PEGN 624 Project
S2009
Jeff Brown
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Definitions

Initialization

Data

Grid Constants

```
In[1]:= Δx = 100 ;(*ft*)

In[2]:= Δy = 100; (*ft*)

In[3]:= Δz = 20; (*ft*)

In[307]:= Δt = 1; (* day *)

In[554]:= VR = Δx * Δy * Δz

Out[554]= 200 000

In[555]:= VRdt = VR / Δt

Out[555]= 200 000
```

Tolerances

```
In[312]:= ϵP = 1. * 10^(-3)
          ϵSo = ϵSg = 1. * 10^(-4)
          ϵTs = ϵT = 1. * 10^(-1)
          ϵG = 1. * 10^(-2)

Out[312]= 0.001

Out[313]= 0.0001

Out[314]= 0.1

Out[315]= 0.01
```

Rock Properties

```
In[4]:= km = 3000; (*md*)

In[5]:= ϕm = 0.3;

In[6]:= convertk = 0.006328

Out[6]= 0.006328
```

```
In[7]:=  convertq = 5.6146
Out[7]=  5.6146

In[11]:= convertT = 459.67
Out[11]= 459.67

In[87]:= convertU = (144/778.26)
Out[87]= 0.185028
```

Molecular Weight

```
In[82]:= MWo = 250;

In[83]:= MWg = MWw = 18;
```

Thermal Properties

```
In[304]:= ρR = 170 (* lbm/ft^3 *)
Out[304]= 170

In[305]:= CR = 0.206 (* btu/(lbm °F) *)
Out[305]= 0.206

In[306]:= kT = 1.6 * 24 (* btu/(day ft °F) *)
Out[306]= 38.4

In[694]:= TR = 100;
```

Ts

Correlations: T in °F; P in psia

```
In[198]:= calcTs[P_] := 116 (P)^0.224

In[199]:= calcTs[P_, T_] := calcTs[P]

In[200]:= calcTs[100]
Out[200]= 325.43

In[201]:= calcTs[600]
Out[201]= 486.143
```

Viscosities

Correlations: T in °F

```
In[202]:= calcμo[T_] := 3.62431 * 10^(-4) Exp[8485.4 / (T + convertT)]

In[203]:= calcμo[P_, T_] := calcμo[T]
```

In[204]:= `{calcμo[100], calcμo[400]}`

Out[204]= `{1392.36, 7.01363}`

In[205]:= `calcμw[T_] :=` $\frac{2.185}{\left(0.04012\,T + 5.1547 * 10^{(-6)} * (T^{\wedge}2) - 1\right)}$

In[206]:= `calcμw[P_, T_] := calcμw[T]`

In[207]:= `{calcμw[100], calcμw[400]}`

Out[207]= `{0.713226, 0.137657}`

In[208]:= `calcμg[T_] := 8.822 * 10^(-6) (T + convertT)^1.116`

In[209]:= `calcμg[P_, T_] := calcμg[T]`

In[210]:= `{calcμg[100], calcμg[400]}`

Out[210]= `{0.0102863, 0.0166066}`

- **Molar Density**

Correlations: T in °F; P in psia

In[211]:= `calcξo[P_, T_] := 1.683 * (0.1987 - 7.5885 * 10^(-5) * (T + convertT)) * Exp[10^(-5) * (P - 14.7)]`

In[212]:= `{calcξo[100, 100], calcξo[100, 400], calcξo[600, 100], calcξo[600, 400]}`

Out[212]= `{0.263159, 0.224811, 0.264478, 0.225938}`

In[213]:= `calcξw[P_, T_] := 22.14 * (0.202 - 7.725 * 10^(-5) * (T + convertT)) * Exp[4 * 10^(-6) * (P - 118)]`

In[214]:= `{calcξw[100, 100], calcξw[100, 400], calcξw[600, 100], calcξw[600, 400]}`

Out[214]= `{3.51481, 3.00176, 3.52185, 3.00777}`

May need to use other correlation where P > 1000.

In[215]:= `calcξg[P_] :=` $\frac{P^{\wedge}0.9585}{363.9 * 18}$

In[216]:= `calcξg[P_, T_] := calcξg[P]`

In[217]:= `{calcξg[100], calcξg[600]}`

Out[217]= `{0.0126109, 0.0702431}`

- **Molar Density Derivatives**

In[140]:= `calcDξoDP[P_, T_] := 10^(-5) * calcξo[P, T]`

In[141]:= `{calcDξoDP[100, 100], calcDξoDP[100, 400], calcDξoDP[600, 100], calcDξoDP[600, 400]}`

Out[141]= $\{2.63159 \times 10^{-6}, 2.24811 \times 10^{-6}, 2.64478 \times 10^{-6}, 2.25938 \times 10^{-6}\}$

In[142]:= `D[calcξo[P, T], T]`

Out[142]= $-0.000127714\, e^{\frac{-14.7+P}{100000}}$

In[143]:= `calcDξoDT[P_, T_] := -0.00012771445500000002` e^((-14.7`+P)/100000)`

In[144]:= `{calcDξoDT[100, 100], calcDξoDT[100, 400], calcDξoDT[600, 100], calcDξoDT[600, 400]}`

Out[144]= {-0.000127823, -0.000127823, -0.000128464, -0.000128464}

In[145]:= `calcDξwDP[P_, T_] := 4 * 10^(-6) * calcξw[P, T]`

In[146]:= `{calcDξwDP[100, 100], calcDξwDP[100, 400], calcDξwDP[600, 100], calcDξwDP[600, 400]}`

Out[146]= {0.0000140593, 0.000012007, 0.0000140874, 0.0000120311}

In[147]:= `D[calcξw[P, T], T]`

Out[147]= $-0.00171032\, e^{\frac{-118+P}{250000}}$

In[148]:= `calcDξwDT[P_, T_] := -0.0017103150000000002` e^((-118+P)/250000)`

In[149]:= `{calcDξwDT[100, 100], calcDξwDT[100, 400], calcDξwDT[600, 100], calcDξwDT[600, 400]}`

Out[149]= {-0.00171019, -0.00171019, -0.00171362, -0.00171362}

In[150]:= `calcDξgDP[P_] := 0.9585 * (P^(0.9585 - 1))/(363.9 * 18)`

In[151]:= `calcDξgDP[P_, T_] := calcDξgDP[P]`

In[152]:= `{calcDξgDP[100], calcDξgDP[600]}`

Out[152]= {0.000120875, 0.000112213}

In[626]:= `calcDξgDT[P_, T_] := 0`

Specific Gravities

In[293]:= `convertdensity = 0.4335275 (*psi/ft->g/cm^3*)`

Out[293]= 0.433528

In[294]:= `baseρw = 62.42796`

Out[294]= 62.428

In[295]:= `calcγo[P_, T_] := (MWo calcξo[P, T])/(baseρw) * convertdensity`

In[296]:= `{calcγo[100, 100], calcγo[100, 400], calcγo[600, 100], calcγo[600, 400]}`

