TERM PROJECT

Version: 2009-11-19 (there may be other versions later)

The term project involves a 2-D formulation and a 1-D solution of the flow equations described in the next several pages. The term project should stand on it's own – feel free to include portions of homework problems. Include anything that you feel will help us know that you understand the material.

1) Describe the finite difference formulation for each of the four options in 2-D. Include pictures of 0 and non-zero portions of matrix equations.
2) Present the results of your 1-D simulations for option 1, 2, and 4, including a comparison of the different techniques.
3) Provide a copy of the source code you used for this project

Option 1: IMPES; Implicit Pressure, Sequential Explicit Saturation
Solve for P_o^{n+1} *using an implicit matrix equation, then solve for* S_w^{n+1} *using an explicit equation.*
Option 2: Partially Implicit; Implicit Pressure, Sequential Saturation
Solve for P_o^{n+1} *using an implicit matrix equation, then solve for* S_w^{n+1} *using an implicit matrix equation with* P_{cow}^{n+1}.
Option 3: Fully Implicit; Iterative Saturation
Solve for $P_o^{\ell+1}$*, using* S_w^n*. Next, solve for* $S_w^{\ell+1}$ *using* $P_o^{\ell+1}$*. Iterate, alternating between solving for* P_o *and* S_w.
Option 4: Fully Implicit; Coupled
Simultaneously solve for P_o *and* S_w.

Option 1: IMPES; Implicit Pressure, Sequential Explicit Saturation

Solve for P_o^{n+1} *using an implicit matrix equation, then solve for* S_w^{n+1} *using an explicit equation.*

Pressure Equation: $\nabla \cdot k\left(\lambda_T^n \nabla P_o^{n+1} - (\lambda_w^n \gamma_w^n + \lambda_o^n \gamma_o^n)\nabla D - \lambda_w^n \nabla P_{cow}^n\right) + \hat{q}_T^{n+1} = \phi C_T^n \dfrac{\partial P_o}{\partial t}$

Saturation equation, explicit (IMPES):

$$\nabla \cdot k\lambda_w^n \left(\nabla P_o^{[n+1]} - \gamma_w^n \nabla D - \nabla P_{cow}^n\right) + \hat{q}_w^{[n+1]} = \phi S_w^n (C_\phi + C_w)\frac{\partial P_o}{\partial t} + \phi \frac{\partial S_w}{\partial t}$$

Option 2: Partially Implicit; Implicit Pressure, Sequential Saturation

Solve for P_o^{n+1} *using an implicit matrix equation, then solve for* S_w^{n+1} *using an implicit matrix equation with* P_{cow}^{n+1}.

Pressure Equation: $\nabla \cdot k\left(\lambda_T^n \nabla P_o^{n+1} - (\lambda_w^n \gamma_w^n + \lambda_o^n \gamma_o^n)\nabla D - \lambda_w^n \nabla P_{cow}^n\right) + \hat{q}_T^{n+1} = \phi C_T^n \dfrac{\partial P_o}{\partial t}$

Saturation equation, partially implicit:

$$\nabla \cdot k\lambda_w^n \left(\nabla P_o^{[n+1]} - \gamma_w^n \nabla D - \frac{\partial P_{cow}^n}{\partial S_w}\nabla S_w^{n+1}\right) + \hat{q}_w^{[n+1]} = \phi S_w^{n+1} (C_\phi + C_w)\frac{\partial P_o}{\partial t} + \phi \frac{\partial S_w}{\partial t}$$

Option 3: Fully Implicit; Iterative Saturation

Solve for $P_o^{\ell+1}$, *using* S_w^n. *Next, solve for* $S_w^{\ell+1}$ *using* $P_o^{\ell+1}$. *Iterate, alternating between solving for* P_o *and* S_w.

