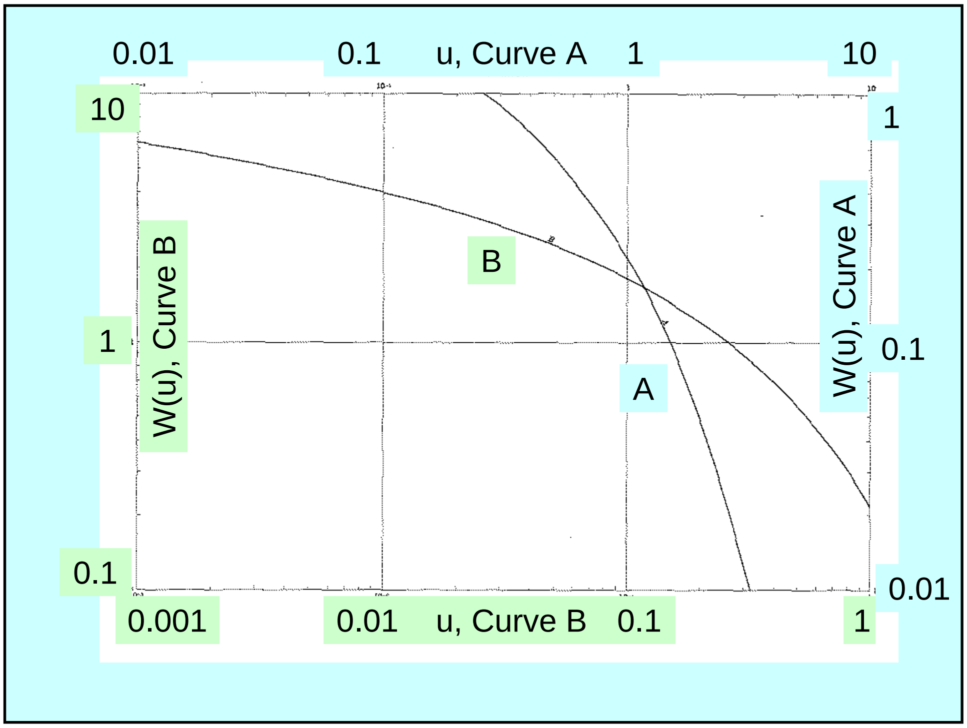
Axes must be kept parallel

Best match of curves is found

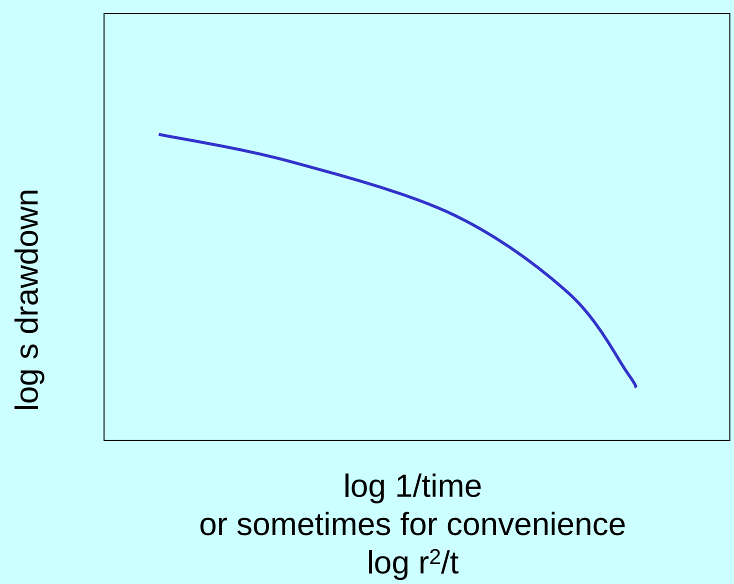
Pick any convenient point

read corresponding W(u), u, s and r^2/t

Solve Theis Equation for T & S



to match the W(u) curve in this format, plot log s vs log 1/t



An example test from Ohio

Pumping well discharges @ 500 GPM

from a 100ft thick aquifer

Observation well is 200 ft away

Plot as s vs r^2/t on same scale of log paper as W(u) vs u

($r = 200$ ft)

