TERM PROJECT

Version: 2009-11-18 (there may be other versions later)

The term project involves a 2-D formulation and a 1-D solution of the flow equations described in the next several pages. The term project should stand on it's own – feel free to include portions of homework problems. Include anything that you feel will help us know that you understand the material.

1) Describe the finite difference formulation for each of the four options in 2-D. Include pictures of 0 and non-zero portions of matrix equations.
2) Present the results of your 1-D simulations for option 1, 2, and 4, including a comparison of the different techniques.
3) Provide a copy of the source code you used for this project

Option 1: IMPES; Implicit Fracture Pressure, Explicit Saturation, Sequential Method

First, solve for fracture pressure P_{of}^{n+1} using the implicit fracture flow equation in matrix form. Next, solve for the matrix pressure P_{om}^{n+1} using an explicit solution of the total matrix flow equation. Next, solve for S_{wf}^{n+1} using an explicit solution of the water saturation equation. Finally, solve for S_{wm}^{n+1} using an explicit solution of the water matrix equation.

Option 2: Implicit Pressure, Partially Implicit Saturation, Sequential Method

First, solve for fracture pressure P_{of}^{n+1} using the implicit fracture flow equation in matrix form. Next, solve for the matrix pressure P_{om}^{n+1} using an explicit solution of the total matrix flow equation. Next, solve for S_{wf}^{n+1} using an implicit matrix solution of the water saturation equation. Finally, solve for S_{wm}^{n+1} using an explicit solution of the water matrix equation.

Option 3: Fully Implicit; Iterative

First, solve for fracture pressure $P_{of}^{\ell+1}$ using the implicit fracture flow equation in matrix form. Next, solve for the matrix pressure $P_{om}^{\ell+1}$ using an explicit solution of the total matrix flow equation. Next, solve for $S_{wf}^{\ell+1}$ using an implicit matrix solution of the water saturation equation. Finally, solve for $S_{wm}^{\ell+1}$ using an explicit solution of the water matrix equation. Repeat the iterations until convergence is obtained, calculating first $P_{o}^{\ell+1}$ using the values of P_{om}^{ℓ}, S_{wf}^{ℓ}, and S_{wm}^{ℓ}. Next, solve for $P_{om}^{\ell+1}$ using the values of $P_{o}^{\ell+1}$, S_{wf}^{ℓ}, and S_{wm}^{ℓ}. Third, solve for $S_{wf}^{\ell+1}$ using the values of $P_{o}^{\ell+1}$, $P_{om}^{\ell+1}$, and S_{wm}^{ℓ}. Finally, solve for $S_{wm}^{\ell+1}$ using the values of $P_{o}^{\ell+1}$, $P_{om}^{\ell+1}$, and $S_{wf}^{\ell+1}$.

Option 4: Fully Implicit; Simultaneous

Solve simultaneously for P_{of}^{n+1}, P_{om}^{n+1}, S_{wf}^{n+1}, and S_{wm}^{n+1} using a Newton-Raphson formulation.

Option 1: IMPES; Implicit Fracture Pressure, Sequential Matrix Pressure, Sequential Saturation

First, solve for fracture pressure P_{of}^{n+1} using the implicit fracture flow equation in matrix form. Next, solve for the matrix pressure P_{om}^{n+1} using an explicit solution of the total matrix flow equation. Next, solve for S_{wf}^{n+1} using an explicit solution of the water saturation equation. Finally, solve for S_{wm}^{n+1} using an explicit solution of the water matrix equation.

