ALL GROUND-WATER HYDROLOGY WORK IS MODELING

A Model is a representation of a system.

Modeling begins when one formulates a concept of a hydrologic system, continues with application of, for example, Darcy’s Law or the Theis equation to the problem, and may culminate in a complex numerical simulation.

MODELS can be used BENEFICIALLY and for DECEPTION

GROUND WATER MODELING

WHY MODEL?

• To make predictions about a ground-water system’s response to a stress

• To understand the System

• To design field studies

• Use as a thinking tool
Characterize the system

Governing equation of Ground Water Flow:

\[ \frac{\partial}{\partial x} \left( K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial h}{\partial z} \right) - W = S \frac{\partial h}{\partial t} \]

Geometry

Material Properties (K, S, T, \( \Phi_e \), D, R, etc)

Boundary Conditions (Head, Flux, Concentration etc)

Stresses (changing boundary conditions)

Boundary Types

<table>
<thead>
<tr>
<th>infinite source/sink</th>
<th>Specified Head/Concentration: a special case of constant head (ABC, EFG)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant Head /Concentration: could replace (ABC, EFG)</td>
</tr>
<tr>
<td>calculated head</td>
<td>Specified Flux: could be recharge across (CD)</td>
</tr>
<tr>
<td>maintains flux</td>
<td>No Flow (Streamline): a special case of specified flux (HI)</td>
</tr>
<tr>
<td>infinite source/sink</td>
<td>Head Dependent Flux: could replace (ABC, EFG)</td>
</tr>
<tr>
<td>adjusts position</td>
<td>Free Surface: water-table, phreatic surface (CD)</td>
</tr>
<tr>
<td>adjusts length</td>
<td>Seepage Face: h = z; pressure = atmospheric at ground surface (DE)</td>
</tr>
</tbody>
</table>

What about A-H and G-I?
Every boundary must be defined. There aren’t any open ended options.
If you are using a model and not required to enter a condition, then some condition is assumed, most likely flux=0.
ALL IMPORTANT MECHANISMS & PROCESSES MUST BE INCLUDED IN THE MODEL, OR RESULTS WILL BE INVALID.

TYPES OF MODELS

CONCEPTUAL
PHYSICAL
ANALOG
EMPIRICAL
GRAPHICAL
MATHEMATICAL
  • SIMPLE - ANALYTICAL
  • COMPLEX - NUMERICAL
CONCEPTUAL MODEL

Geometry
Material Properties
Boundary Conditions
General Flow Patterns
System Stresses (usually BCs)

CONCEPTUAL MODEL + FLOW EQUATIONS
= QUANTITATIVE MODEL OF FLOW SYSTEM
PHYSICAL MODEL
Geometry
Materials
Boundary Conditions

ANALOG MODEL
Geometry
Material Properties
Boundary Conditions

Electrical analog model of the Champaign-Urbana Illinois area ground-water system (circa 1960)
The top panel is a circuit of resistors and capacitors representing the regional model.
Measuring the voltage at various locations in the circuit is equivalent to measuring head in the aquifer.
The middle level includes a local model of a portion of the regional model at both the same scale and twice the scale.
The lower area includes the controls for imposing current on the model.
These models are very difficult to calibrate because each change of material properties involves removing and re-soldering the resistors and capacitors.
EMPIRICAL MODEL
A Mathematical Fit to Data Unrelated to Process Equations

e.g. Manning’s Equation

\[ V = \frac{1.49 \ R^{2/3} \ S^{1/2}}{n} \]

where:
- \( V \) = average velocity in fps
- \( R \) = hydraulic radius (flow area \([\text{ft}^2]/\text{wetted perimeter}[\text{ft}]\))
- \( S \) = slope of energy gradient
- \( n \) = Manning friction factor

GRAPHICAL MODEL - FLOW NET

Geometry
Material Properties
Boundary Conditions
ANALYTICAL MODEL

Closed form algebraic solution

Geometry
Material Properties
Boundary Conditions

Recall Dupuit, Flow to fixed heads with recharge:

\[
\begin{align*}
  h_x &= \sqrt{h_1^2 - \left(\frac{h_1^2 - h_2^2}{L}\right)x} + \frac{w}{K} (L - x) \\
  q_x &= \frac{K\left(h_1^2 - h_2^2\right)}{2L} - w \left(\frac{L}{2} - x\right) \\
  d &= \frac{L}{2} - \frac{K}{w} h_1^2 - h_2^2
\end{align*}
\]

