# Gravity-Drainage and Oil-Reinfiltration Modeling in Naturally Fractured Reservoir Simulation

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#### Summary

The gravity-drainage and oil-reinfiltration processes that occur in the gas-cap zone of naturally fractured reservoirs (NFRs) are studied through single porosity refined grid simulations. A stack of initially oil-saturated matrix blocks in the presence of connate water surrounded by gas-saturated fractures is considered; gas is provided at the top of the stack at a constant pressure under gravity-capillary dominated flow conditions. An in-house reservoir simulator, SIMPUMA-FRAC, and two other commercial simulators were used to run the numerical experiments; the three simulators gave basically the same results.

Gravity-drainage and oil-reinfiltration rates, along with average fluid saturations, were computed in the stack of matrix blocks through time. Pseudofunctions for oil reinfiltration and gravity drainage were developed and considered in a revised formulation of the dual-porosity flow equations used in the fractured reservoir simulation.

The modified dual-porosity equations were implemented in SIMPUMA-FRAC (Galindo-Nava 1998; Galindo-Nava et al. 1998), and solutions were verified with good results against those obtained from the equivalent single porosity refined grid simulations. The same simulations—considering gravity drainage and oil reinfiltration processes—were attempted to run in the two other commercial simulators, in their dual-porosity mode and using available options. Results obtained were different among them and significantly different from those obtained from SIMPUMA-FRAC.

#### Introduction

One of the most important aspects in the numerical simulation of fractured reservoirs is the description of the processes that occur during the rock-matrix/fracture fluid exchange and the connection with the fractured network. This description was initially done in a simplified manner and therefore incomplete (Gilman and Kazemi 1988; Saidi and Sakthikumar 1993).

Experiments and theoretical and numerical studies (Saidi and Sakthikumar 1993; Horie et al. 1998; Tan and Firoozabadi 1990; Coats 1989) have allowed to understand that there are mechanisms and processes, such as oil reinfiltritation or oil imbibition and capillary continuity between matrix blocks, that were not taken into account with sufficient detail in the original dual-porosity formulations to model them properly and that modify significantly the oil-production forecast and the ultimate recovery in an NFR.

The main idea of this paper is to study in further detail the oil reinfiltration process that occurs in the gas invaded zone (gas cap zone) in an NFR and to evaluate its modeling to implement it in a dual-porosity numerical simulator.

We consider the reservoir to be a stack of matrix blocks (sugar cubes) according to the Warren and Root (1963) conceptual dualporosity model, and oil reinfiltration occurs when the oil confined in the upper blocks is expelled out of matrix blocks through fractures and it reinfiltrates in the blocks below. This block-toblock oil flow occurs mainly because of the competition of the capillary and viscous forces.

In the second part of the paper, we used a single-porosity simulator to build a fine grid in the space occupied by the stack of matrix blocks and fractures allocating the particular characteristics and properties of each medium to the different portions that these systems occupy in the grid.

The processes that occur during the numerical experiment were studied. The capillary forces act only on the matrix blocks with a value of zero in the fractures and the viscous forces are canceled out through the introduction of a very low gas injection rate through the top face of the stack; a flow process driven by capillary and gravity forces only is established in this fashion (Vicencio-Fuentes 1998; Ortega-Galindo 2000). This is done because a 100% gas saturation must be guaranteed in the fractures while maintaining a constant pressure system to avoid a pressure change that would indicate the action of viscous forces.

In the fine-grid simulation, average gas and oil saturations are computed as time goes by for each one of the matrix blocks in the stack. Drainage and reinfiltration rates are computed through each one of the matrix-block faces and their dependencies on the matrix-block average gas saturations are established. Then the pseudofunctions that are required in the modified dual porosity formulation are calculated.

For the second part of the study, using the modified dual porosity simulator SIMPUMA-FRAC, a coarse grid is built with same dimensions as the single porosity fine grid and the gravity drainage is simulated by using the matrix-fracture transfer pseudofunctions that had been previously generated. Hence, the modified-dual-porosity simulator should reproduce the average behavior observed in the fine grid for the stack of blocks in the single-porosity model.

A comparison is also made with two commercial simulators in their single- and dual-porosity formulations and also with what is claimed to be their gravity drainage options.

#### **Problem Formulation**

Let us consider the 3D flow of oil and gas in the presence of connate water in some part of a NFR made of matrix blocks surrounded by fractures (**Figs. 1a and 1b**). Let us consider also that at time zero in the stack the p (oil or gas phase), pressure distribution is given by a supposed fluid gravitational equilibrium; matrix blocks are saturated by oil and connate water; and fractures are 100% saturated by gas. The reservoir lateral boundaries are not permeable; the upper and lower boundaries are open to flow and sources or sinks do not exist in that part of the reservoir. The zero-flow or impermeable-lateral-boundary condition is the result of the fluid-flow symmetry, which exists among the matrix blocks that make up the reservoir (Fig. 1a). Let us consider that the z direction coincides with the vertical direction and that the grid is orthogonal, the latter situation implies the gravity force will be present only in the z flow direction.

