PEGN 620A: Naturally Fractured Reservoir
Homework 9:Solution

Given Equations

Flow in fracture

$$\nabla \cdot \{k_{f,eff}[\lambda_{tf}^n \nabla P_{of}^{n+1} - (\lambda_{wf}^n \gamma_w^n + \lambda_{of}^n \gamma_o^n)\nabla D - \lambda_{wf}^n \nabla P_{cwof}^n]\} - \tau_t^{n+1} + \hat{q}_{tf} = (\phi c_t)_f \frac{\partial P_{of}}{\partial t} \quad (1)$$

Transfer function

$$\tau_t^{n+1} = \sigma k_m[\lambda_{tf}^n(P_{of}^{n+1} - P_{om}^{n+1}) + \frac{\sigma_z}{\sigma}(\lambda_{wf/m}^n \gamma_w^n + \lambda_{om/f}^n \gamma_o^n)(h_{wf}^n - h_{wm}^n) + \lambda_{wf/m}^n(P_{cwom}^n - P_{cwof}^n)] \quad (2)$$

Flow in matrix

$$\tau_t^{n+1} = (\phi c_t)_m \frac{\partial P_{om}}{\partial t} \quad (3)$$

Discretize and rearrange Eq (3) we get

$$\tau_t^{n+1} = (\phi c_t)_m \frac{P_{i,om}^{n+1} - P_{i,om}^n}{\Delta t}$$

$$P_{i,om}^{n+1} = \tau_t^{n+1} \frac{\Delta t}{(\phi c_t)_m} + P_{i,om}^n$$

Substitute $P_{i,om}^{n+1}$ into Eq (2) and rearrange

$$\tau_t^{n+1} = \alpha\sigma k_m[\lambda_{tf}^n(P_{i,of}^{n+1} - (\tau_t^{n+1}\frac{\Delta t}{(\phi c_t)_m} + P_{i,om}^n)) + \frac{\sigma_z}{\sigma}(\lambda_{wf/m}^n \gamma_w^n + \lambda_{om/f}^n \gamma_o^n)(h_{wf}^n - h_{wm}^n) + \lambda_{wf/m}^n(P_{cwom}^n - P_{cwof}^n)]$$

$$\tau_t^{n+1}\left(1 + \frac{\alpha\sigma k_m \lambda_{tf}^n \Delta t}{(\phi c_t)_m}\right) = \alpha\sigma k_m[\lambda_{tf}^n(P_{i,of}^{n+1} - P_{i,om}^n) + \frac{\sigma_z}{\sigma}(\lambda_{wf/m}^n \gamma_w^n + \lambda_{om/f}^n \gamma_o^n)(h_{wf}^n - h_{wm}^n) + \lambda_{wf/m}^n(P_{cwom}^n - P_{cwof}^n)]$$

$$\tau_t^{n+1} = \frac{\alpha\sigma k_m}{\left(1+\frac{\alpha\sigma k_m \lambda_{tf}^n \Delta t}{(\phi c_t)_m}\right)} [\lambda_{tf}^n (P_{i,of}^{n+1} - P_{i,om}^n) + \frac{\sigma_z}{\sigma}(\lambda_{wf/m}^n \gamma_w^n + \lambda_{om/f}^n \gamma_o^n)(h_{wf}^n - h_{wm}^n) + \lambda_{wf/m}^n (P_{cwom}^n - P_{cwof}^n)] \tag{4}$$

Multiply Eq (1) by Vi, where $V_i = \Delta x_i \Delta y_i \Delta z_i$ and rearrange

$$\overbrace{V_i \nabla \cdot k_{f,eff} \lambda_{tf}^n \nabla P_{of}^{n+1}}^{Pressure-term} - \overbrace{V_i \nabla \cdot k_{f,eff} (\lambda_{wf}^n \gamma_w^n + \lambda_{of}^n \gamma_o^n) \nabla D}^{Gravity-term} - \overbrace{V_i \nabla \cdot k_{f,eff} \lambda_{wf}^n \nabla P_{cwof}^n}^{Capillary-term} - V_i \tau_t^{n+1} + q_{tf} = \overbrace{V_i (\phi c_t)_f \frac{\partial P_{of}}{\partial t}}^{Accumulation-term} \tag{5}$$