A steady state solution to the flow equations in 1D with Boundary Conditions:
Bottom, no-flow (fixed flux = 0) head is calculated such that flow will parallel boundary
Top, fixed flux = recharge head/gradient are calculated to accommodate that recharge - e.g. High recharge >> High heads
Sides, h1 and h2 are fixed heads flux is calculated to accommodate those heads - e.g. a high h1 will shift the divide to the left of the problem domain and produce large influx that joins the recharge and discharges to the right, if h2 is very low, that influx will be even higher
Recall Toth developed a solution for steady-state flow in a 2D section from a divide to a stream. He solved the Laplace equation:

$$\frac{\partial^2 h}{\partial x^2} - \frac{\partial^2 h}{\partial z^2} = 0$$

**Boundaries**

- Left: $\frac{\partial h}{\partial x}(0, z) = 0$
- Right: $\frac{\partial h}{\partial x}(s, z) = 0$
- Lower: $\frac{\partial h}{\partial z}(x, 0) = 0$
- Upper water table: $h(x, z_u) = z$

What are the Boundary Conditions?

His result:

Fig. 3. Two-dimensional theoretical potential distributions and flow patterns for different depths to the horizontal impermeable boundary.
What are the Boundary Conditions?

Theis Equation

\[ s = h_0 - h = \frac{Q}{4\pi T} W(u) \]

Drawdown given \( Q, T, S, r, t \)

\[ u = \frac{r^2 S}{4Tt} \quad \text{or} \quad \frac{r^2}{t} = \frac{4T}{S} u \]

\( s = \) drawdown [L]
\( h_0 = \) initial head @ \( r \) [L]
\( h = \) head at \( r \) at time \( t \) [L]
\( t = \) time since pumping began [T]
\( r = \) distance from pumping well [L]
\( Q = \) discharge rate [L^3/T]
\( T = \) transmissivity [L^2/T]
\( S = \) Storativity [ ]

\[ W(u) = \int_{0}^{\infty} \frac{e^{-u}}{u} du + \left[ -0.5772 - \ln u + \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} + \ldots \right] \]

Pumping near a stream, we are interested in depletion of stream flow
\( q_p \ [L^3/T] = \text{rate of stream depletion at time } t \text{ measured from start of pumping} \)

\[
q_p = Q \text{ erfc } \left( \frac{a}{\sqrt{4tT/S}} \right)
\]

- \( a \) = perpendicular distance between well and stream [L]
- \( t \) = time since pumping began [T]
- \( T \) = aquifer transmissivity (\( K \cdot \text{thickness} \)) [\( L^2/T \)]
- \( S \) = aquifer storage coefficient [dimensionless]
- \( Q \) = pumping rate [\( L^3/T \)]
- \( \text{erfc} \) = the complimentary error function [dimensionless]

\( v_p \ [L^3] \) is the total volume of stream depletion since pumping began

\[
v_p = Q t \left[ \left( \frac{a^2}{2tT/S} + 1 \right) \text{ erfc } \left( \frac{a}{\sqrt{4tT/S}} \right) - \left( \frac{a}{\sqrt{4tT/S}} \right) \left( \frac{2e^{-a^2/(4tT/S)}}{\sqrt{\pi}} \right) \right]
\]

\( Q t \) = is the total volume pumped since pumping began [\( L^3 \)]
Rate and volume of stream depletion after pumping stops is determined by the superposition where drawdown is summed with drawup from an injection image well that starts when pumping stops.

\[ t = \text{time from start to stop of pumping} \quad [T] \]

\[ t' = \text{time since pumping stopped} \quad [T] \]

\[ q_r \quad [L^3/T] \text{is the residual rate of stream depletion at time } t + t' \]

\[
q_r = Q \operatorname{erfc}\left( \frac{1}{\sqrt{4(\xi + \xi')^{1/8}}} \right) - q_r \operatorname{erfc}\left( \frac{1}{\sqrt{4(\xi')^{1/8}}} \right)
\]

\[ V_r \quad [L^3] \text{is the total volume of stream depletion since pumping began} \]