#### **Differential Flow Equations**

The differential equations that describe the flow of fluids in a NFR, considering the reinfiltration and gravity drainage are (Kazemi et al. 1976):

Fracture Flow Equations

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Fig. 1—Single-porosity stack of matrix blocks (a) and dual-porosity stack of matrix blocks with multiphase flow developed by Kazemi et al. (1976).

$$\nabla \cdot [\lambda_o(\nabla p_o - \gamma_o \nabla z)] + \tau^*_{omf} + q^*_{oDmf} - q^*_{oRfm} = \frac{\partial}{\partial t} (\phi b_o S_o).$$
<sup>(1)</sup>

Gas:

$$\nabla \cdot \left[\lambda_g \left(\nabla p_g - \gamma_g \nabla z\right) + R_s \lambda_o (\nabla p_o - \gamma_o \nabla z)\right] + \tau^*_{gmf} + \tau^*_{omf} R_s$$
$$+ q^*_{gNmf} + \left(q^*_{oDmf} R_s\right) - \left(q^*_{oRfm} R_s\right) = \frac{\partial}{\partial t} \left[\phi b_g S_g + \phi b_o R_s S_o\right].$$

Matrix/Fracture Flow Equations

Oil:

$$-\tau_{omf}^* - q_{oDmf}^* + q_{oRfm}^* = \frac{\partial}{\partial t} (\phi b_o S_o)_m.$$
(3)

Gas:

$$-\tau_{gmf}^{*} - \tau_{omf}^{*}R_{s} - q_{gNmf}^{*} - \left(q_{oDmf}^{*}R_{s}\right) + \left(q_{oRfm}^{*}R_{s}\right)$$
$$= \frac{\partial}{\partial t} \left[\phi b_{g}S_{g} + \phi b_{o}R_{s}S_{o}\right]_{m}$$
$$0 < x < x_{e}, \quad 0 < y < y_{e}, \quad 0 < z < z_{e}, \quad t > 0. \dots \dots \dots (4)$$

Viscous, gravity, and capillary effects are considered in the development of the multiphase flow gas/oil with connate-water equations, for matrix and matrix/fractures and for heterogeneities and anisotropy present in the fractured porous media as well. The term for matrix-fracture fluid exchange caused by viscous forces,  $\tau_{pmf}^*$ , p = o, g, is based on the Warren and Root (Warren and Root 1963; Barenblat et al. 1960) theory extension from mono phase to

Note that Eq. 5 considers the pressure difference in the oil phase, between matrix and fractures for  $\tau_{pmf}^*$ , instead of pressure difference in the gas phase, because capillary pressure effects are taken into account through the exchange terms for oil gravity drainage and oil reinfiltration,  $q_{oDmf}$  and  $q_{oRfm}$ , respectively. These terms act as sink or source terms in the fracture equations in the entire space and time domain of the problem (Thomas et al. 1983). The remaining exchange matrix-fracture terms definitions that appear in Eqs. 1 through 4 will be described later.

The  $\sigma$  coefficient is a shape factor that considers the matrix block area exposed to fluid exchange between matrix and fracturesper-volume unit and a characteristic length associated to matrixfracture flow (Galindo Nava 1998; Warren and Root 1963; Kazemi et al. 1976; Thomas et al. 1983; Rodríguez de la Garza 1987; Galindo Nava and Rodríguez de la Garza 1998).

The *p*, oil or gas, phase relative permeabilities for the matrix are function of matrix gas saturation,  $S_{gm}$ ,

$$k_{rpm} = k_{rpm}(S_{gm}).$$
 (6)

According to Eqs. 1 and 2, there are two equations for fractures with these unknowns or primary variables:  $p_o, p_g, S_o, S_g$ , hence, there are four unknowns. The same thing happens for Eqs. 3 and 4, there are two equations and four unknowns:  $p_{om}, p_{gm}, S_{om}, S_{gm}$ . Therefore two additional equations are needed for each medium. In this way, there will be a consistent set of four equations with four unknowns for each medium and it will be possible to be solved. The primary variables or unknowns for this case were:  $p_o, S_g, p_{om}, S_{em}$ .

The additional equations needed are the constraint equations and the capillary pressure equations.