Out[296]= {0.456872, 0.390298, 0.459163, 0.392254}

In[297]:= `calcγw[P_, T_] := (MWw calcξw[P, T])/(baseρw) * convertdensity`

In[298]:= `{calcγw[100, 100], calcγw[100, 400], calcγw[600, 100], calcγw[600, 400]}`

Out[298]= {0.439352, 0.37522, 0.440231, 0.375971}

In[299]:= `calcγg[P_, T_] := (MWw calcξg[P, T])/(baseρw) * convertdensity`

```
In[300]:= {calcγg[100, 100], calcγg[100, 400], calcγg[600, 100], calcγg[600, 400]}

Out[300]= {0.00157636, 0.00157636, 0.00878038, 0.00878038}
```

- **Heat Capacities**

Correlations: T in °F; P in psia

```
In[153]:= calcCpo[T_] := 57.88 + 6.03 * 10^(-2) (T + convertT)

In[154]:= calcCpo[P_, T_] := calcCpo[T]

In[155]:= {calcCpo[100], calcCpo[400]}

Out[155]= {91.6281, 109.718}

In[156]:= calcCpw[T_] := 41.809 - 7.8289 * 10^(-2) (T + convertT) + 6.2788 * 10^(-5) (T + convertT)^2

In[157]:= calcCpw[P_, T_] := calcCpw[T]

In[158]:= {calcCpw[100], calcCpw[400]}

Out[158]= {17.6601, 20.9087}

In[159]:= calcCpg[T_] := 7.136 + 1.4667 * 10^(-3) (T + convertT) + 2.55 * 10^(-8) (T + convertT)^2

In[160]:= calcCpg[P_, T_] := calcCpg[T]

In[161]:= {calcCpg[100], calcCpg[400]}

Out[161]= {7.96486, 8.41572}
```

- **Enthalpies**

Correlations: T in °F; P in psia

```
In[162]:= Integrate[calcCpo[To], {To, 32, 77}]

Out[162]= 3999.8

In[163]:= Integrate[calcCpo[To], {To, 77, T}]
```

Out[163]= $-6769.81 + 85.5981\,T + 0.03015\,T^2$

In[164]:= calcHo[T_] :=
 $3999.8002950000005` + -6769.813127000001` + 85.59810100000001`\,T + 0.030150000000000003`\,T^2$

```
In[165]:= calcHo[P_, T_] := calcHo[T]

In[166]:= Integrate[calcCpw[To], {To, 32, T}]
```

Out[166]= $-600.997 + 19.0888\,T - 0.0102827\,T^2 + 0.0000209293\,T^3$

In[167]:= calcHwless[T_] := $-600.9972648597292` +$
 $19.088780570813203`\,T - 0.010282740039999995`\,T^2 + 0.000020929333333333336`\,T^3$

```
In[168]:= calcHwless[P_, T_] := calcHwless[T]

In[169]:= Integrate[calcCpw[To], {To, 32, calcTs[P]}]
```

Out[169]= $-600.997 + 2214.3\,P^{0.224} - 138.365\,P^{0.448} + 32.6685\,P^{0.672}$

In[170]:= **calcHwmore[P_] := −600.9972648597292` + 2214.2985462143315` P^0.224` − 138.36454997823992` P^0.448` + 32.66851268266667` P^0.672`**

$$\text{calcHwmore}[P\_] := -600.9972648597292` + 2214.2985462143315`\,P^{0.224`} - 138.36454997823992`\,P^{0.448`} + 32.66851268266667`\,P^{0.672`}$$

In[171]:= **calcHwmore[P_, T_] := calcHwmore[P]**

In[172]:=

$$\text{calcHgmore}[P\_, T\_] := \text{calcHwmore}[P] + 1318\left(\left(\frac{T}{116}\right)\text{^}4.464\right)\text{^}-0.08774$$

■ Internal Energies

In[190]:=

$$\text{calcUo}[P\_, T\_] := \text{calcHo}[T] - \text{convertU}\,\frac{P}{\text{MWo calc}\xi\text{o}[P, T]}$$

In[191]:= **{calcUo[100, 100], calcUo[100, 400], calcUo[600, 100], calcUo[600, 400]}**

Out[191]= {6091.02, 36 292.9, 6089.62, 36 291.3}

In[192]:=

$$\text{calcUwless}[P\_, T\_] := \text{calcHwless}[T] - \text{convertU}\,\frac{P}{\text{MWw calc}\xi\text{w}[P, T]}$$

In[193]:= **{calcUwless[100, 100], calcUwless[100, 400], calcUwless[600, 100], calcUwless[600, 400]}**

Out[193]= {1225.69, 6728.41, 1224.23, 6726.7}

In[194]:=

$$\text{calcUwmore}[P\_, T\_] := \text{calcHwmore}[P] - \text{convertU}\,\frac{P}{\text{MWw calc}\xi\text{w}[P, T]}$$

In[195]:= **{calcUwmore[100, 500], calcUwmore[100, 1000], calcUwmore[600, 500], calcUwmore[600, 1000]}**

Out[195]= {5243.04, 5242.88, 8651.16, 8650.21}

In[196]:=

$$\text{calcUgmore}[P\_, T\_] := \text{calcHgmore}[P, T] - \text{convertU}\,\frac{P}{\text{MWg calc}\xi\text{g}[P]}$$

In[197]:= **{calcUgmore[100, 500], calcUgmore[100, 1000], calcUgmore[600, 500], calcUgmore[600, 1000]}**

Out[197]= {5905.58, 5728.77, 9309.22, 9132.4}

■ Internal Energies Derivatives

In[259]:= **D[calcUo[P, T], T]**

Out[259]=

$$85.5981 + 0.0603\,T - \frac{3.3371\times10^{-8}\,e^{\frac{14.7-P}{100000}}\,P}{(0.1987 - 0.000075885\,(459.67 + T))^2}$$

In[260]:=

$$\text{calcDUoDT}[P\_, T\_] := 85.59810100000001` + 0.060300000000000006`\,T - \frac{3.337102883149041`\text{*^-8}\,e^{\frac{14.7`-P}{100000}}\,P}{(0.1987` - 0.00007588500000000001`\,(459.67` + T))^2}$$

In[261]:= **{calcDUoDT[100, 100], calcDUoDT[100, 400], calcDUoDT[100, 1000], calcDUoDT[600, 100], calcDUoDT[600, 400], calcDUoDT[600, 1000]}**