(note the \( t \) in the \( Q_t \) of the second term is \( t' \))

\[ V_r \quad [L^3] \text{accounts for depletion after pumping stops} \]

\[
V_r = Q \left( t + t' \right) \left[ \left( \sqrt[8]{\xi + \xi'} + 1 \right) \operatorname{erfc}\left( \frac{1}{\sqrt{4(\xi + \xi')^{1/8}}} \right) - \left( \sqrt[8]{\xi + \xi'} \right) \left( \frac{e^{-a^2/(4(s + s') T/8)}}{\sqrt{\pi}} \right) \right] - Q t_s \left[ \left( \sqrt[8]{\xi_s + \xi_s} + 1 \right) \operatorname{erfc}\left( \frac{1}{\sqrt{4\xi_s^{1/8}}} \right) - \left( \sqrt[8]{\xi_s} \right) \left( \frac{e^{-a^2/(4s' T/8)}}{\sqrt{\pi}} \right) \right]
\]
Recall Analytical Transport models, e.g. for 1D slug source:

What are the boundary conditions in this case?
What type of conditions are needed that we did not need before?
What properties are needed here that weren’t needed in the previous models we have discussed today?

\[ C(x=vt + X, y=Y, z=Z) = \frac{M}{8\pi t} e^{-\frac{X^2}{4D_t} - \frac{Y^2}{4D_t} - \frac{Z^2}{4D_t}} \]

Analytical Solution for transport in
1D flow field
continuous source
3D spreading

What are the boundary conditions in this case?
NUMERICAL MODELING DISCRETIZES THE SYSTEM  

e.g. Finite Difference Modeling

Each block has only ONE AVERAGED value of each Property, Boundary Condition, State Variable

The model on the right represents vertical variation of head and concentration using only 2 points. How many does the model on the left use?

NUMERICAL FLOW MODELING

DISCRETIZE

Write equations of GW Flow between each node
- Darcy’s Law
- Conservation of Mass

Define
- Material Properties
- Boundary Conditions
- Initial Conditions
- Stresses (varying conditions)

At each node either H or Q is known, the other is unknown
- n equations & n unknowns
- solve simultaneously with matrix algebra

Result
- H at each known Q node
- Q at each known H node

Calibrate
- Steady State
- Transient

Validate
- Sensitivity
- Predictions

Similar Process for Transport Modeling only Concentration and Flux is unknown
**NUMERICAL MODELING DISCRETIZES THE SYSTEM**

**e.g. Finite Difference Modeling**

Each block has only ONE AVERAGED value of each Property, Boundary Condition, State Variable. To get more detail, Use more blocks.

**NUMERICAL MODELING – Finite Element Modeling**
NUMERICAL MODELING – Finite Element Modeling

More flexibility in designing Grid

Examples of Model Results with links to animations

Flow Model
Hanford Reservation
Waste Water Disposal
http://inside.mines.edu/~epoeter/_GW/24Modeling/flow_43_96.avi

Flow and Transport Model
East Texas Land Fill Plume
http://inside.mines.edu/~epoeter/_GW/24Modeling/etex2.avi

North-South cross section through plume
MODFLOW

Block Centered 3D Finite Difference Ground Water Flow Model

Developed by McDonald & Harbaugh at USGS in 1983 - enhanced many times since then

Public Domain

Most widely used Saturated Porous Media Flow model

Many features available

MODFLOW:

Input Files → MODFLOW Executable → With Packages provided, runs calculations Based on Model Set-up → Output

MODFLOW uses text file input and output. GUI’s can facilitate your work by creating the input files, running the program, and reading the output files, through a graphical interface Graphical User Interface
Some MODFLOW Boundary Condition Packages