Discretize Eq (5) for 1-D problem term by term and rearrange

- Pressure term

$$V_i \frac{\partial}{\partial x}(k_{f,eff} \lambda_{tf}^n \nabla P_{of}^{n+1}) \overset{discretize}{\rightarrow} \alpha \Delta y_i \Delta z_i \left[(k_{f,eff} \lambda_{tf}^n)_{i+\frac{1}{2}} (\frac{P_{of,i+1}^{n+1} - P_{of,i}^{n+1}}{\Delta x_{i+\frac{1}{2}}}) - (k_{f,eff} \lambda_{tf}^n)_{i-\frac{1}{2}} (\frac{P_{of,i}^{n+1} - P_{of,i-1}^{n+1}}{\Delta x_{i-\frac{1}{2}}}) \right] \tag{6}$$

Rearrange

$$T_{x,i+\frac{1}{2}}^n P_{of,i+1}^{n+1} + T_{x,i-\frac{1}{2}}^n P_{of,i-1}^{n+1} - (T_{x,i+\frac{1}{2}}^n + T_{x,i-\frac{1}{2}}^n) P_{of,i}^{n+1} \tag{7}$$

where

$$T_{x,i+\frac{1}{2}}^n = \alpha (k_{f,eff} \lambda_{tf}^n \frac{\Delta y \Delta z}{\Delta x})_{i+\frac{1}{2}}$$

$$T_{x,i-\frac{1}{2}}^n = \alpha (k_{f,eff} \lambda_{tf}^n \frac{\Delta y \Delta z}{\Delta x})_{i-\frac{1}{2}}$$

$$\alpha = 0.006328$$

- Gravity term

$$V_i \frac{\partial}{\partial x}(k_{f,eff} (\lambda_{wf}^n \gamma_w^n + \lambda_{of}^n \gamma_o^n)^n \nabla D) \overset{discretize}{\rightarrow}$$

$$\alpha \Delta y_i \Delta z_i \left[(k_{f,eff}(\lambda_{wf}^n \gamma_w^n + \lambda_{of}^n \gamma_o^n))_{i+\frac{1}{2}} (\frac{D_{i+1} - D_i}{\Delta x_{i+\frac{1}{2}}}) - (k_{f,eff}(\lambda_{wf}^n \gamma_w^n + \lambda_{of}^n \gamma_o^n))_{i-\frac{1}{2}} (\frac{D_i - D_{i-1}}{\Delta x_{i-\frac{1}{2}}}) \right] \tag{8}$$

Rearrange

$$(T_{xw}^n \gamma_w^n + T_{xo}^n \gamma_o^n)_{i+\frac{1}{2}} (D_{i+1} - D_i) - (T_{xw}^n \gamma_w^n + T_{xo}^n \gamma_o^n)_{i+\frac{1}{2}} (D_i - D_{i-1}) \tag{9}$$

where

$$T_{xw,i+\frac{1}{2}}^n = \alpha(k_{f,eff} \lambda_{wf}^n \frac{\Delta y \Delta z}{\Delta x})_{i+\frac{1}{2}}$$

$$T_{xw,i-\frac{1}{2}}^n = \alpha(k_{f,eff} \lambda_{wf}^n \frac{\Delta y \Delta z}{\Delta x})_{i-\frac{1}{2}}$$

$$T_{xo,i+\frac{1}{2}}^n = \alpha(k_{f,eff} \lambda_{of}^n \frac{\Delta y \Delta z}{\Delta x})_{i+\frac{1}{2}}$$

$$T_{xo,i-\frac{1}{2}}^n = \alpha(k_{f,eff} \lambda_{of}^n \frac{\Delta y \Delta z}{\Delta x})_{i-\frac{1}{2}}$$

$$\alpha = 0.006328$$

- Capillary term
  $V_i \nabla \cdot k_{f,eff} \lambda_{wf}^n \nabla P_{cwof}^n \overset{discretize}{\rightarrow}$

$$\alpha \Delta y_i \Delta z_i \left[(k_{f,eff} \lambda_{wf}^n)_{i+\frac{1}{2}} (\frac{P_{cwof,i+1}^n - P_{cwof,i}^n}{\Delta x_{i+\frac{1}{2}}}) - (k_{f,eff} \lambda_{wf}^n)_{i-\frac{1}{2}} (\frac{P_{cwof,i}^n - P_{cwof,i-1}^n}{\Delta x_{i-\frac{1}{2}}}) \right] \tag{10}$$