Out[261]= {91.628, 109.718, 145.898, 91.6273, 109.717, 145.896}

In[262]:= `D[calcUo[P, T], P]`

Out[262]=
$$-\frac{0.000439758\, e^{\frac{14.7-P}{100000}}}{0.1987-0.000075885\,(459.67+T)}+\frac{4.39758\times 10^{-9}\, e^{\frac{14.7-P}{100000}}\, P}{0.1987-0.000075885\,(459.67+T)}$$

In[263]:=
$$\text{calcDUoDP}[P\_,T\_]:=-\frac{0.0004397579077747962\text{`}\, e^{\frac{14.7\text{`}-P}{100000}}}{0.1987\text{`}-0.00007588500000000001\text{`}\,(459.67\text{`}+T)}+\frac{4.397579077747962\text{`*\^{}-9}\, e^{\frac{14.7\text{`}-P}{100000}}\, P}{0.1987\text{`}-0.00007588500000000001\text{`}\,(459.67\text{`}+T)}$$

In[264]:= `{calcDUoDP[100, 100], calcDUoDP[100, 400], calcDUoDP[100, 1000], calcDUoDP[600, 100], calcDUoDP[600, 400], calcDUoDP[600, 1000]}`

Out[264]= {-0.00280961, -0.00328886, -0.0049918, -0.0027816, -0.00325607, -0.00494204}

In[265]:= `D[calcUwless[P, T], T]`

Out[265]=
$$19.0888-0.0205655\,T+0.000062788\,T^2-\frac{3.58663\times 10^{-8}\, e^{\frac{118-P}{250000}}\, P}{(0.202-0.00007725\,(459.67+T))^2}$$

In[266]:=
$$\text{calcDUwlessDT}[P\_,T\_]:=85.59810100000001\text{`}+0.06030000000000006\text{`}\,T-\frac{3.337102883149041\text{`*\^{}-8}\, e^{\frac{14.7\text{`}-P}{100000}}\, P}{(0.1987\text{`}-0.00007588500000000001\text{`}\,(459.67\text{`}+T))^2}$$

In[267]:= `{calcDUwlessDT[100, 100], calcDUwlessDT[100, 400], calcDUwlessDT[600, 100], calcDUwlessDT[600, 400]}`

Out[267]= {91.628, 109.718, 91.6273, 109.717}

In[268]:= `D[calcUwless[P, T], P]`

Out[268]=
$$-\frac{0.000464288\, e^{\frac{118-P}{250000}}}{0.202-0.00007725\,(459.67+T)}+\frac{1.85715\times 10^{-9}\, e^{\frac{118-P}{250000}}\, P}{0.202-0.00007725\,(459.67+T)}$$

In[269]:=
$$\text{calcDUwlessDP}[P\_,T\_]:=-\frac{0.00046428821563847613\text{`}\, e^{\frac{118-P}{250000}}}{0.202\text{`}-0.00007725000000000001\text{`}\,(459.67\text{`}+T)}+\frac{1.8571528625539045\text{`*\^{}-9}\, e^{\frac{118-P}{250000}}\, P}{0.202\text{`}-0.00007725000000000001\text{`}\,(459.67\text{`}+T)}$$

In[270]:= `{calcDUwlessDP[100, 100], calcDUwlessDP[100, 400], calcDUwlessDP[600, 100], calcDUwlessDP[600, 400]}`

Out[270]= {-0.00292341, -0.00342307, -0.00291173, -0.0034094}

In[271]:= `D[calcUwmore[P, T], T]`

Out[271]=
$$-\frac{3.58663\times 10^{-8}\, e^{\frac{118-P}{250000}}\, P}{(0.202-0.00007725\,(459.67+T))^2}$$

In[272]:=
$$\text{calcDUwmoreDT}[P\_,T\_]:=-\frac{3.586626465807229\text{`*\^{}-8}\, e^{\frac{118-P}{250000}}\, P}{(0.202\text{`}-0.00007725000000000001\text{`}\,(459.67\text{`}+T))^2}$$

In[273]:=
```
{calcDUwmoreDT[100, 500], calcDUwmoreDT[100, 1000],
 calcDUwmoreDT[600, 500], calcDUwmoreDT[600, 1000]}
```

Out[273]= {-0.000219387, -0.000450395, -0.00131369, -0.00269697}

In[274]:=
```
D[calcUwmore[P, T], P]
```

Out[274]=
$$\frac{496.003}{P^{0.776}} - \frac{61.9873}{P^{0.552}} + \frac{21.9532}{P^{0.328}} - \frac{0.000464288\, e^{\frac{118-P}{250000}}}{0.202 - 0.00007725\,(459.67 + T)} + \frac{1.85715 \times 10^{-9}\, e^{\frac{118-P}{250000}}\, P}{0.202 - 0.00007725\,(459.67 + T)}$$

In[275]:=
$$\text{calcDUwmoreDP}[P\_, T\_] := \frac{496.0028743520103`}{P^{0.776`}} - \frac{61.98731839025149`}{P^{0.552`}} + \frac{21.953240522752004`}{P^{0.32799999999999996`}} - \frac{0.00046428821563847613`\, e^{\frac{118-P}{250000}}}{0.202` - 0.00007725000000000001`\,(459.67` + T)} + \frac{1.8571528625539045`*\verb|^|-9\, e^{\frac{118-P}{250000}}\, P}{0.202` - 0.00007725000000000001`\,(459.67` + T)}$$

In[276]:=
```
{calcDUwmoreDP[100, 500], calcDUwmoreDP[100, 1000],
 calcDUwmoreDP[600, 500], calcDUwmoreDP[600, 1000]}
```

Out[276]= {13.88, 13.8784, 4.33952, 4.33796}

In[277]:=
```
D[calcUgmore[P, T], T]
```

Out[277]=
$$-\frac{3322.2\, T^{3.464}}{\left(T^{4.464}\right)^{1.08774}}$$

In[278]:=
$$\text{calcDUgmoreDT}[P\_, T\_] := -\frac{3322.203982900616`\, T^{3.4640000000000004`}}{\left(T^{4.464`}\right)^{1.08774`}}$$

In[279]:=
```
{calcDUgmoreDT[100, 500], calcDUgmoreDT[100, 1000],
 calcDUgmoreDT[600, 500], calcDUgmoreDT[600, 1000]}
```

Out[279]= {-0.582569, -0.22203, -0.582569, -0.22203}

In[280]:=
```
D[calcUgmore[P, T], P]
```

Out[280]=
$$-\frac{2.79427}{P^{0.9585}} + \frac{496.003}{P^{0.776}} - \frac{61.9873}{P^{0.552}} + \frac{21.9532}{P^{0.328}}$$