Time Since Pumping Began, t		Drawdown in Observation Well, $h_0 - h$	r^2/t
Minutes	Days	Feet	Feet²/Day
0	0	0.00	∞
1.0	6.96×10^{-4}	0.66	5.76×10^{7}
1.5	1.02×10^{-3}	0.87	3.84×10^{7}
2.0	1.39×10^{-3}	0.99	2.88×10^{7}
2.5	1.74×10^{-3}	1.11	2.30×10^{7}
3.0	2.09×10^{-3}	1.21	1.92×10^{7}
4	2.78×10^{-3}	1.36	1.44×10^{7}
5	3.48×10^{-3}	1.49	1.15×10^{7}
6	4.17×10^{-3}	1.59	9.6×10^{6}
8	5.57×10^{-3}	1.75	7.2×10^{6}
10	6.96×10^{-3}	1.86	5.76×10^{6}
12	8.33×10^{-3}	1.97	4.80×10^{6}
14	9.72×10^{-3}	2.08	4.1×10^{6}
18	1.25×10^{-2}	2.20	3.2×10^{6}
24	1.67×10^{-2}	2.36	2.4×10^{6}
30	2.09×10^{-2}	2.49	1.92×10^{6}
40	2.78×10^{-2}	2.65	1.44×10^{6}
50	3.48×10^{-2}	2.78	1.15×10^{6}
60	4.17×10^{-2}	2.88	9.6×10^{5}
80	5.57×10^{-2}	3.04	7.2×10^{5}
100	6.96×10^{-2}	3.16	5.76×10^{5}
120	8.33×10^{-2}	3.28	4.8×10^{5}
150	1.02×10^{-1}	3.42	3.84×10^{5}
180	1.25×10^{-1}	3.51	3.2×10^{5}
210	1.46×10^{-1}	3.61	2.74×10^{5}
240	1.67×10^{-1}	3.67	2.4×10^{5}

Jacob simplified the Theis equation for large t and small r using only the first 2 terms of the W(u) function

$$s = h_o - h = \frac{Q}{4\pi T} W(u) \qquad u = \frac{r^2 S}{4Tt}$$

$$W(u) = \int_u^{\infty} \frac{e^{-u}}{u} du = [-0.5772 - \ln u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} + \ldots\ldots]$$

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S = Storativity [ ]

$$s = \frac{2.3Q}{4\pi T} \log \frac{2.25Tt}{r^2 S}$$

Simplified expression (note constant due to switch from ln to log)

Jacob’s simplified Theis equation

$$s = h_o - h = \frac{Q}{4\pi T} W(u)$$

$$W(u) = \int_u^\infty \frac{e^{-u}}{u} du = [-0.5772 - \ln u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} + \ldots\ldots]$$

$$u = \frac{r^2 S}{4Tt}$$

Can you get to

$$s = \frac{2.3Q}{4\pi T} \log \frac{2.25Tt}{r^2 S}$$

$$s = \frac{Q}{4\pi T}(-0.5772 - \ln u) \qquad note: \quad \ln u = 2.3 \log u$$

$$s = \frac{Q}{4\pi T}(-2.3 \log 1.7822 - 2.3 \log u)$$

$$s = \frac{2.3Q}{4\pi T}(-\log 1.7822 - \log u) \quad note: \quad -\log u = \log \frac{1}{u}$$

$$s = \frac{2.3Q}{4\pi T}\left(-\log 1.7822 + \log \frac{1}{u}\right) \quad note: \quad \log(a-b) = \log \frac{a}{b}$$

$$s = \frac{2.3Q}{4\pi T}\left(\log \frac{1}{u} - \log 1.7822\right) = \frac{2.3Q}{4\pi T} \log\left(\frac{\frac{1}{u}}{1.7822}\right)$$

$$note: \quad u = \frac{r^2 S}{4Tt} \qquad s = \frac{2.3Q}{4\pi T} \log\left(\frac{\frac{1}{\frac{r^2 S}{4Tt}}}{1.7822}\right) = \frac{2.3Q}{4\pi T} \log\left(\frac{4Tt}{1.7822 r^2 S}\right)$$

