# **PEGN 620A Naturally Fractured Reservoir Fall 2009**

## HW #3

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## Due: September 17, 2009

PEGN 620A Naturally Fractured Reservoirs – Engineering & Reservoir Simulation Fall 2009

**HW#3**

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(a) Assuming matrix blocks are vertical cylinders, show that .

(b) Assuming matrix blocks are spherical, show that .

**Deliverables:**

- Detailed derivation

- Assumptions

**Hints:**

Diffusivity equations (pseudo-steady state conditions)

*For cylindrical coordinates:*

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*For spherical coordinates:*

****

**The answers to the question above-mentioned can be shown as follows:**

**(A) Assuming matrix blocks are vertical cylinders:**

Assuming that the radius of the cylinder is *r*e , the diameter of the cylinder is *h*m, and the height of the cylinder is *h*, which can be shown in Fig.1.

*r*e=*h*m/2

*h*

0

θ

Figure 1 The sketch of the vertical cylinder

The diffusivity equation for cylindrical coordinates under the pseudo-steady state conditions is as follows:

 (A-1)

In order to deduce the formulation of *p*m, we can integrate the two sides of the diffusivity equation in terms of the volume for vertical cylinder:

**** (A-2)

 (A-3)

 (A-4)

 (A-5)

From the deduction from Eq.A-2 to Eq.A-5, we can obtain the formulation of *p*m:

 (A-6)

According to the pseudo-steady state conditions, we should solve the average of *p*m, i.e.,

 (A-7)

 (A-8)

The final equation of  obtained is as follows:

 (A-9)

In view of the pseudo-steady state conditions, the subsurface flow rate is shown in the following equation:

 (A-10)

Substitute the Eq. A-10 into Eq. A-9, thus

 (A-11)

So the shape factor is , i.e. .

**(B) Assuming matrix blocks are spherical:**

Assuming that the radius of the sphere is *r*e, the diameter of the sphere is *h*m, in order to calculate the volume of the assuming sphere, we consider the micro volume composed by the micro increase of d*r*, d*θ*, d*φ*, which is show in Fig.2, then the volume of the sphere can be integrate.

*z*

d*r*

*r*sin*φdθ*

*rsinφ*

d*θ*

*r*

*r*d*φ*

d*φ*

*φ*

*x*

*θ*

d*θ*

*y*

Fig.2 The sketch of the micro unit in the sphere

The diffusivity equation for spherical coordinates under the pseudo-steady state conditions is as follows:

**** (B-1)

In order to deduce the formulation of *p*m, we can integrate the two sides of the diffusivity equation in terms of the volume for vertical cylinder:

 (B-2)

**** (B-3)

**** (B-4)

**** (B-5)

From the deduction from Eq. B-2 to Eq. B-5, we can obtain the formulation of *p*m:

 (B-6)

According to the pseudo-steady state conditions, we should solve the average of *p*m, i.e.,

 (B-7)

**** (B-8)

The final equation of  obtained is as follows:

**** (B-9)

In view of the pseudo-steady state conditions, the subsurface flow rate is shown in the following equation:

 (B-10)

Substitute the Eq. B-10 into Eq. B-9, thus

 (B-11)

So the shape factor is , i.e. .