- Specified Head Boundaries
  - CHD - Time-Variant Specified Head Package
  - FHB - Flow and Head Boundary Package
- Specified Flux Boundaries
  - FHB - Flow and Head Boundary Package
  - RCH - Recharge Package
  - WEL - Well Package
- Head Dependent Flux Boundary Packages
  - DAF, DAFF - DAFLOW with MODFLOW
  - DRN - Drain Package
  - DRT - Drain Return Package
  - ETS - Evapotranspiration Segment's Package
  - EVT - Evapotranspiration Package
  - GNB - General-Head Boundary Package
  - LK - Lake Package
  - MNW1 - Multi-Node, Drawdown-Limited Well Package
  - RES - Reservoir Package
  - RIV - River Package
  - SFR - Streamflow-Routing Package
  - STR - Stream Package
  - UZF - Unsaturated Zone Flow Package

MODFLOW Head-dependent Flux Boundary Condition Packages

RIVER package

For each river reach in each cell:
MODFLOW requires that the user input Conductance, which is all of Darcy's Law except the head difference for Head Dependent Flux boundaries.

\[ Q = KA \frac{dh}{dl} \]

Conductance = \( KA/\text{thickness} \)

then MODFLOW calculates the flow as:
\[ Q = \text{Conductance} \cdot dh \]

Conductance of the river bed is calculated as:
\[ K_v = \frac{\text{Area (the plan view area, L*W)}}{\text{thickness}} \]

\[ dh = (\text{Stage-input-by-user} - \text{Head-in-cell-calculated-by-MODFLOW}) \]

Unless head in cell is less than bottom of river, then
\[ dh = (\text{Stage-input-by-user} - \text{elevation-of-bottom-of-river}) \]
MODFLOW Head-dependent Flux Boundary Condition Packages

**RIVER package**

- \( Q = \text{Conductance } dh \)
- i.e. CRIV \( dh \)
- \( dh \) is limited to stage - bottom of sediment when bottom is above the water table

\[ dh = (\text{bottom of sediment} - \text{water table}) \]

**DRAIN package**

- ONLY allows OUTFLOW
- \( Q = \text{Conductance } dh \)
- i.e. CD \( dh \)
- When head is above the associated elevation

\[ dh = (\text{drain elevation} - \text{head in cell}) \]

Unless head in cell is less than drain elevation, then \( dh \) is set to 0
MODFLOW Head-dependent Flux Boundary Condition Packages

**DRAIN package**

For each drain in each cell:

MODFLOW requires that the user input Conductance, which is all of Darcy’s Law except the head difference for Head Dependent Flux boundaries.

\[ Q = KA \frac{dh}{dl} \]

Conductance = \( KA/\text{thickness} \)

then MODFLOW calculates the flow as:

\[ Q = \text{Conductance} \, dh \]

Conductance of the drain is calculated as:

\[ K \text{ of material over which gradient is calculated} \times \frac{\text{Area}}{\text{thickness}} \]

Area may be the cylindrical area midway between where the heads used for the gradient are located \(*\) length of the drain.

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MODFLOW Flux Boundary Condition Packages

**Recharge package**

For recharge, a rate is specified for each cell.

MODFLOW calculates

\[ Q = \text{rate} \times \text{cell area} \]

Q is specified & forced in (or out) unless the cell “goes dry” in which case the modeler may choose to apply it to a lower layer.

Heads increase as recharge is increased.
When pumping a well in a MODFLOW grid cell, Q is specified & withdrawn or injected unless the cell “goes dry.” Calculated drawdown represents the average drawdown in the cell, not the actual drawdown in the well.

MODFLOW assigns one head value to each cell.

Next we will use MODFLOW and GMS (Ground Water Modeling System) to get a feel for ground water modeling. The model will be used to design withdrawals from an existing well field to meet regulatory requirements on drawdown and change in stream flow. If the simulated head is lower than the desired head the error bar will drop down from the center, otherwise it will be above the center. If the difference is <= the specified 95% intervals the bar will be green, if the difference is greater than that but less than twice that it will be yellow and beyond that it will be red. This feature is usually used for calibrating a model but here it is a useful tool for accomplishing your design.
Locations of Production Wells
Where you can adjust withdrawal rate

Experiment with MODFLOW
Using the GMS (Ground Water Modeling System)

GUI (Graphical User Interface)

Download the Example Files and Associated Write-up on the class web page for this lecture
http://inside.mines.edu/~epoeter/_GW/24Modeling/Modeling.htm