## **Constraint Equations**

$S_o + S_g + S_{wc} = 1,$		 (7)
$S_{om} + S_{gm} + S_{wcm} =$	1	 

#### **Finite-Difference Flow Equations**

The nonlinear partial differential Eqs. 1 through 4 are solved numerically. For that, they are discretized by employing the finite-difference method. The flow term in the equations is approximated by central differences in space and the accumulation terms are approximated by backward difference in time.

From Eqs. 1 and 2, applying the finite-difference method and multiplying each side of the equations by  $Vr_{i,j,k}$  the following finite-difference flow equations are obtained for oil and gas, correspondingly:

Fracture Flow Equations

$$\Delta [T_o(\Delta p_o - \gamma_o \Delta D)]_{i,j,k}^{n+1} + (\tau_{omf})_{i,j,k}^{n+1} + (q_{oDmf})_{i,j,k}^{n+1} - (q_{oRfm})_{i,j,k+1}^{n+1}$$
$$= \frac{Vr_{i,j,k}}{\Delta t} \Delta_t [\phi b_o (1 - S_g - S_{wc})]_{i,j,k}, \qquad (11)$$

Matrix/Fracture Flow Equations

The transmissibility that appears in the finite-difference equations for fractures is defined for the p, oil or gas phase as,

The transmissibility that appears in the flow finite-difference equations for matrix-fracture that come from Eq. 5 is defined for the p, oil or gas phase as,

For parallelepiped matrix blocks, the shape factor is defined as follows:

$$\sigma = 4\left(\frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2}\right).$$
 (17)

Relative permeabilities from matrix/fracture flow equations are evaluated at upstream conditions depending on the way the matrix blocks and fractures exchange fluids. The pressure dependent functions in these transmissibilities are evaluated at the existing pressure at the matrix blocks (Galindo-Nava. 1998; Thomas et al. 1983; Rodríguez de la Garza 1987; Galindo-Nava et al. 1998).

#### **Reinfiltration Pseudofunction**

The drainage oil rate at surface conditions is given by:

where  $q_{oDmf_{i,j,k}}^{**} = q_{oDmf_{i,j,k}}^{**}(S_{gm_{i,j,k}})$  is the matrix-blocks-drainage oil rate in the gridblock *i*, *j*, *k* at reservoir-conditions-per-bulk-volume unit, which is a function of matrix blocks gas saturation.  $q_{oDmf_{i,j,k}}^{**}$  is obtained trough fine-grid simulations in a stack of matrix blocks, as suggested by Tan and Firoozabadi (1995). Notice that  $S_{g_{i,j,k}}$  was introduced in the Eq. 18 to account for the matrix blocks that are really exposed to gravity drainage when fractures in the *i*, *j*, *k* simulation gridblock are partially filled with gas.

The reinfiltration oil rate at surface conditions in matrix blocks in the i, j, k simulation gridblock can be obtained from:

where  $q_{okfm_{i,j,k}}^{**}$  is the reinfiltration oil rate at reservoir conditions in matrix blocks in the *i*, *j*, *k* simulation-gridblock-per-bulk-volume unit, given by:

$$q_{oRfm_{i,j,k}}^{**} = \frac{-\left(\sigma_{R}^{*}K_{eo}^{*}\right)_{i,j,k} \left[p_{om_{i,j,k}} - p_{o_{i,j,k-1}} - \left(\gamma_{o}\Delta z\right)_{i,j,k-\frac{1}{2}}\right]}{\left(\mu_{o}\Delta z\right)_{i,j,k-\frac{1}{2}}},$$
(20)

where  $\sigma_R^*$  is the reinfiltration shape factor, defined as the area of matrix blocks in the simulation gridblock *i*, *j*, *k* exposed to reinfiltration per bulk volume unit; and  $K_{eo}^* = K_{eo}^*(S_{gm_{i,k}})$  is the effective oil reinfiltration pseudopermeability. The reinfiltration pseudofunction  $(\sigma_R^* K_{eo}^*)_{i,j,k}$  can be obtained from fine grid simulations in a stack of matrix blocks, as described below.

The net exchange of free gas rate between the matrix blocks and fractures  $q_{gNmf_{LLR}}$  because of gravity and capillary effects is:

The above equation implies that the net oil exchange between matrix blocks and fractures at reservoir conditions because of capillary and gravity effects produces an equal exchange of free gas in the opposite direction.