Fracture Pressure Equation:

$$\nabla\cdot\left(k_{f,eff}\left(\lambda_{Tf}^{n}\nabla P_{of}^{n+1}-\left(\lambda_{wf}^{n}\gamma_{w}^{n}+\lambda_{of}^{n}\gamma_{o}^{n}\right)\nabla D-\lambda_{wf}^{n}\nabla P_{cwo}^{n}\right)\right)-\tau_{T}^{n+1}+\hat{q}_{Tf}^{n+1}=\phi_{f}C_{Tf}\frac{P_{of}^{n+1}-P_{of}^{n}}{\Delta t}$$

Total Transfer:

$$\tau_{T}^{n+1}=\sigma k_{m}\left(\lambda_{Tf}^{n}(P_{of}^{n+1}-P_{om}^{n+1})+\frac{\sigma_{z}}{\sigma}(\lambda_{wf/m}^{n}\gamma_{w}^{n}+\lambda_{om/f}^{n}\gamma_{o}^{n})(h_{wf}^{n}-h_{wm}^{n})+\lambda_{wf/m}^{n}(P_{cwom}^{n}-P_{cwof}^{n})\right)$$

Total Matrix flow:

$$\tau_{T}^{n+1}=\phi_{m}C_{Tm}\frac{P_{om}^{n+1}-P_{om}^{n}}{\Delta t}$$

Saturation equation, explicit (IMPES):

$$\nabla\cdot\left(k_{f,eff}\left(\lambda_{wf}^{n}\nabla P_{of}^{[n+1]}-\left(\lambda_{wf}^{n}\gamma_{w}^{n}\right)\nabla D-\lambda_{wf}^{n}\nabla P_{cwo}^{n}\right)\right)-\tau_{w}^{[n+1]}+\hat{q}_{wf}^{[n+1]}=\phi_{f}S_{wf}^{n+1}(C_{\phi f}+C_{wf})\frac{P_{of}^{[n+1]}-P_{of}^{n}}{\Delta t}+\phi\frac{S_{wf}^{n+1}-S_{wf}^{n}}{\Delta t}$$

Water Transfer:

$$\tau_{w}^{[n+1]}=\sigma k_{m}\left(\lambda_{wf}^{n}(P_{of}^{[n+1]}-P_{om}^{[n+1]})+\frac{\sigma_{z}}{\sigma}(\lambda_{wf/m}^{n}\gamma_{w}^{n})(h_{wf}^{n}-h_{wm}^{n})+\lambda_{wf/m}^{n}(P_{cwom}^{n}-P_{cwof}^{n})\right)$$

Water Total Matrix flow:

$$\tau_{w}^{[n+1]}=\phi_{m}S_{wm}^{n+1}(C_{\phi m}+C_{wm})\frac{P_{om}^{[n+1]}-P_{om}^{n}}{\Delta t}+\phi\frac{S_{wm}^{n+1}-S_{wm}^{n}}{\Delta t}$$

Option 2: Partially Implicit; Implicit Pressure, Sequential Saturation

First, solve for fracture pressure P_{of}^{n+1} *using the implicit fracture flow equation in matrix form. Next, solve for the matrix pressure* P_{om}^{n+1} *using an explicit solution of the total matrix flow equation. Next, solve for* S_{wf}^{n+1} *using an implicit matrix solution of the water saturation equation. Finally, solve for* S_{wm}^{n+1} *using an explicit solution of the water matrix equation.*

Fracture Pressure Equation:

$$\nabla\cdot\left(k_{f,eff}\left(\lambda_{Tf}^{n}\nabla P_{of}^{n+1}-\left(\lambda_{wf}^{n}\gamma_{w}^{n}+\lambda_{of}^{n}\gamma_{o}^{n}\right)\nabla D-\lambda_{wf}^{n}\nabla P_{cwo}^{n}\right)\right)-\tau_{T}^{n+1}+\hat{q}_{Tf}^{n+1}=\phi_{f}C_{Tf}\frac{P_{of}^{n+1}-P_{of}^{n}}{\Delta t}$$

Total Transfer:

$$\tau_{T}^{n+1}=\sigma k_{m}\left(\lambda_{Tf}^{n}(P_{of}^{n+1}-P_{om}^{n+1})+\frac{\sigma_{z}}{\sigma}(\lambda_{wf/m}^{n}\gamma_{w}^{n}+\lambda_{om/f}^{n}\gamma_{o}^{n})(h_{wf}^{n}-h_{wm}^{n})+\lambda_{wf/m}^{n}(P_{cwom}^{n}-P_{cwof}^{n})\right)$$

