HW#6

Assigned: Tuesday, September 29, 2009

Due: Tuesday, October 13, 2009

a) Derive the numerical dispersion for the 1-D dispersion equation with advection/conduction only.

Start with:

$$-u\frac{\partial C}{\partial x} = \frac{\partial C}{\partial t} \longrightarrow -u\frac{C_i^{n+1} - C_{i-1}^{n+1}}{\Delta x} = \frac{C_i^{n+1} - C_i^n}{\Delta t}$$

Expand using the backward difference in time:

$$C_i^{n+1} = C_i^n - \Delta t\left(\frac{\partial C}{\partial t}\right)_i^{n+1} + \frac{(\Delta t)^2}{2!}\left(\frac{\partial^2 C}{\partial t^2}\right)_i^{n+1} + \mathcal{O}[(\Delta t)^3]$$

b) For a three-node 1-D Buckley Leverett problem, show that the following formulation is *not* conservative. Use one boundary node for injection at the left edge, with a different Δx at each node. Use Δx_1 for the first node, not $\Delta x_1/2$.

Start with:

$$-u_T\frac{\partial \hat{f}_w}{\partial x} = \frac{\partial S_w}{\partial t} \longrightarrow -u_T\left(\frac{\partial \hat{f}_w}{\partial S_w}\right)_{i-1}^{n+1}\frac{S_{wi}^{n+1} - S_{w,i-1}^{n+1}}{\Delta x_i} = \frac{S_{wi}^{n+1} - S_{wi}^n}{\Delta t}$$