When the above equations are introduced, the pseudocomponent p exchange rate because of viscous effects, (i.e., because of the pressure differences between matrix and fractures in Eq. 5) is modified this way:

$$\tau_{pmf_{i,j,k}} = T_{pmf_{i,j,k}} (p_{om} - p_o)_{i,j,k}. \qquad (22)$$

# Finite-Difference Equations Linearization: Newton-Raphson Method

Eqs. 11 through 14 to each one of the *i*, *j*, *k* grid nodes at each time level n+1 is applied give a nonlinear system with 4IJK equations with 4IJK unknowns,  $(p_o, S_g, p_{om}, S_{gm})_{i,j,k}$ , i=1, 2, ..., I; j=1,2, ..., J;

 $k=1, 2, \ldots, K$ , whose solution can be obtained using a full implicit formulation through Newtonian iteration; hence, the following residual functions are defined.

#### Fracture Flow Equations

Oil:

Gas:

$$F_{g_{ij,k}}^{n+1} = \Delta \Big[ T_g \Big( \Delta p_g + \Delta P_{cgo} - \gamma_g \Delta D \Big) \Big]_{i,j,k}^{n+1} \\ + \Delta [T_o R_s (\Delta p_o - \gamma_o \Delta D)]_{i,j,k}^{n+1} + \Big( \tau_{gmf} \Big)_{i,j,k}^{n+1} + \Big( \tau_{omf} R_s \Big)_{i,j,k}^{n+1} \\ + \Big( q_{gNmf} \Big)_{i,j,k}^{n+1} + \Big( q_{oDmf} R_s \Big)_{i,j,k}^{n+1} - \Big( q_{oRfm} R_s \Big)_{i,j,k+1}^{n+1} \\ - \frac{V r_{i,j,k}}{\Delta t} \Delta_t \Big[ \phi b_g S_g + \phi b_o \big( 1 - S_g - S_{wc} \big) \Big]_{i,j,k} = 0.$$
(24)

Matrix/Fracture Flow Equations

Oil:

$$F_{om_{i,j,k}}^{n+1} = -\left(\tau_{omf}\right)_{i,j,k}^{n+1} - \left(q_{oDmf}\right)_{i,j,k}^{n+1} + \left(q_{oRfm}\right)_{i,j,k-1}^{n+1} \dots \dots (25) - \frac{Vr_{i,j,k}}{\Delta t} \Delta_t \left[\phi \, b_o \left(1 - S_g - S_{wc}\right)\right]_{m_{i,j,k}} = 0$$

Gas:

First, it can be noticed that residual functions for fractures  $F_{p_{i,j,k}}^{n+1}$  and matrix  $F_{pm_{i,j,k}}^{n+1}$  have in general the following dependency on the unknowns:

$$F_{p_{i,j,k}}^{n+1} = F_p \left[ \left( p_o, S_g \right)_{i,j,k-1}, \left( p_o, S_g \right)_{i,j-1,k}, \left( p_o, S_g \right)_{i-1,j,k}, \left( p_o, S_g \right)_{i,j,k}, \\ \left( p_o, S_g \right)_{i+1,j,k}, \left( p_o, S_g \right)_{i,j+1,k}, \left( p_o, S_g \right)_{i,j,k+1}, \left( p_{om}, S_{gm} \right)_{i,j,k}, \\ \left( p_{om}, S_{gm} \right)_{i,j,k+1} \right]^{n+1}.$$
(27)

And

$$F_{pm_{i,j,k}}^{n+1} = F_{pm} \Big[ (p_o, S_g)_{i,j,k-1}, (p_o, S_g)_{i,j,k}, (p_{om}, S_{gm})_{i,j,k} \Big]^{n+1}.$$
(28)

Note that in Eq. 27 there is an additional term for fractures with respect to the traditional dual-porosity formulation that inserts matrix unknowns in the i,j,k+1 node while in the matrix/fracture flow equations of Eq. 28, there is an additional term that inserts fracture unknowns in the i,j,k-1 node. The residual functions in Eqs. 27 and 28 are expanded from a truncated Taylor series that holds only the lower order terms, an iterative Newton-Raphson scheme is established to solve the fully implicit finite-difference flow equations, Eqs. 23 through 26, which leads to the following system of linear equations:

