HW#6

Assigned: Tuesday, September 29, 2009

Due: Tuesday, October 13, 2009

a) Derive the numerical dispersion for the 1-D dispersion equation with advection/conduction only.

Start with:

\[-u \frac{\partial C}{\partial x} = \frac{\partial C}{\partial t} \quad \longrightarrow \quad -u \frac{C_{i}^{n+1} - C_{i-1}^{n+1}}{\Delta x} = \frac{C_{i}^{n+1} - C_{i}^{n}}{\Delta t}\]

Expand using the backward difference in time:

\[C_{i}^{n+1} = C_{i}^{n} - \Delta t \left( \frac{\partial C}{\partial t} \right)_{i}^{n+1} + \frac{(\Delta t)^2}{2!} \left( \frac{\partial^2 C}{\partial t^2} \right)_{i}^{n+1} + O((\Delta t)^3)\]

b) For a three-node 1-D Buckley Leverett problem, show that the following formulation is not conservative. Use one boundary node for injection at the left edge, with a different $\Delta x$ at each node. Use $\Delta x_1$ for the first node, not $\Delta x_1/2$.

Start with:

\[-u_r \frac{\partial f_w}{\partial x} = \frac{\partial S_w}{\partial t} \quad \longrightarrow \quad -u_r \left( \frac{\partial f_w}{\partial S_w} \right)_{i-1}^{n+1} \frac{S_{w,i}^{n+1} - S_{w,i-1}^{n+1}}{\Delta x_i} = \frac{S_{w,i}^{n+1} - S_{w,i}^{n}}{\Delta t}\]