In[281]:=
$$\text{calcDUgmoreDP}[P\_, T\_] := -\frac{2.7942672114717433`}{P^{0.9585`}} + \frac{496.0028743520103`}{P^{0.776`}} - \frac{61.98731839025149`}{P^{0.552`}} + \frac{21.953240522752004`}{P^{0.32799999999999996`}}$$

In[282]:=
```
{calcDUgmoreDP[100, 500], calcDUgmoreDP[100, 1000],
 calcDUgmoreDP[600, 500], calcDUgmoreDP[600, 1000]}
```

Out[282]= {13.8498, 13.8498, 4.33706, 4.33706}

▪ Compressibilities

In[301]:=
```
cϕ = 10 * 10^(-6); (* psi^-1*)
```

▪ Source Terms

In[568]:=
```
flowsource = 35000. / 18
```

Out[568]= 1944.44

In[572]:=
```
heatsource = 1000. * flowsource
```

Out[572]= 1.94444×10^{6}

Capillary Pressure, Rel Perm

Rel Perm

Saturations

In[484]:=
```
Swrm = 0.20;
Sowrm = 0.30;
Sogrm = 0.20;
Sgcm = 0.05;
```

Rel Perm

In[488]:=
```
krwmstar = 0.2;
nwm = 3.0;
```

In[490]:=
```
krowmstar = 0.6;
nowm = 4.0;
```

In[492]:=
```
krgmstar = 0.5;
ngm = 3.0;
```

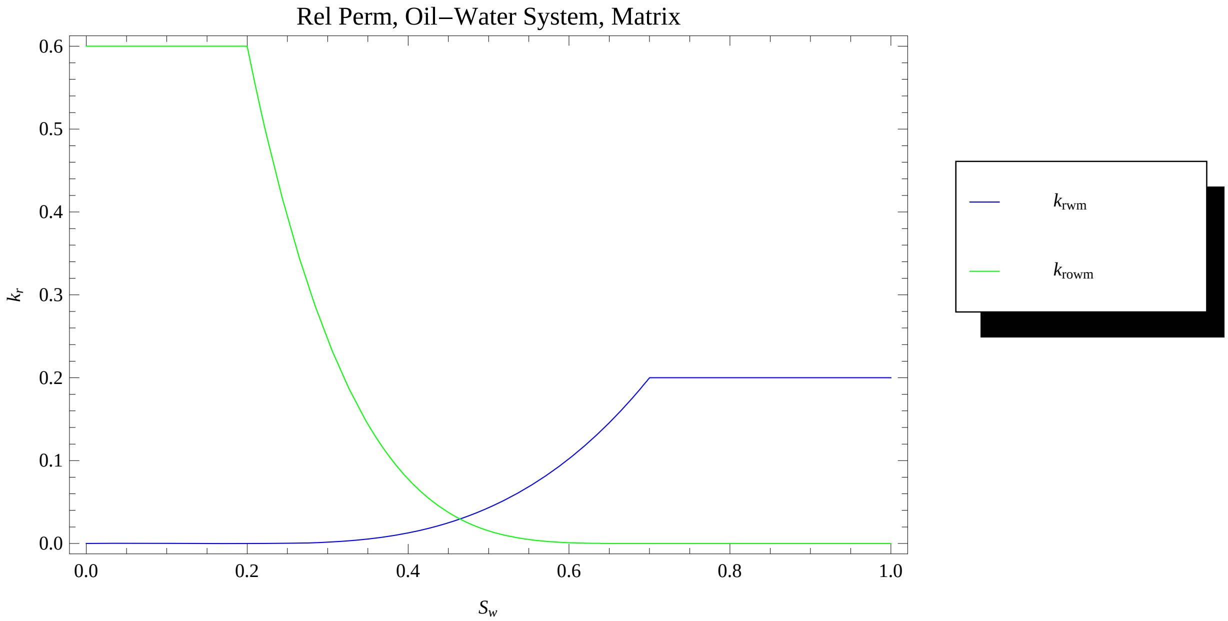
In[494]:=
```
computekrwm[Sw_] :=
 Which[Sw ≤ Swrm, 0,
  Sw ≥ (1 - Sowrm), krwmstar,
  True,
  krwmstar * ((Sw - Swrm)/(1 - Swrm - Sowrm)) ^nwm
 ]
```

In[495]:=
```
computekrowm[So_] :=
 Which[So ≤ Sowrm, 0,
  So ≥ (1 - Swrm), krowmstar,
  True,
  krowmstar * ((So - Sowrm)/(1 - Swrm - Sowrm)) ^nowm
 ]
```

In[496]:=
```
computekrgm[Sg_] :=
 Which[Sg ≤ Sgcm, 0,
  Sg ≥ (1 - Swrm - Sogrm), krgmstar,
  True,
  krgmstar * ((Sg - Sgcm)/(1 - Swrm - Sogrm - Sgcm)) ^ngm
 ]
```

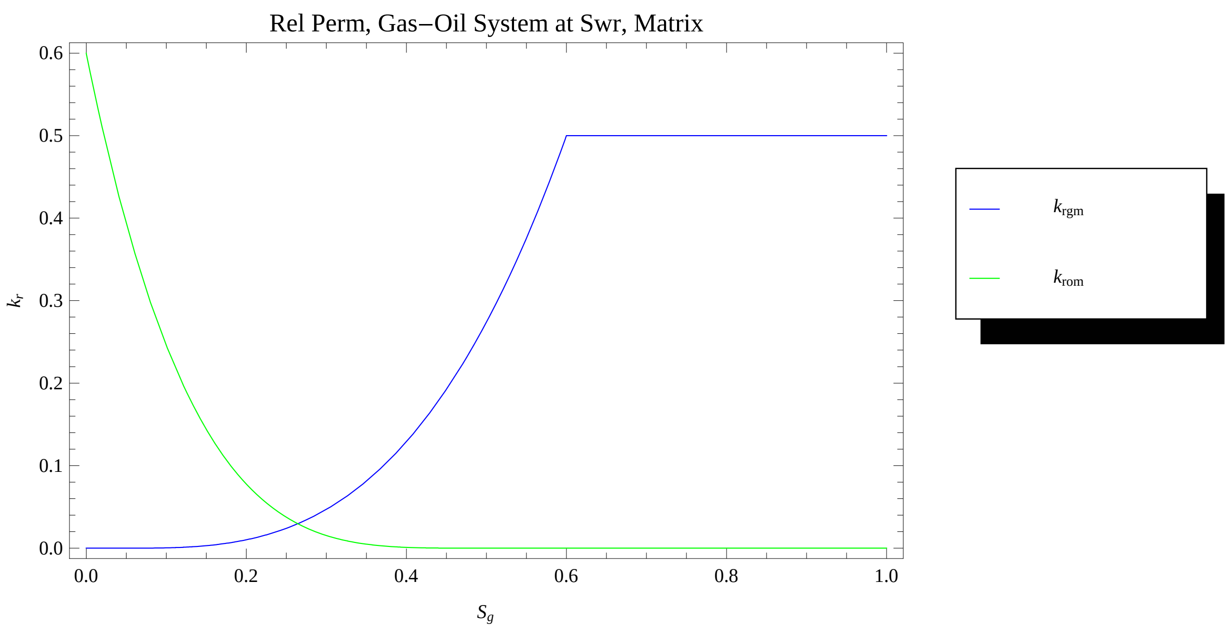
In[497]:=
```
Plot[{computekrwm[Sw], computekrowm[1 - Sw]}, {Sw, 0, 1}, Frame → True,
 FrameLabel → {"Sw", "kr"}, PlotLabel → "Rel Perm, Oil-Water System, Matrix",
 PlotStyle → {Blue, Green}, ImageSize → 600, PlotLegend → {"krwm", "krowm"},
 LegendPosition → {.9, 0}, LegendSize → {.5, .3}]
```