Fracture Flow Equations

$$\begin{split} \sum_{lmn} \left\{ \left( \frac{\partial F_{p_{ij,k}}}{\partial p_{olmn}} \right)^{(\nu)} \delta p_{olmn}^{(\nu+1)} + \left( \frac{\partial F_{p_{ij,k}}}{\partial S_{g_{lmn}}} \right)^{(\nu)} \delta S_{glmn}^{(\nu+1)} \right\} \\ &+ \left( \frac{\partial F_{p_{ij,k}}}{\partial p_{om_{ij,k}}} \right)^{(\nu)} \delta p_{om_{ij,k}}^{(\nu+1)} + \left( \frac{\partial F_{p_{ij,k}}}{\partial S_{gm_{ij,k}}} \right)^{(\nu)} \delta S_{gm_{ij,k}}^{(\nu+1)} \\ &+ \left( \frac{\partial F_{p_{ij,k}}}{\partial p_{om_{ij,k+1}}} \right)^{(\nu)} \delta p_{om_{ij,k+1}}^{(\nu+1)} + \left( \frac{\partial F_{p_{ij,k}}}{\partial S_{gm_{ij,k+1}}} \right)^{(\nu)} \delta S_{gm_{ij,k+1}}^{(\nu+1)} = -F_{p_{ij,k}}^{(\nu)} \\ lmn = i, j, k - 1; i, j - 1, k; i - 1, j, k; i, j, k; i + 1, j, k; \\ i, j + 1, k; i, j, k + 1; p = o, g. \end{split}$$

Matrix/Fracture Flow Equations

Eqs. 29 and 30 now constitute a linear system whose unknowns are the pressure and saturation matrix and fracture iterative changes,  $(\delta p_o, \delta S_g, \delta p_{om}, \delta S_{gm})_{i,j,k}^{(\nu+1)}$  at the iteration level ( $\nu$ +1). The matrix structure for the system of equations of a normal

The matrix structure for the system of equations of a normal ordering generates a heptadiagonal matrix. Taking just one row, Eqs. 29 and 30 can be written as:

where  $h_{i,j,k}$ ,  $f_{i,j,k}$ ,  $c_{i,j,k}$ ,  $a_{i,j,k}$ ,  $b_{i,j,k}$ ,  $e_{i,j,k}$ , and  $g_{i,j,k}$  are 2×2 order submatrix containing the derivatives of the residual functions for each *i*, *j*, *k* gridblock in the simulation grid with respect to the unknowns in *i*, *j*, *k*-1; *i*, *j*-1, *k*; *i*-1, *j*, *k*; *i*, *j*, *k*; *i*+1, *j*, *k*; *i*, *j*+1, *k*; *i*, *j*, *k*+1, respectively. The *ff* and *fm* subscripts indicate the fracture residual function derivatives with respect to the fracture and matrix unknowns, respectively. Similarly, the *mf* and *mm* subscripts indicate the matrix residual function derivatives with respect to the fracture and matrix unknowns, respectively. The subvectors  $\delta u_{fi,j,k}$  and  $\delta u_{mi,j,k}$  are of order 2 and contain the fracture and matrix unknowns iterative changes for the *i*,*j*,*k* simulation gridblock.  $F_{i,j,k}$  and  $F_{m,i,j,k}$  are subvectors or order 2 that contain the oil and gas residual functions for the simulation *i*, *j*, *k* gridblock for fracture and matrix, respectively.

Eq. 31 can also be written as follows:

Fracture Flow Equations

$$\begin{aligned} h_{ff_{i,j,k}}^{(v)} \delta u_{f_{i,j,k-1}}^{(v+1)} + f_{ff_{i,j,k}}^{(v)} \delta u_{f_{i,j-1,k}}^{(v+1)} + c_{ff_{i,j,k}}^{(v)} \delta u_{f_{i-1,j,k}}^{(v+1)} + a_{ff_{i,j,k}}^{(v)} \delta u_{f_{i,j,k}}^{(v+1)} \\ + a_{fm_{i,j,k}}^{(v)} \delta u_{m_{i,j,k}}^{(v+1)} + b_{ff_{i,j,k}}^{(v)} \delta u_{f_{i+1,j,k}}^{(v+1)} + e_{ff_{i,j,k}}^{(v)} \delta u_{f_{i,j+1,k}}^{(v+1)} \\ + g_{ff_{i,j,k}}^{(v)} \delta u_{f_{i,j,k-1}}^{(v+1)} + g_{fm_{i,j,k}}^{(v)} \delta u_{m_{i,j,k+1}}^{(v+1)} = -F_{i,j,k}^{(v)} \quad \dots \dots \dots \dots (32) \end{aligned}$$

#### Matrix/Fracture Flow Equations

$$h_{mf_{i,j,k}}^{(\nu)} \delta u_{f_{i,j,k-1}}^{(\nu+1)} + a_{mf_{i,j,k}}^{(\nu)} \delta u_{f_{i,j,k}}^{(\nu+1)} + a_{mm_{i,j,k}}^{(\nu)} \delta u_{m_{i,j,k}}^{(\nu+1)} = -F_{m_{i,j,k}}^{(\nu)} \dots (33)$$

Now  $\delta u_{m_{i,j,k}}$  can be solved in terms of  $\delta u_{f_{i,j,k-1}}$  and  $\delta u_{f_{i,j,k}}$  from Eq. 33 as follows:

where

$$F_{m_{ij,k}}^{(\nu)} = \left(a_{mm_{ij,k}}^{(\nu)}\right) F_{m_{ij,k}}^{(\nu)}.$$
(37)

Substituting Eq. 34 in Eq. 32 matrix unknowns from fracture equations are eliminated and the system of equations is reduced as in the normal dual-porosity approach. The reduced system is:

The stars in  $h^*$ ,  $a^*$ ,  $g^*$ , and  $F^*$  denote the modified submatrixes and subvectors that result after Eq. 34 coupling in the Eq. 32.