Total Matrix flow:

$$\tau_{T}^{n+1}=\phi_{m}C_{Tm}\frac{P_{om}^{n+1}-P_{om}^{n}}{\Delta t}$$

Saturation equation, explicit:

$$\nabla\cdot\left(k_{f,eff}\lambda_{wf}^{n}\left(\nabla P_{of}^{[n+1]}-\gamma_{w}^{n}\nabla D-\frac{\partial P_{cowf}^{n}}{\partial S_{wf}}\nabla S_{wf}^{n+1}\right)\right)-\tau_{w}^{[n+1]}+\hat{q}_{wf}^{[n+1]}=\phi_{f}S_{wf}^{n}(C_{\phi f}+C_{wf})\frac{P_{of}^{[n+1]}-P_{of}^{n}}{\Delta t}+\phi\frac{S_{wf}^{n+1}-S_{wf}^{n}}{\Delta t}$$

Water Transfer:

$$\tau_{w}^{[n+1]}=\sigma k_{m}\left(\lambda_{wf}^{n}(P_{of}^{[n+1]}-P_{om}^{[n+1]})+\frac{\sigma_{z}}{\sigma}(\lambda_{wf/m}^{n}\gamma_{w}^{n})(h_{wf}^{n}-h_{wm}^{n})+\lambda_{wf/m}^{n}(P_{cwom}^{n}-P_{cwof}^{n})\right)$$

Water Total Matrix flow:

$$\tau_{w}^{[n+1]}=\phi_{m}S_{wm}^{n+1}(C_{\phi m}+C_{wm})\frac{P_{om}^{[n+1]}-P_{om}^{n}}{\Delta t}+\phi\frac{S_{wm}^{n+1}-S_{wm}^{n}}{\Delta t}$$

Option 3: Fully Implicit; Iterative

First, solve for fracture pressure $P_{of}^{\ell+1}$ using the implicit fracture flow equation in matrix form. Next, solve for the matrix pressure $P_{om}^{\ell+1}$ using an explicit solution of the total matrix flow equation. Next, solve for $S_{wf}^{\ell+1}$ using an implicit matrix solution of the water saturation equation. Finally, solve for $S_{wm}^{\ell+1}$ using an explicit solution of the water matrix equation. Repeat the iterations until convergence is obtained, calculating first $P_{o}^{\ell+1}$ using the values of P_{om}^{ℓ}, S_{wf}^{ℓ}, and S_{wm}^{ℓ}. Next, solve for $P_{om}^{\ell+1}$ using the values of $P_{o}^{\ell+1}$, S_{wf}^{ℓ}, and S_{wm}^{ℓ}. Third, solve for $S_{wf}^{\ell+1}$ using the values of $P_{o}^{\ell+1}$, $P_{om}^{\ell+1}$, and S_{wm}^{ℓ}. Finally, solve for $S_{wm}^{\ell+1}$ using the values of $P_{o}^{\ell+1}$, $P_{om}^{\ell+1}$, and $S_{wf}^{\ell+1}$.

Fracture Pressure Equation:

$$\nabla\cdot\left(k_{f,eff}\left(\lambda_{Tf}^{[n+1]}\nabla P_{of}^{n+1}-\left(\lambda_{wf}^{[n+1]}\gamma_{w}^{[n+1]}+\lambda_{of}^{[n+1]}\gamma_{o}^{[n+1]}\right)\nabla D-\lambda_{wf}^{[n+1]}\nabla P_{cwo}^{[n+1]}\right)\right)-\tau_{T}^{n+1}+\hat{q}_{Tf}^{n+1}=\phi_{f}C_{Tf}^{[n+1]}\frac{P_{of}^{n+1}-P_{of}^{n}}{\Delta t}$$

Total Transfer:

$$\tau_{T}^{n+1}=\sigma k_{m}\left(\lambda_{Tf}^{[n+1]}(P_{of}^{n+1}-P_{om}^{n+1})+\frac{\sigma_{z}}{\sigma}(\lambda_{wf/m}^{[n+1]}\gamma_{w}^{[n+1]}+\lambda_{om/f}^{[n+1]}\gamma_{o}^{[n+1]})(h_{wf}^{[n+1]}-h_{wm}^{[n+1]})+\lambda_{wf/m}^{[n+1]}(P_{cwom}^{[n+1]}-P_{cwof}^{[n+1]})\right)$$