Out[497]=



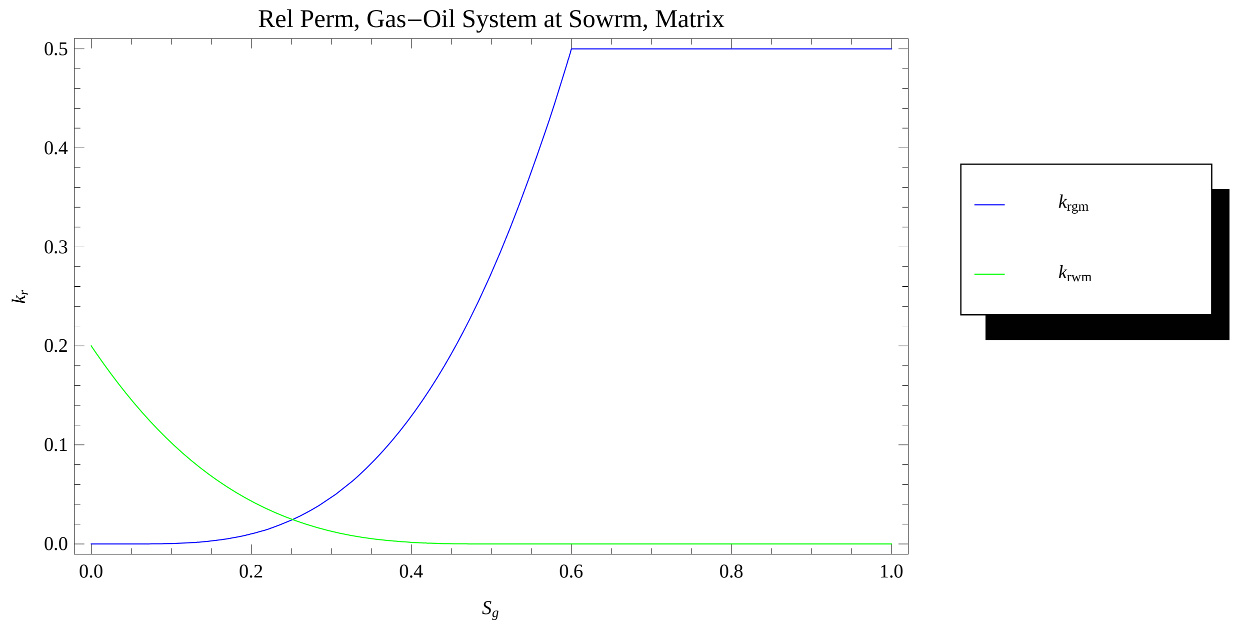
In[498]:=
```
Plot[{computekrgm[Sg], computekrowm[1 - Sg - Swrm]}, {Sg, 0, 1}, Frame → True,
 FrameLabel → {"Sg", "kr"}, PlotLabel → "Rel Perm, Gas-Oil System at Swr, Matrix",
 PlotStyle → {Blue, Green}, ImageSize → 600, PlotLegend → {"krgm", "krom"},
 LegendPosition → {.9, 0}, LegendSize → {.5, .3}]
```

Out[498]=



In[501]:=
```
Plot[{computekrgm[Sg], computekrwm[1 - Sg - Sowrm]}, {Sg, 0, 1}, Frame → True,
 FrameLabel → {"Sg", "kr"}, PlotLabel → "Rel Perm, Gas-Oil System at Sowrm, Matrix",
 PlotStyle → {Blue, Green}, ImageSize → 600, PlotLegend → {"krgm", "krwm"},
 LegendPosition → {.9, 0}, LegendSize → {.5, .3}]
```

Out[501]=



- **Capillary Pressure**

Capillary Pressure

In[507]:=
```
Pcowstar = 5;
Pcogstar = 10;
Pcogentry = 3;
ncw = 1.76;
ncg = 1.20;
```

In[512]:=
```
Pcogm1[Sg_] := Pcogentry + Pcogstar * ((Sg - Sgcm) / (1 - Swrm - Sogrm - Sgcm))^ncg
```

