HW#5

Assigned: January 22, 2009
Due: January 29, 2009

a) Expand the following equations in three dimensions (x-, y- and z-directions)

$$\nabla \cdot \left[\frac{k}{\mu} \left(\nabla p - \gamma \nabla D \right) \right] + \hat{q} = \frac{1}{M} \frac{\partial p}{\partial t} - \beta_{T,f} \frac{\partial T}{\partial t} - \alpha_p \frac{\partial \varepsilon_b}{\partial t} \quad (5.1)$$

$$G \nabla^2 \vec{d}_s + \left(G + \lambda \right) \nabla \left(\nabla \cdot \vec{d}_s \right) + \vec{\gamma}_b = -\alpha_p \nabla p - 3 \beta_{T,b} K_b \nabla T + \rho_f \frac{d^2 \vec{d}_s}{dt^2} \quad (5.2)$$

$$\nabla \cdot \left(k_T \nabla T \right) - \nabla \cdot \left(\rho_f \vec{v} H \right) + \rho H \hat{q} = \left[\phi \rho_f c_{v,f} + (1-\phi) \rho_s c_{v,s} \right] \frac{\partial T}{\partial t} - 3 \beta_{T,b} K_b T_R \frac{\partial \varepsilon_b}{\partial t} \quad (5.3)$$

<u>Background Information:</u>

- Divergence of a vector $\vec{f}$ is equal to a scalar

$$\nabla \cdot \vec{f} = \frac{\partial}{\partial x}(f_x) + \frac{\partial}{\partial y}(f_y) + \frac{\partial}{\partial z}(f_z)$$

- Gradient of a scalar f is equal to a vector

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right]^T$$

- Laplacian of a vector $\vec{f}$ can be expressed as:

$$\nabla^2 \vec{f} = \frac{\partial^2 f_x}{\partial x^2} + \frac{\partial^2 f_y}{\partial y^2} + \frac{\partial^2 f_z}{\partial z^2}$$

You have two more components for f_y and f_z following the same form as indicated above.

<u>Clarification on the Terms:</u>

- $\vec{\gamma}_b$ is the gravity vector, only has a z component, $\vec{\gamma}_b = 0$ in x and y direction.
- T_R is the reference temperature (absolute), $T_R = T_{reservior} + 460$
- $\vec{d}_s$ is the displacement vector $[d_{s1}, d_{s2}, d_{s3}]^T$
- $\vec{v}$ is the velocity vector $[v_x, v_y, v_z]^T$

b) Expand equations obtained from Part a) in z-direction only.