**Reinfiltration Pseudofunction Calculation.** In the stack of matrix blocks fine grid simulation under constant pressure conditions, average pressure changes in matrix blocks and surrounded fractures are monitored, and the average matrix blocks gas saturation changes as well. Oil drainage and reinfiltration rates in each matrix block of the stack are also computed as a function of the average matrix gas saturation.

Now under the assumption that the potential gradients in the oil phase established between fractures in i, j, k-1 node and matrix blocks in i, j, k node are similar in both fine grid simulation as well as the dual-porosity formulation, it follows that:

$$q_{oRfm_{i,j,k}}^{**n+1} = -\left(\frac{Akkr_o}{LxLyLz}\right)_{i,j,k}^{n+1} \frac{1}{\left(\mu_{o_{i,j,k}}\right)^{n+1} \Delta z_{i,j,k-\frac{1}{2}}} \times \left(p_{om_{i,j,k}}^{n+1} - p_{o_{i,j,k-1}}^{n+1} - (\gamma_o \Delta z)_{i,j,k-\frac{1}{2}}^{n+1}\right) \dots \dots \dots (39)$$

Therefore, we can calculate:

$$\left(\sigma_{R}^{*}K_{eo}^{*}\right)_{i,j,k}^{n+1} = \left(\frac{Akkr_{o}}{LxLyLz}\right)_{i,j,k}^{n+1}$$

$$= -\frac{q_{oRfm_{i,j,k}}^{**n+1} \left(\mu_{o_{i,j,k}}\right)^{n+1} \Delta z_{i,j,k-\frac{1}{2}}}{\left(p_{om_{i,j,k}}^{n+1} - p_{o_{i,j,k-1}}^{n+1} - (\gamma_{o}\Delta z)_{i,j,k-\frac{1}{2}}^{n+1}\right)}. \quad \dots \dots \dots \dots (40)$$

Through  $q_{oRfm_{ij,k}}^{**}(S_{gm_{ij,k}})$ , we establish the dependency of  $(\sigma_R^*K_{eo}^*)_{i,i,k}$  on the average gas saturation of i,j,k matrix block.

Stack of Matrix Block Simulation: Single Porosity Model. Because the flow in a stack of matrix blocks surrounded by fractures is symmetrical, the single-porosity model was built considering only one quarter of the stack of matrix blocks as shown in Fig. 1a. This domain was discretized through a tridimensional grid  $4 \times 4 \times 67$  gridblocks in the *x*, *y*, *z* directions, respectively *i* = 1,2,3,4; *j* = 1,2,3,4; *k* = 1,2, . . .,67, totaling 1,072 gridblocks, Fig. 1a.

The gridblocks in planes i = 1, j = 1, 2, 3, 4; k = 1, 2, ..., 67, and j = 4; i = 1, 2, 3, 4; k = 1, 2, ..., 67, make up the lateral fractures of the fracture medium. The contiguous gridblocks to matrix gridblocks i = 2, 3, 4; j = 2, 3, 4; in each of the layers k = 1, 12, 23, 34, ..., 67, as it can be seen in Fig. 1a, corresponds to horizontal fractures between blocks.

In the experiment to study the oil gravity drainage and oil reinfiltration, some initial and boundary conditions are set in such a way that they are representative of these processes. The initial conditions are that at zero time  $S_{om}=1-S_{wc}$ ,  $S_{gm}=0$ , in matrix and  $S_g=1$ ,  $S_o=0$  in fractures. For this experiment the  $S_{om}$  was 0.8, the  $S_{wcm}$  was 0.2 and there is no water in fractures.

Also because of symmetry the boundary conditions for the lateral walls of the stack are closed to flow of oil or gas. At the top face of the stack an extremely low constant gas injection rate was set, for this case  $q_{gi}$ =7 scf/day to prevent any flow by pressure gradient so that the viscous forces are neglected; at the bottom face of the stack the boundary conditions set are gas and oil production at a constant pressure of 995 psia; with this condition there are not compressibility effects in the matrix and flow is dominated by capillary and gravity forces. Then the experiment is started and the oil gravity drainage and reinfiltration processes are observed in the stack of matrix blocks through time.