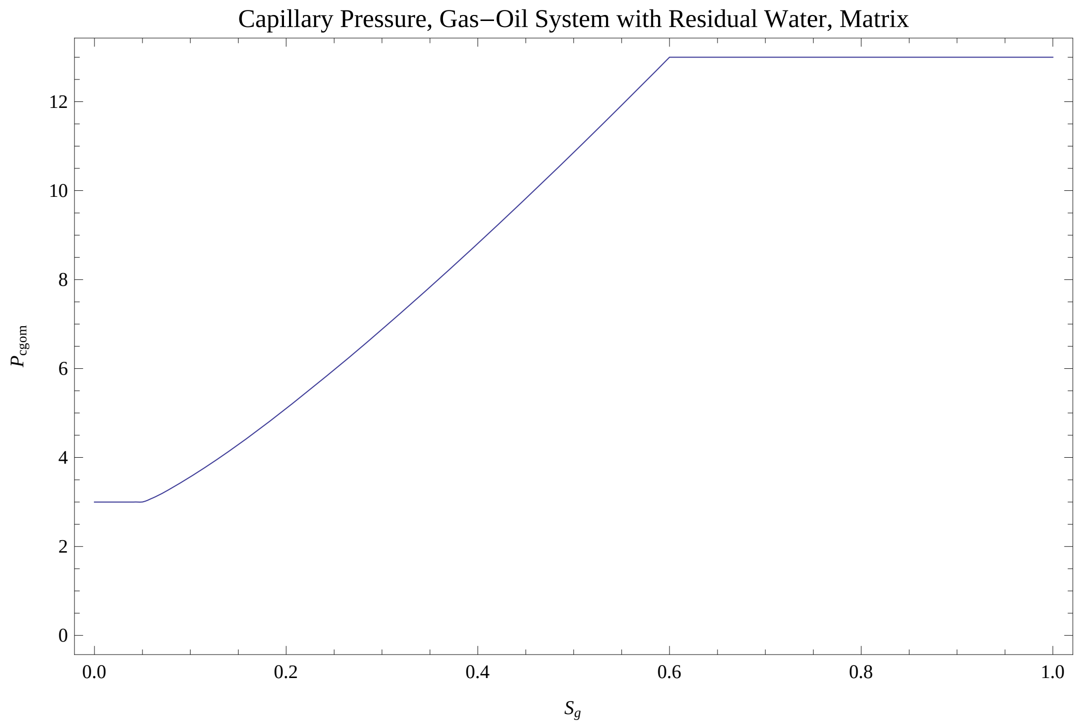
In[513]:=
```
Pcowm1[Sw_] := Pcowstar * ((1 - Sowrm - Sw) / (1 - Sowrm - Swrm))^ncw
```

In[516]:=
```
computePcogm[Sg_] :=
 Module[{},
  Which[Sg ≤ Sgcm, Pcogentry,
   Sg ≥ (1 - Swrm - Sogrm), Pcogentry + Pcogstar,
   True, Pcogm1[Sg]
  ]]
```

In[520]:=
```
computePcowm[Sw_] :=
 Module[{},
  Which[Sw ≤ Swrm, Pcowstar,
   Sw ≥ (1 - Sowrm), 0,
   True, Pcowm1[Sw]
  ]]
```

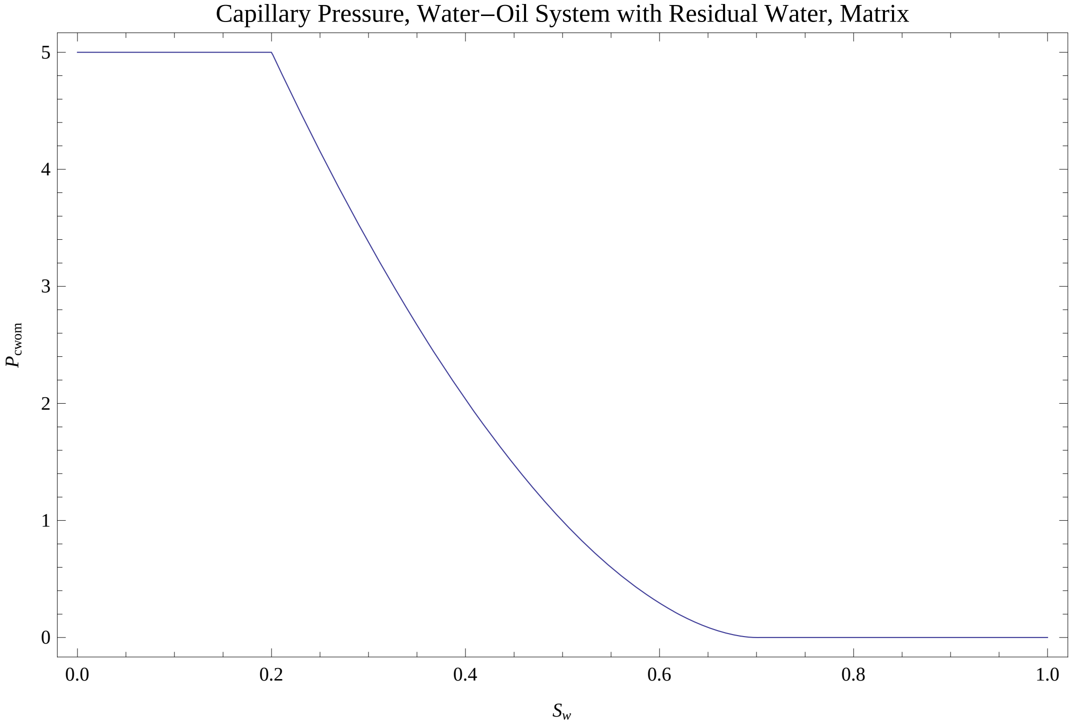
In[517]:=
```
Plot[computePcogm[Sg], {Sg, 0, 1}, Frame → True, FrameLabel → {"Sg", "Pcgom"},
 PlotLabel → "Capillary Pressure, Gas-Oil System with Residual Water, Matrix",
 ImageSize → 500, PlotRange → All, AxesOrigin → {0, 0}]
```

Out[517]=



In[521]:=
```
Plot[computePcowm[Sw], {Sw, 0, 1}, Frame → True, FrameLabel → {"Sw", "Pcwom"},
 PlotLabel → "Capillary Pressure, Water-Oil System with Residual Water, Matrix",
 ImageSize → 500, PlotRange → All, AxesOrigin → {0, 0}]
```

Out[521]=



■ Transmissibilities & Fractional Flows

■ Transmissibilities

use upstream only here since k, Δx are constant

In[536]:=
```
calcTw[P_, T_, So_, Sg_, Sw_] :=
 convertk * km * computekrwm[Sw] / calcμw[P, T] * (Δy Δz) / Δx
```

In[537]:=
```
calcTo[P_, T_, So_, Sg_, Sw_] :=
 convertk * km * computekrowm[So] / calcμo[P, T] * (Δy Δz) / Δx
```

In[538]:=
```
calcTg[P_, T_, So_, Sg_, Sw_] :=
 convertk * km * computekrgm[Sg] / calcμg[P, T] * (Δy Δz) / Δx
```

In[539]:=
```
a1 = {500, 100, 0.8, 0, 0.2}
```

Out[539]=
```
{500, 100, 0.8, 0, 0.2}
```

In[540]:=
```
{calcTw @@ a1, calcTo @@ a1, calcTg @@ a1}
```

Out[540]=
```
{0, 0.163612, 0}
```

In[541]:=
```
a2 = {500, 500, 0.7, 0.1, 0.2}
```
Out[541]=
```
{500, 500, 0.7, 0.1, 0.2}
```
In[542]:=
```
{calcTw @@ a2, calcTo @@ a2, calcTg @@ a2}
```
Out[542]=
```
{0, 37.2111, 7.59617}
```

- **Thermal Transmissibility**

In[614]:=
$$TE = kT \frac{(\Delta y\ \Delta z)}{\Delta x}$$
Out[614]=
```
768.
```