Total Matrix flow:

$$\tau_{T}^{n+1}=\phi_{m}C_{Tm}^{[n+1]}\frac{P_{om}^{n+1}-P_{om}^{n}}{\Delta t}$$

Saturation equation, explicit:

$$\nabla\cdot\left(k_{f,eff}\lambda_{wf}^{n+1}\left(\nabla P_{of}^{[n+1]}-\gamma_{w}^{[n+1]}\nabla D-\frac{\partial P_{cowf}^{n}}{\partial S_{wf}}\nabla S_{wf}^{n+1}\right)\right)-\tau_{w}^{[n+1]}+\hat{q}_{wf}^{[n+1]}=\phi_{f}S_{wf}^{n+1}(C_{\phi f}+C_{wf})\frac{P_{of}^{[n+1]}-P_{of}^{n}}{\Delta t}+\phi\frac{S_{wf}^{n+1}-S_{wf}^{n}}{\Delta t}$$

Water Transfer:

$$\tau_{w}^{[n+1]}=\sigma k_{m}\left(\lambda_{wf}^{[n+1]}(P_{of}^{[n+1]}-P_{om}^{[n+1]})+\frac{\sigma_{z}}{\sigma}(\lambda_{wf/m}^{[n+1]}\gamma_{w}^{[n+1]})(h_{wf}^{[n+1]}-h_{wm}^{[n+1]})+\lambda_{wf/m}^{[n+1]}(P_{cwom}^{[n+1]}-P_{cwof}^{[n+1]})\right)$$

Water Total Matrix flow:

$$\tau_{w}^{[n+1]}=\phi_{m}S_{wm}^{n+1}(C_{\phi m}+C_{wm})\frac{P_{om}^{[n+1]}-P_{om}^{n}}{\Delta t}+\phi\frac{S_{wm}^{n+1}-S_{wm}^{n}}{\Delta t}$$

Option 4: Fully Implicit; Simultaneous

Solve simultaneously for P_{of}^{n+1}, P_{om}^{n+1}, S_{wf}^{n+1}, *and* S_{wm}^{n+1}.

Water Saturation equation:

$$\nabla\cdot\left(k_{f,eff}\lambda_{wf}^{n+1}\left(\nabla P_{of}^{n+1}-\gamma_w^n\nabla D-\frac{\partial P_{cowf}^n}{\partial S_{wf}}\nabla S_{wf}^{n+1}\right)\right)-\tau_w^{n+1}+\hat{q}_{wf}^{n+1}=\phi_f S_{wf}^{n+1}(C_{\phi f}+C_{wf})\frac{P_{of}^{n+1}-P_{of}^n}{\Delta t}+\phi\frac{S_{wf}^{n+1}-S_{wf}^n}{\Delta t}$$

Water Transfer:

$$\tau_w^{n+1}=\sigma k_m\left(\lambda_{wf/m}^{n+1}(P_{of}^{n+1}-P_{om}^{n+1})+\frac{\sigma_z}{\sigma}(\lambda_{wf/m}^{n+1}\gamma_w^n)(h_{wf}^{n+1}-h_{wm}^{n+1})+\lambda_{wf/m}^{n+1}(P_{cwom}^{n+1}-P_{cwof}^{n+1})\right)$$

Water Matrix flow:

$$\tau_w^{n+1}=\phi_m S_{wm}^{n+1}(C_{\phi m}+C_{wm})\frac{P_{om}^{n+1}-P_{om}^n}{\Delta t}+\phi\frac{S_{wm}^{n+1}-S_{wm}^n}{\Delta t}$$

Oil Saturation equation:

$$\nabla\cdot\left(k_{f,eff}\lambda_{of}^{n+1}\left(\nabla P_{of}^{n+1}-\gamma_o^n\nabla D\right)\right)-\tau_o^{n+1}+\hat{q}_{of}^{n+1}=\phi_f S_{of}^{n+1}(C_{\phi f}+C_{wf})\frac{P_{of}^{n+1}-P_{of}^n}{\Delta t}+\phi\frac{S_{of}^{n+1}-S_{of}^n}{\Delta t}$$