$$s = \frac{2.3Q}{4\pi T} \log\left(\frac{2.25Tt}{r^2 S}\right)$$

$$s = \frac{2.3Q}{4\pi T} \log \frac{2.25Tt}{r^2 S}$$

Since Q r T & S are constant, s vs log t is a straight line:

a graph of s (drawdown) vs log(t) (time)

can be described by a straight line

$$y = s \qquad x = \log(t) \qquad y = mx + b$$

$$m = slope = \frac{2.3Q}{4\pi T} \qquad b = intercept = \frac{2.3Q}{4\pi T} \log \frac{2.25T}{r^2 S}$$

$$s = \frac{2.3Q}{4\pi T} \log(t) + \frac{2.3Q}{4\pi T} \log \frac{2.25T}{r^2 S}$$

For Jacob's simplification of the Theis equation we find an easy way to estimate T:

$$s = \frac{2.3Q}{4\pi T}\log(t) + \frac{2.3Q}{4\pi T}\log\frac{2.25T}{r^2 S}$$

if we consider the slope in terms of

Δs (change in drawdown over one log cycle of t)

$$\frac{\Delta s}{\log(t)} = \frac{2.3Q}{4\pi T} \quad \text{given log(t) for one log cycle of t is 1}$$

$$\Delta s = \frac{2.3Q}{4\pi T} \quad \text{and rearranging to solve for T:}$$

$$T = \frac{2.3Q}{4\pi \Delta s}$$

at the time intercept t_o the straight line approximation of $s = 0$

$$0 = \frac{2.3Q}{4\pi T}\log(t_o) + \frac{2.3Q}{4\pi T}\log\frac{2.25T}{r^2 S} \qquad \text{pulling out } \frac{2.3Q}{4\pi T}:$$

$$0 = \frac{2.3Q}{4\pi T}\left(\log(t_o) + \log\frac{2.25T}{r^2 S}\right)$$

with zero on the left hand side the constant is irrelevant

$$0 = \left(\log(t_o) + \log\frac{2.25T}{r^2 S}\right) \qquad \text{subtract } \log\frac{2.25T}{r^2 S} \text{ from both sides}$$

$$-\log\frac{2.25T}{r^2 S} = \log(t_o) \qquad \text{exponentiate both sides}$$

$$\frac{r^2 S}{2.25T} = t_o \qquad \text{rearrange to solve for S}$$

$$S = \frac{2.25Tt_o}{r^2}$$

In short:
Jacob's straight line method for u<0.03

$$T = \frac{2.3Q}{4\pi \Delta h} \qquad S = \frac{2.25\,T t_o}{r^2}$$

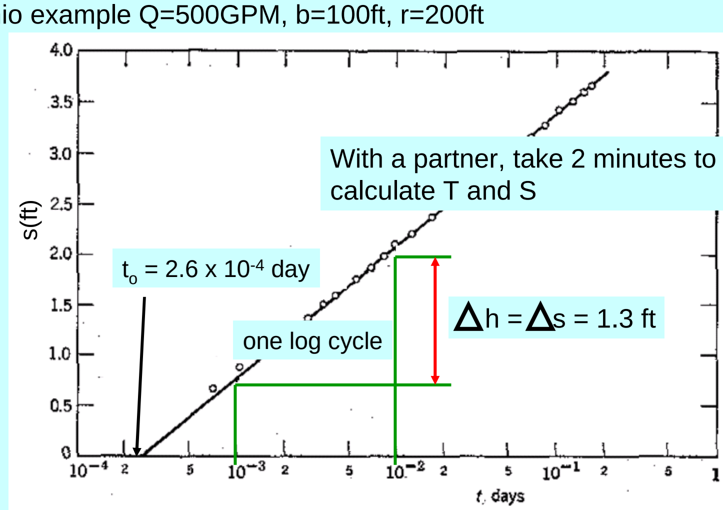
where:

$\Delta h = \Delta s$ = drawdown over 1 log cycle of time

t_o = time intercept for zero drawdown

For the Ohio example Q=500GPM, b=100ft, r=200ft

Plot s vs t



$$T = \frac{2.3Q}{4\pi \Delta h} \qquad S = \frac{2.25\,T t_o}{r^2}$$