- **Thermal Boundaries**

In[695]:=
$$calcqb[T\_] := -kT * \Delta x * \Delta y * \frac{(T - TR)}{Sqrt\left[Pi\left(\frac{kT}{\rho R * CR}\right)\Delta t\right]}$$
In[614]:=
$$TE = kT \frac{(\Delta y\ \Delta z)}{\Delta x}$$
Out[614]=
```
768.
```

- **Fractional Flow**

use upstream only here since k, Δx are constant

In[588]:=
$$calc\lambda w[P\_, T\_, So\_, Sg\_, Sw\_] := \frac{computekrwm[Sw]}{calc\mu w[P, T]}$$
In[589]:=
$$calc\lambda o[P\_, T\_, So\_, Sg\_, Sw\_] := \frac{computekrowm[So]}{calc\mu o[P, T]}$$
In[590]:=
$$calc\lambda g[P\_, T\_, So\_, Sg\_, Sw\_] := \frac{computekrgm[Sg]}{calc\mu g[P, T]}$$
In[591]:=
```
calcλtot[P_, T_, So_, Sg_, Sw_] :=
 calcλw[P, T, So, Sg, Sw] + calcλo[P, T, So, Sg, Sw] + calcλg[P, T, So, Sg, Sw]
```
In[592]:=
```
calcffw[P_, T_, So_, Sg_, Sw_] :=
 calcλw[P, T, So, Sg, Sw] / calcλtot[P, T, So, Sg, Sw]
```
In[593]:=
```
calcffo[P_, T_, So_, Sg_, Sw_] :=
 calcλo[P, T, So, Sg, Sw] / calcλtot[P, T, So, Sg, Sw]
```
In[594]:=
```
calcffg[P_, T_, So_, Sg_, Sw_] :=
 calcλg[P, T, So, Sg, Sw] / calcλtot[P, T, So, Sg, Sw]
```
In[595]:=
```
a1 = {500, 100, 0.8, 0, 0.2}
```
Out[595]=
```
{500, 100, 0.8, 0, 0.2}
```

In[596]:=
```
{calcffw @@ a1, calcffo @@ a1, calcffg @@ a1}
```
Out[596]=
```
{0, 1., 0}
```
In[597]:=
```
a2 = {500, 500, 0.7, 0.1, 0.2}
```
Out[597]=
```
{500, 500, 0.7, 0.1, 0.2}
```
In[598]:=
```
{calcffw @@ a2, calcffo @@ a2, calcffg @@ a2}
```
Out[598]=
```
{0, 0.83047, 0.16953}
```

- **Thermodynamic Constraints**

In[698]:=
```
calcG[P_, T_, So_, Sg_, Sw_] :=
```
$$\left(\frac{T}{116}\right)^{4.464} * \left(\frac{Sw}{Sw + \epsilon G}\right) - P$$

In[700]:=
```
calcDGDP[P_, T_, So_, Sg_, Sw_] :=
 -1
```
In[705]:=
```
calcDGDT[P_, T_, So_, Sg_, Sw_] :=
```
$$\frac{4.464}{116}\left(\frac{T}{116}\right)^{3.464} * \left(\frac{Sw}{Sw + \epsilon G}\right)$$

In[715]:=
```
calcDGDSo[P_, T_, So_, Sg_, Sw_] :=
```
$$\left(\frac{T}{116}\right)^{4.464} * \left(\frac{-\epsilon G}{(Sw + \epsilon G)^2}\right)$$

At T=100F, P=500

- **Conditions**

In[551]:=
```
a1 = {P1 = 500 (*P*), T1 = 100 (*T*), So1 = 0.8 (*So*), Sg1 = 0 (*Sg*), Sw1 = 0.2 (*Sw*)}
```
Out[551]=
```
{500, 100, 0.8, 0, 0.2}
```
In[544]:=
```
a11 = Take[a1, 2]
```
Out[544]=
```
{500, 100}
```
In[559]:=
```
T1
```
Out[559]=
```
100
```
In[731]:=
```
Ts1 = calcTs[P1, T1]
```
Out[731]=
```
466.689
```

Thus T1 < Ts

- **Water Equation**

Term 1,9,11 (δP)

```
In[631]:= W1 = (calcTw @@ a1) * (calcξw @@ a11)
Out[631]= 0

In[632]:= W9 = -W1
Out[632]= 0

In[633]:= W11 = W1
Out[633]= 0
```

Term 2,10,12 (δP)

```
In[634]:= W2 = (calcTg @@ a1) * (calcξg @@ a11)
Out[634]= 0

In[635]:= W10 = -W2
Out[635]= 0

In[636]:= W12 = -W2
Out[636]= 0
```

Term 3 (δP)

```
In[637]:= W3 = -VRdt * ϕm * cϕ * Sw1 * calcξw[P1, T1] + VRdt * ϕm * Sw1 * calcDξwDP[P1, T1]
Out[637]= -0.253472
```

Term 4 (δT)

```
In[638]:= W4 = -VRdt * ϕm * Sw1 * calcDξwDT[P1, T1]
Out[638]= 20.5552
```

Term 5 (δSo)

```
In[639]:= W5 = VRdt * ϕm * calcξw[P1, T1]
Out[639]= 211 227.

In[640]:= W6 = 0
Out[640]= 0
```

Term 7 (ℓ)

```
In[641]:= W7 = VRdt * (ϕm * Sw1 * calcξw[P1, T1])
Out[641]= 42 245.3
```

Term 8 (n)

```
In[642]:= W8 = -VRdt * (ϕm * Sw1 * calcξw[P1, T1])
Out[642]= -42 245.3
```

Term 13, 14 : 0 since ΔD = 0

```
In[643]:= W13 = W14 = 0
Out[643]= 0
```

Term 15, cell 1 injection

```
W15a = -flowsource
Out[647]= 1944.44
```

Term 15, cell N production

```
In[648]:= W15c =
  -(-flowsource * (calcffw @@ a1) * (calcξw @@ a11) - flowsource * (calcffg @@ a1) * (calcξg @@ a11))
Out[648]= 0
```

- **Oil Equation**

Term 1,7 (δP)

```
In[658]:= O1 = (calcTo @@ a1) * (calcξo @@ a11)
Out[658]= 0.0432286
In[659]:= O7 = -O1
Out[659]= -0.0432286
```

Term 2 (δP)

```
In[660]:= O2 = -VRdt * ϕm * So1 * (cϕ * calcξo[P1, T1] + calcDξoDP[P1, T1])
Out[660]= -0.253645
```

Term 3 (δT)

```
In[661]:= O3 = -VRdt * ϕm * So1 * calcDξoDT[P1, T1]
Out[661]= 6.16012
```