The average gas-saturation results for each one of the six matrix blocks are presented in Fig. 2 for the three simulators, SIMPUMA-FRAC and the two commercial simulators. In Fig. 2 it can be seen the three simulators give equivalent results for the single-porosity simulation. Block 1 is the one at the top of the stack while Block 6 is at the bottom of the stack.

Figs. 3 and 4 exhibit the drainage and reinfiltration pseudofunction curves obtained by the SIMPUMA-FRAC simulator in its single-porosity formulation for this stack of six matrix blocks as a function of the average matrix gas saturation, which are introduced later in the same simulator but with the modified dual-porosity formulation.

# Flow Simulations in the Stack: Modified Dual Porosity Simulator

The problem is transformed from a 1,072-gridblock single-porosity simulation to an equivalent six-gridblock dual-porosity simulation with the same initial and boundary conditions set for the single-porosity simulation; then the SIMPUMA-FRAC is run in its modified dual-porosity mode. The two other commercial simulators are also run in their dual-porosity formulation for gravity drainage.

The dimensions for the single- and dual-porosity simulations are shown in Fig. 1.

The petrophysical properties are converted from single porosity to the equivalent dual-porosity properties (Thomas et al. 1983; Gilman and Kazemi 1983; Ladrón de Guevara-Torres 2006; Galindo Nava and Rodríguez de la Garza 1998):

$$\phi_{me} = \frac{\phi_m V_m}{V_T} = 0.119725$$
  
 $\phi_{fe} = \frac{V_f}{V_T} = 0.002288$   
 $k_{fe} = k_m + k_f \phi_{fe} = 1.6516$  darcies

**Fig. 5** shows the SIMPUMA-FRAC simulator results in its modified dual-porosity formulation compared to the single-porosity results. In Fig. 5, a good agreement between both single- and dual-porosity simulation runs, except at early time for the lower blocks. This disagreement is caused by the fact that in the dualporosity model, the fractures were not fully saturated with gas at early times as in the case of the single-porosity model.

Furthermore, for the modified dual-porosity formulation, we found that there are two important parameters in order to achieve a good match with the results obtained from the single-porosity



Fig. 2—Average gas saturation for each one of the six matrix blocks.

model: gas injection rate on top of the stack and the endpoints of the drainage and reinfiltration pseudofunction curves at  $S_g=0$ , as shown in **Figs. 6 and 7.** Regarding the gas-injection rate, a compromise had to be made between keeping the flow in the stack dominated by gravity and capillary forces as well as the vertical fractures fully saturated with gas, a condition that was not fulfilled at early times (where the mismatch can be seen in Fig. 5). Notice in Figs. 3 and 4 that the endpoint of the pseudocurves at  $S_g = 0$  are not well defined and had to be extrapolated for these curves to be used in the dual porosity model, as seen in Figs. 6 and 7.

Finally, **Figs. 8 and 9** show the dual-porosity results obtained with the commercial simulators using their own gravity-drainage option and the comparison with their own single-porosity simulations. As can be seen, there is a big disagreement between results.

## Conclusions

 A modification was done to the traditional dual-porosity formulation and was included in an in-house dual-porosity simulator, SIMPUMA-FRAC, to take into account oil gravity drainage and reinfiltration in NFR. This modification consisted of adding fluid exchange terms between the matrix and fractures in the flow equations to take into account these processes.

- The modified dual-porosity SIMPUMA-FRAC simulator solution was verified with the one obtained with the single-porosity model and achieved a good agreement between results.
- 3. A test was designed for the modified dual-porosity model through a stack of matrix blocks originally oil and connate water saturated surrounded by fractures filled with gas getting the solution from a single-porosity model.
- 4. It was found by comparing the results that the SIMPUMA-FRAC response was much better than the two commercial simulators in the equivalent versions for oil drainage and reinfiltration.
- 5. It was shown that effectively the oil flow in a stack of oil saturated matrix blocks surrounded by fractures filled with gas is a local oil exchange process from an upper to a lower block through the horizontal fracture planes, even when there is no capillary continuity with a negligible contribution to the fracture lateral planes.



Fig. 3—Reinfiltration pseudofunction vs. average gas saturation for the stack of six matrix blocks in the SIMPUMA-FRAC simulator.



Fig. 4—Drainage pseudofunction vs. average gas saturation for the stack of six matrix blocks in the SIMPUMA-FRAC simulator.