Rearrange

$$T_{xw,i+\frac{1}{2}}^n (P_{cwof,i+1}^n - P_{cwof,i}^n) - T_{xw,i-\frac{1}{2}}^n (P_{cwof,i}^n - P_{cwof,i-1}^n) \tag{11}$$

where

$$T_{xw,i+\frac{1}{2}}^n = \alpha(k_{f,eff} \lambda_{wf}^n \frac{\Delta y \Delta z}{\Delta x})_{i+\frac{1}{2}}$$

$$T_{xw,i-\frac{1}{2}}^n = \alpha(k_{f,eff} \lambda_{wf}^n \frac{\Delta y \Delta z}{\Delta x})_{i-\frac{1}{2}}$$

$$\alpha = 0.006328$$

- Accumulation term

$$V_i(\phi c_t)_f \frac{\partial P_{of}}{\partial t} \overset{discretize}{\rightarrow} V_i(\phi c_t)_f \frac{P_{i,of}^{n+1} - P_{i,of}^{n}}{\Delta t} \tag{12}$$

Substitute Eq(4), (7), (9), (11) and (12) into Eq (5) and rearrange

$$T^n_{x,i+\frac{1}{2}} P^{n+1}_{of,i+1} + T^n_{x,i-\frac{1}{2}} P^{n+1}_{of,i-1} - \left(T^n_{x,i+\frac{1}{2}} + T^n_{x,i-\frac{1}{2}} + V_i \frac{(\phi c_t)_f}{\Delta t} + V_i \frac{\alpha\sigma k_m \lambda^n_{tf}}{1 + \frac{\alpha\sigma k_m \lambda^n_{tf} \Delta t}{(\phi c_t)_m}} \right) P^{n+1}_{of,i} = RHS \tag{13}$$

$$RHS = \left\{ \begin{array}{c} V_i \frac{\alpha\sigma k_m}{\left(1+\frac{\alpha\sigma k_m \lambda^n_{tf}\Delta t}{(\phi c_t)_m}\right)} \left[-\lambda^n_{tf} P^n_{i,om} + \frac{\sigma_z}{\sigma}(\lambda^n_{wf/m}\gamma^n_w + \lambda^n_{om/f}\gamma^n_o)(h^n_{wf} - h^n_{wm}) + \lambda^n_{wf/m}(P^n_{cwom} - P^n_{cwof}) \right] \\ +(T^n_{xw}\gamma^n_w + T^n_{xo}\gamma^n_o)_{i+\frac{1}{2}}(D_{i+1} - D_i) - (T^n_{xw}\gamma^n_w + T^n_{xo}\gamma^n_o)_{i+\frac{1}{2}}(D_i - D_{i-1}) \\ +T^n_{xw,i+\frac{1}{2}}(P^n_{cwof,i+1} - P^n_{cwof,i}) - T^n_{xw,i-\frac{1}{2}}(P^n_{cwof,i} - P^n_{cwof,i-1}) \\ -V_i \frac{(\phi c_t)_f}{\Delta t} P^n_{i,of} - q_{tf} \end{array} \right\}$$

where

$$T^n_{x,i+\frac{1}{2}} = (k_{f,eff}\lambda^n_{tf}\frac{\Delta y \Delta z}{\Delta x})_{i+\frac{1}{2}}$$

$$T^n_{x,i-\frac{1}{2}} = (k_{f,eff}\lambda^n_{tf}\frac{\Delta y \Delta z}{\Delta x})_{i-\frac{1}{2}}$$

$$T^n_{xw,i+\frac{1}{2}} = \alpha(k_{f,eff}\lambda^n_{wf}\frac{\Delta y \Delta z}{\Delta x})_{i+\frac{1}{2}}$$

$$T^n_{xw,i-\frac{1}{2}} = \alpha(k_{f,eff}\lambda^n_{wf}\frac{\Delta y \Delta z}{\Delta x})_{i-\frac{1}{2}}$$

$$T^n_{xo,i+\frac{1}{2}} = \alpha(k_{f,eff}\lambda^n_{of}\frac{\Delta y \Delta z}{\Delta x})_{i+\frac{1}{2}}$$

$$T^n_{xo,i-\frac{1}{2}} = \alpha(k_{f,eff}\lambda^n_{of}\frac{\Delta y \Delta z}{\Delta x})_{i-\frac{1}{2}}$$

$$\alpha = 0.006328$$