Oil Transfer:

$$\tau_o^{n+1}=\sigma k_m\left(\lambda_{om/f}^{n+1}(P_{of}^{n+1}-P_{om}^{n+1})+\frac{\sigma_z}{\sigma}(\lambda_{om/f}^{n+1}\gamma_o^n)(h_{wf}^{n+1}-h_{wm}^{n+1})\right)$$

Oil Matrix flow:

$$\tau_o^{n+1}=\phi_m S_{om}^{n+1}(C_{\phi m}+C_{wm})\frac{P_{om}^{n+1}-P_{om}^n}{\Delta t}+\phi\frac{S_{om}^{n+1}-S_{om}^n}{\Delta t}$$

Capillary pressure and relative permeability equations:

$$k_{row} = k_{row}^* \left(\frac{S_o - S_{owr}}{1 - S_{wr} - S_{owr}} \right)^{n_{ow}} \qquad k_{rw} = k_{rw}^* \left(\frac{S_w - S_{wr}}{1 - S_{wr} - S_{owr}} \right)^{n_w}$$

$$P_{cow1} = \alpha \ln \left[\frac{1 - S_w - S_{owr}}{1 - S_{wx} - S_{owr}} \right] \qquad P_{cow2} = -\alpha \ln \left[\frac{S_w - S_{wr}}{S_{wx} - S_{wr}} \right]$$

$$P_{cow} = \begin{cases} P_{cow,max}, & S_w \le S_{wr} \\ P_{cow,min}, & S_w \ge (1 - S_{owr}) \\ \min[P_{cow2}, P_{cow,max}], & S_{wr} \le S_w \le S_{wx} \\ \max[P_{cow1}, P_{cow,min}], & (1 - S_{owr}) \ge S_w \ge S_{wx} \end{cases}$$

Data:

$k_{rwm}^* = 0.1 \quad k_{rowm}^* = 0.7 \quad n_{wm} = 1.5 \quad n_{owm} = 2.5 \quad S_{owrm} = 0.30 \quad S_{wrm} = 0.25$

$k_{rwf}^* = 0.8 \quad k_{rowf}^* = 0.8 \quad n_{wf} = 2.0 \quad n_{owf} = 2.0 \quad S_{owrf} = 0.05 \quad S_{wrf} = 0.05$

$\alpha_m = 0.8; \; S_{wxm} = 0.5 \quad P_{cowm}^{\max} = +5\,psi \quad P_{cowm}^{\min} = -5\,psi$

$\alpha_f = 0.08; \; S_{wxf} = 0.5 \quad P_{cowf}^{\max} = +0.5\,psi \quad P_{cowf}^{\min} = -0.5\,psi$

$\mu_w = 0.6cp \quad \mu_o = 2.4cp \quad \gamma_w = 0.433\,psi/ft \quad \gamma_o = 0.433 * 0.8\,psi/ft$

$k_m = 10md \quad \phi_m = 0.20 \quad C_{Tm} = 10 \cdot 10^{-6}\,psi^{-1}$

$w_f = 50\mu m$ (convert to feet; use to calculate ϕ_f and k_f) $u_{Tf} = 25\,ft/day$

$L_x = 10\,feet \quad L_y = 10\,feet \quad L_z = 30\,feet$

$\Delta x = 100\,feet \quad \Delta y = 100\,feet \quad \Delta z = 30\,feet \quad IMAX = 10$

$P_{of}^{init} = 3000\,psia$, $P_{WB,f}^{all} = 2500\,psia$, $S_{wf}^{init} = S_{wfr}, S_{wm}^{init} = S_{wmr}$

$C_{\phi f} = 5 \cdot 10^{-6}\,psi^{-1}$, $C_{\phi m} = 3 \cdot 10^{-6}\,psi^{-1}$, $C_w = 4 \cdot 10^{-6}\,psi^{-1}$, $C_o = 10 \cdot 10^{-6}\,psi^{-1}$