Pressure Equation:

$$\nabla \cdot k\left(\lambda_T^{[n+1]} \nabla P_o^{n+1} - (\lambda_w^{[n+1]} \gamma_w^n + \lambda_o^n \gamma_o^{[n+1]})\nabla D - \lambda_w^{[n+1]} \nabla P_{cow}^{[n+1]}\right) + \hat{q}_T^{n+1} = \phi C_T^{[n+1]} \frac{P_o^{n+1} - P_o^n}{\Delta t}$$

Saturation equation, fully implicit:

$$\nabla \cdot k\lambda_w^{n+1} \left(\nabla P_o^{[n+1]} - \gamma_w^{[n+1]} \nabla D - \frac{\partial P_{cow}^{[n+1]}}{\partial S_w}\nabla S_w^{n+1}\right) + \hat{q}_w^{[n+1]} = \phi S_w^{n+1} (C_\phi + C_w)\frac{P_o^{[n+1]} - P_o^n}{\Delta t} + \phi \frac{S_w^{n+1} - S_w^n}{\Delta t}.$$

Option 4: Fully Implicit; Coupled

Simultaneously solve for P_o *and* S_w.

Water Saturation equation, fully implicit:

$$\nabla \cdot k\lambda_w^{n+1} \left(\nabla P_o^{n+1} - \gamma_w^n \nabla D - \frac{\partial P_{cow}^{n+1}}{\partial S_w}\nabla S_w^{n+1}\right) + \hat{q}_T^{n+1} = \phi S_w^{n+1} (C_\phi + C_w)\frac{P_o^{n+1} - P_o^n}{\Delta t} + \phi \frac{S_w^{n+1} - S_w^n}{\Delta t}$$

Oil Saturation equation, fully implicit:

$$\nabla \cdot k\lambda_o^{n+1} \left(\nabla P_o^{n+1} - \gamma_o^n \nabla D\right) + \hat{q}_T^{n+1} = \phi S_o^{n+1} (C_\phi + C_o)\frac{P_o^{n+1} - P_o^n}{\Delta t} + \phi \frac{S_o^{n+1} - S_o^n}{\Delta t}$$

Capillary pressure and relative permeability equations:

$$k_{row} = k_{row}^{*}\left(\frac{S_o - S_{owr}}{1 - S_{wr} - S_{owr}}\right)^{n_{ow}} \qquad k_{rw} = k_{rw}^{*}\left(\frac{S_w - S_{wr}}{1 - S_{wr} - S_{owr}}\right)^{n_w}$$

$$P_{cow1} = \alpha \ln\left[\frac{1 - S_w - S_{owr}}{1 - S_{wx} - S_{owr}}\right] \qquad P_{cow2} = -\alpha \ln\left[\frac{S_w - S_{wr}}{S_{wx} - S_{wr}}\right]$$

$$P_{cow} = \begin{cases} P_{cow,max}, & S_w \le S_{wr} \\ P_{cow,min}, & S_w \ge (1 - S_{owr}) \\ \min[P_{cow2}, P_{cow,max}], & S_{wr} \le S_w \le S_{wx} \\ \max[P_{cow1}, P_{cow,min}], & (1 - S_{owr}) \ge S_w \ge S_{wx} \end{cases}$$

Data:

$NX = 10$, $NY = 10$, $\Delta x = 10\, ft$, $\Delta y = 10\, ft$, $\Delta z = 10\, ft$

$k_{rw}^{*} = 0.1$ $\quad k_{row}^{*} = 0.7$ $\quad n_w = 1.5$ $\quad n_{ow} = 2.5$ $\quad S_{orw} = 0.30$ $\quad S_{wr} = 0.25$

$\alpha = 0.8$; $S_{wx} = 0.5$ $\quad P_{cow,max} = +5\, psi$ $\quad P_{cow,min} = -5\, psi$

$\mu_w = 0.6cp$ $\quad \mu_o = 2.4cp$ $\quad \gamma_w = 0.433\, psi/ft$ $\quad \gamma_o = 0.433 * 0.8\, psi/ft$

$\phi = 0.20$, $q_{INJ} = 200\, ft^3/day$, $k = 100md$

$P_i = 3000\, psia$, $P_w = 2500\, psia$, $S_{wi} = 0.25$

$C_\phi = 3 \cdot 10^{-6}\, psi^{-1}$, $C_w = 4 \cdot 10^{-6}\, psi^{-1}$, $C_o = 10 \cdot 10^{-6}\, psi^{-1}$