# Nomenclature

- b = 1/B: inverse of the volume factor, cm<sup>3</sup> @ cs/cm<sup>3</sup> @ cy
- cs = standard conditions
- cy = reservoir conditions
- $\hat{D}$  = datum, cm
- F = residual function in the Newton-Raphson method
- k = matrix permeability, Darcy
- $k_r$  = relative permeability, fraction
- $L_f$  = fracture length, cm
- $L_l$  = length of the simulation gridblock in *l* direction or length of the matrix blocks in *l* direction, cm
- NB = number of blocks

- $p_p$  = fracture pressure for p pseudocomponent, atm
- $\dot{P_c}$  = fracture capillary pressure, atm
- $P_{cm}$  = matrix capillary pressure, atm
- $p_{fondo}$  = pressure at the bottom of the stack (boundary condition), atm
  - $p_{mp}$  = matrix pressure for the *p*, atm

 $q_{gNmf_{i,j,k}}$  = net free gas exchange rate because of oil drainage and reinfiltration at standard conditions, cm<sup>3</sup> @ cs/s

 $q_{oDmf}^{**}$  = oil drainage rate for matrix blocks to fractures at reservoir conditions per unit bulk volume, (cm<sup>3</sup> @ cy/cm<sup>3</sup>)/s

 $q_{oDmf_{i,j,k}}$  = oil drainage rate for matrix blocks to fractures at standard conditions, cm<sup>3</sup> @ cs/s



# Dual Porosity vs. Single Porosity Model Results

Fig. 5—Average gas saturation vs. time for each one of the six matrix blocks.



Fig. 6—Reinfiltration pseudofunction vs. average gas saturation for the stack of six matrix blocks in the SIMPUMA-FRAC simulator.

- $q_{oRfm}^{**}i, j, k =$  oil reinfiltration rate for matrix blocks to fractures at reservoir conditions per unit bulk volume, (cm<sup>3</sup> @ cy/cm<sup>3</sup>)/s
  - $R_{\rm s}$  = solubility ratio of gas in oil, cm<sup>3</sup> @ cs/cm<sup>3</sup> @ cs
  - S = saturation, fraction
  - t = time, s
  - $T = \text{transmissibility}, (\text{cm}^3/\text{s})/(\text{atm})$
  - $u_e$  = outer boundary in direction u, cm. u = x, y, z
  - $Vr = bulk volume, cm^3$
  - x, y, z = cartesian coordinates

- z = vertical depth, cm
- $\nabla$  = Vector differential operator (nabla symbol)
- $\gamma$  = specific weight,  $g_f/cm^3$
- $\Delta$  = central differences space operator, 1/cm
- $\Delta_t$  = regressive differences time operator, 1/s
- $\Delta t = \text{time step, s}$
- $\delta$  = iterative change
- $\lambda$  = mobility, (cm<sup>3</sup> @ cy/cm<sup>3</sup> @ c.s.) darcy/cp
- $\mu$  = viscosity, cp
- $\sigma$  = shape factor



Fig. 7—Drainage pseudofunction vs. average gas saturation for the stack of six matrix blocks in the SIMPUMA-FRAC simulator.



Fig. 8—Average gas saturation vs. time for each one of the matrix blocks for the commercial Simulator E in dual- and singleporosity mode.

- $\tau_{mf}$  = matrix-fracture exchange rate because of viscous flow at standard conditions, cm<sup>3</sup> @ cs/s
- $\tau_{mf}^*$  = matrix-fracture exchange rate because of viscous flow at standard conditions per bulk volume, (cm<sup>3</sup> @ cs/cm<sup>3</sup>)/s
  - $\phi$  = porosity, fraction

Subscript:

- f = fracture, fractures
- ff = fracture residual function derivatives with respect to the fracture
- fm = fracture residual function derivatives with respect to matrix unknowns
- g = gas

i, j, k = the *i*th, *j*th, *k*th block or *i*th, *j*th, *k*th gridcell

$$m = matrix$$

*mf* = matrix residual function derivatives with respect to the fracture



Fig. 9—Average gas saturation vs. time for each one of the matrix blocks for the commercial Simulator A in dual- and singleporosity mode.

# Superscript:

- n = time level
- (v) = iteration level

- *mm* = matrix residual function derivatives with respect to the matrix unknowns
  - o = oil
  - p = gas or oil pseudocomponent
  - U = number of gridblocks in the *x*,*y*,*z* direction. U = *IJK*
  - w = water in fractures
- wc = connate water in matrix

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# **SI Metric Conversion Factors**

atm × 1.013 250*	E + 05 = Pa
$cp \times 1.0^*$	$E - 03 = Pa \cdot s$
psi × 6.894 757	E + 00 = kPa
Service and for story in accord	

\*Conversion factor is exact.

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