If we only consider advection and start with a "point" of material with $C_o=1000\text{mg/l}$

A point has no volume so can it have a concentration? So why do we say a “point”?

$K = 0.1 \text{ cm/sec}$
$dh = 10 \text{ cm}$
$dl = 100 \text{ cm}$
$\phi = 0.2$

How long will it take for the material to move 50 cm?
What will the concentration be at that location at that time?

<table>
<thead>
<tr>
<th>in the down gradient direction</th>
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<tbody>
<tr>
<td>$v = \frac{Kdh}{\phi dl} = \frac{0.1 \text{ cm}}{\text{sec}} \cdot \frac{10 \text{ cm}}{0.2 \cdot 100 \text{ cm}} = 0.05 \text{ cm/sec}$</td>
</tr>
</tbody>
</table>

$d = vt$
$t = \frac{d}{v} = \frac{50 \text{ cm}}{0.05 \text{ cm/sec}} = 1000 \text{ sec} = 0.28 \text{ hr}$

$C = C_o = 1000 \frac{\text{mg}}{\text{l}}$

Suppose that source enters the up gradient end of a column
At a continuous concentration of $C_o=1000\text{mg/l}$

$K = 0.1 \text{ cm/sec}$
$dh = 10 \text{ cm}$
$dl = 100 \text{ cm}$
$\phi = 0.2$

Dispersivity $\alpha_x = 5 \text{ cm}$

What will the concentration be at 50 cm after 1000 sec?

average linear velocity

$\bar{v} = \frac{Kdh}{\phi dl} = \frac{0.1 \text{ cm}}{\text{sec}} \cdot \frac{10 \text{ cm}}{0.2 \cdot 100 \text{ cm}} = 0.05 \text{ cm/sec}$

distance traveled in 1000 sec?

$d = \bar{v}t = 0.05 \frac{\text{cm}}{\text{sec}} \cdot 1000 \text{ sec} = 50 \text{ cm}$

By inspection we know that the concentration should be $0.5C_o=500\text{mg/l}$
But let’s carry out the calculation
Experiment with the spreadsheet
http://inside.mines.edu/~epoeter/_GW/22ContamTrans/C1d.xls

Note the values of C using only the first term and then both terms at times and locations where your intuition allows you to know the concentration.

When is use of the second term important? When does excel cause it to be in error?

Try  \( x = 50, 49, 51, 0, 100 \) then 10, 30, 200

Consider other times
TRY 10000sec  \( x = 500, 490, 510, 0, 1000 \)
then \( x = 100, 200, 300, 400, 450 \), compare 450 to 550 ?symmetrical?

Where and when can you know the correct C?
The second term is important for calculating \( C \) @ early times near the source.

\[
\bar{v} = 0.05 \frac{cm}{sec} \quad x = 0.05 \frac{cm}{sec} \quad 1000sec = 50cm \quad so \quad X = Y = Z = 0 \quad and \quad we \quad want \quad C_{\text{max}}
\]

\[
\begin{align*}
D_x &= \bar{v}\alpha x + D^* = 0.05 \frac{cm}{sec} \cdot 5cm + 1 \times 10^{-10} \frac{m^2}{sec} \cdot \frac{1000cm^2}{1m^2} = 0.25 \frac{cm^2}{sec} \\
D_y &= \bar{v}\alpha y + D^* = 0.05 \frac{cm}{sec} \cdot \frac{1}{5} \cdot 5cm + 1 \times 10^{-10} \frac{m^2}{sec} \cdot \frac{1000cm^2}{1m^2} = 0.05 \frac{cm^2}{sec} \\
D_z &= \bar{v}\alpha y + D^* = 0.05 \frac{cm}{sec} \cdot \frac{1}{10} \cdot 5cm + 1 \times 10^{-10} \frac{m^2}{sec} \cdot \frac{1000cm^2}{1m^2} = 0.025 \frac{cm^2}{sec}
\end{align*}
\]

\[
C = \frac{M}{8(\pi)^{3/2}\sqrt{D_x D_y D_z}}
\]

\[
C = \frac{1000mg}{8(\pi 1000sec)^{3/2}\sqrt{0.25 \frac{cm^2}{sec} \cdot 0.05 \frac{cm^2}{sec} \cdot 0.025 \frac{cm^2}{sec}}} = 0.0402 \frac{mg}{cm^3} \cdot \frac{1000cm^3}{l} = 40.2 \frac{mg}{l} \sim 40 \frac{mg}{l}
\]