Term 4 (δSo)

```
In[662]:= O4 = -VRdt * ϕm * calcξo[P1, T1]
Out[662]= -15 852.8
```

Term 5 (ℓ)

```
In[663]:= O5 = VRdt * (ϕm * So1 * calcξo[P1, T1])
Out[663]= 12 682.2
```

Term 6 (n)

```
In[664]:= O6 = -VRdt * (ϕm * So1 * calcξo[P1, T1])
Out[664]= -12 682.2
```

Term 8 : 0 since ΔD = 0

In[665]:=
```
O8 = 0
```
Out[665]= 0

Term 9, cell N production

In[666]:=
```
O9c = -(-flowsource * (calcffo @@ a1) * (calcξo @@ a11))
```
Out[666]= 513.748

- **Energy Equation, LHS**

Term 1 (δP)

In[668]:=
```
E1 = (calcTw @@ a1) * (calcξw @@ a11) * (calcHwless @@ a11)
```
Out[668]= 0

```
E19 = -E1
```

```
E23 = E1
```

Term 2 (δP)

In[669]:=
```
E2 = (calcTg @@ a1) * (calcξg @@ a11) * (calcHgmore @@ a11)
```
Out[669]= 0

```
E20 = -E2
```

```
E24 = -E2
```

Term 3 (δP)

In[670]:=
```
E3 = (calcTo @@ a1) * (calcξo @@ a11) * (calcHo @@ a11)
```
Out[670]= 263.318

```
E21 = -E3
```

Term 4 (δT)

In[671]:=
```
E4 = TE
```
Out[671]= 768.

```
E22 = -E4
```

Term 5 (δP)

In[672]:=
```
E5 = -VRdt * ϕm * Sg1 * (cϕ * calcξg[P1, T1] * (calcUgmore @@ a11) +
     calcDξgDP[P1, T1] * (calcUgmore @@ a11) + calcξg[P1, T1] * (calcDUgmoreDP @@ a11))
```
Out[672]= 0

Term 6 (δP)

In[673]:=
```
E6 = -VRdt * ϕm * Sw1 * (cϕ * calcξw[P1, T1] * (calcUwless @@ a11) +
     calcDξwDP[P1, T1] * (calcUwless @@ a11) + calcξw[P1, T1] * (calcDUwlessDP @@ a11))
```
Out[673]= -601.12

Term 7 (δP)

In[674]:=
```
E7 = -VRdt * ϕm * So1 * (cϕ * calcξo[P1, T1] * (calcUo @@ a11) +
     calcDξoDP[P1, T1] * (calcUo @@ a11) + calcξo[P1, T1] * (calcDUoDP @@ a11))
```

Out[674]= -1509.32

Term 8 (δP)

In[675]:=
```
E8 = VRdt * ϕm * cϕ * ρR * CR * T1
```

Out[675]= 2101.2

Term 9 (δT)

In[676]:=
```
E9 = -VRdt * ϕm * Sg1 *
   (calcDξgDT[P1, T1] * (calcUgmore @@ a11) + calcξg[P1, T1] * (calcDUgmoreDT @@ a11))
```

Out[676]= 0

Term 10 (δT)

In[677]:=
```
E10 = -VRdt * ϕm * Sw1 *
   (calcDξwDT[P1, T1] * (calcUwless @@ a11) + calcξw[P1, T1] * (calcDUwlessDT @@ a11))
```

Out[677]= -3.84566×10^{6}

Term 11 (δT)

In[678]:=
```
E11 = -VRdt * ϕm * So1 * (calcDξoDT[P1, T1] * (calcUo @@ a11) + calcξo[P1, T1] * (calcDUoDT @@ a11))
```

Out[678]= -1.12453×10^{6}

Term 12 (δT)

In[679]:=
```
E12 = -VRdt * (1 - ϕm) * ρR * CR
```

Out[679]= -4.9028×10^{6}

Term 13 (δSg)

In[680]:=
```
E13 = VRdt * ϕm * (calcξw[P1, T1] calcUwless[P1, T1] - calcξg[P1, T1] calcUgmore[P1, T1])
```

Out[680]= 2.25015×10^{8}

Term 14 (δSo)

```
E14 = VRdt * ϕm * (calcξw[P1, T1] calcUwless[P1, T1] - calcξo[P1, T1] calcUo[P1, T1])
```

Out[681]= 1.6211×10^{8}

- **Energy Equation, RHS**

Term 15 (ℓ)

In[682]:=
```
E15 = VRdt * ϕm * (Sw1 * calcξw[P1, T1] * calcUwless[P1, T1] +
     So1 * calcξo[P1, T1] * calcUo[P1, T1] +
     Sg1 * calcξg[P1, T1] * calcUgmore[P1, T1])
```

Out[682]= 1.28964×10^{8}

Term 16 (n)

In[683]:= **E16 = -E15**

Out[683]= -1.28964×10^{8}

Term 17 (ℓ)

In[684]:= **E17 = VRdt * (1 - ϕm) * ρR * CR * T1**

Out[684]= 4.9028×10^{8}

Term 18 (n)

In[685]:= **E18 = -E17**

Out[685]= -4.9028×10^{8}

Term 25,26,27 =0 since ΔD=0

In[686]:= **E25 = E26 = E27 = 0**

Out[686]= 0

Term 28-29, cell 1 injection

In[691]:= **E28a = -heatsource**

Out[691]= -1.94444×10^{6}

Term 28-29, cell N production

```
In[693]:= E28c = -(-heatsource) * ((calcffw @@ a1) * calcξw[P1, T1] * calcHwless[P1, T1] +
     (calcffo @@ a1) * calcξo[P1, T1] * calcHo[P1, T1] +
     (calcffg @@ a1) * calcξg[P1, T1] * calcHgmore[P1, T1])
```

Out[693]= 3.12939×10^{9}

Term 30, to boundaries

In[697]:= **E30 = -(calcqb[T1] + calcqb[T1])**

Out[697]= 0

- **Thermodynamic Constraints**

Term 1; δP

In[736]:= **G1 = -1;**

Term 2; δSo

In[737]:= **G2 = calcDGDSo[P1, Ts1, So1, Sg1, Sw1]**

Out[737]= -113.334

Term 3; δTs

In[738]:= **G3 = calcDGDT[P1, Ts1, So1, Sg1, Sw1]**

Out[738]= 4.55308

Term 4; (ℓ)

In[740]:=
```
G4 = -(calcG[P1, Ts1, So1, Sg1, Sw1] + computePcowm[Sw1])
```

Out[740]=
```
18.9989
```

- **At T=500F, P=500**