PEGN 624 Project
S2009

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HW #18 Solution, T<Ts (T=100℉, Δt=1 day)

Properties @ n

This section describes the properties which were not defined in HW #17.

Conditions at time n

For HW #18, the conditions for all three grid cells for both time n and time ℓ are the same.

Pressure (psia)

```
P1n
500
```

Temperature (°F)

```
T1n
100
```

So

```
So1n
0.8
```

Sg

```
Sg1n
0
```

Sw

```
Sw1n
0.2
```

Satration Temperature (°F)

```
Ts1n
466.689
```

Use Δt = 1 day for HW #18.

Δt

1

- **Transmissibilities @ n**

$T_w = 0$ because krw=0 @ Sw=Swr

vecTwn[[1]]

0

$T_g = 0$ because krg=0 @ Sg=0

vecTgn[[1]]

0

T_o

vecTon[[1]]

0.163612

- **Molar Densities (ξ) @ n**

ξ_w

vecξwn[[1]]

3.52044

ξ_g

vecξgn[[1]]

0.0589805

ξ_o

vecξon[[1]]

0.264213

- **Enthalpies @ n**

H_w

vecHwn[[1]]

1225.98

H_g (not used, so using the same formula for T ≥ Ts)

vecHgn[[1]]

9592.19

H_o

```
vecHon[[1]]
```

```
6091.3
```

▪ Internal Energies @ n

U_w

```
vecUwn[[1]]
```

```
1224.52
```

U_g

```
vecUgn[[1]]
```

```
9505.05
```

U_o

```
vecUon[[1]]
```

```
6089.9
```

▪ Upstream-weighted Tξ @ n

No harmonic weighting is required since k, Δx, Δy,
and Δz are all constant. $(T\xi)_o$ only is provided since other T's are 0.

```
Transpose[{transindex, fTξon /@ transindex}] // Grid[#, Frame → All] &
```

$\frac{1}{2}$	0
$\frac{3}{2}$	0.0432286
$\frac{5}{2}$	0.0432286
$\frac{7}{2}$	0

▪ Upstream-weighted TξH @ n

$(T\xi H)_o$ only since other T's are 0.

```
Transpose[{transindex, fTξHon /@ transindex}] // Grid[#, Frame → All] &
```

$\frac{1}{2}$	0
$\frac{3}{2}$	263.318
$\frac{5}{2}$	263.318
$\frac{7}{2}$	0

▪ Volumetric Flow

Total volumetric flow (ft^3/day)

```
qtotn
```

552.329

Accumulation @ n

Water accumulation term @ n

```
vecACCWn[[1]]
```

-42245.3

Oil accumulation term @ n

```
vecACCOn[[1]]
```

-12682.2

Energy accumulation term @ n

```
vecACCEn[[1]]
```

-6.19244×10^{8}

Source Terms @ n

Water source terms @ n

```
{vecSOURCEWn} // Grid[#, Frame → All] &
```

-1944.44	0	0

Oil source terms @ n

```
{vecSOURCEOn} // Grid[#, Frame → All] &
```

0	0	145.933

Energy source terms @ n.
The top & bottom boundary fluxes are 0 since T=TR.

```
{vecSOURCEEn} // Grid[#, Frame → All] &
```

-35000000	0	888920.

Properties @ ℓ

This section describes the properties which were not defined in HW #17, and are not defined at time n above.

Molar Density Derivatives (∂ξ/∂P) @ ℓ

$\partial \xi_w / \partial P$

```
vecDξwDPℓ[[1]]
```

0.0000140818

$\partial \xi_g / \partial P$

```
vecDξgDPℓ⟦1⟧
```

0.000113066

$\partial \xi_o / \partial P$

```
vecDξoDPℓ⟦1⟧
```

2.64213×10^{-6}

- **Molar Density Derivatives (∂ξ/∂T) @ ℓ**

$\partial \xi_w / \partial T$

```
vecDξwDTℓ⟦1⟧
```

-0.00171293

$\partial \xi_g / \partial T$

```
vecDξgDTℓ⟦1⟧
```

0

$\partial \xi_o / \partial T$

```
vecDξoDTℓ⟦1⟧
```

-0.000128336

- **Internal Energy Derivatives (∂U/∂P) @ ℓ**

$\partial U_w / \partial P$

```
vecDUwDPℓ⟦1⟧
```

-0.00291406

$\partial U_g / \partial P$

```
vecDUgDPℓ⟦1⟧
```

4.83628

$\partial U_o / \partial P$

```
vecDUoDPℓ⟦1⟧
```

-0.00278719

- **Internal Energy Derivatives (∂U/∂T) @ ℓ**

$\partial U_w / \partial T$

```
vecDUwDTℓ⟦1⟧
```

17.6594

$\partial U_g/\partial T$

```
vecDUgDTℓ⟦1⟧
```

```
-5.47121
```

$\partial U_o/\partial T$

```
vecDUoDTℓ⟦1⟧
```

```
91.6274
```

- **Water Coefficients @ ℓ**

Coefficient of δP

```
vecCδPWℓ⟦1⟧
```

```
-0.591434
```

Coefficient of δT

```
vecCδTWℓ⟦1⟧
```

```
20.5552
```

Coefficient of δSo

```
vecCδSoWℓ⟦1⟧
```

```
211227.
```

Coefficient of δTs

```
vecCδSgWℓ⟦1⟧
```

```
0
```

- **Oil Coefficients @ ℓ**

Coefficient of δP

```
vecCδPOℓ⟦1⟧
```

```
-0.253645
```

Coefficient of δT

```
vecCδTOℓ⟦1⟧
```

```
6.16012
```

Coefficient of δSo

```
vecCδSoOℓ⟦1⟧
```

```
-15852.8
```

Coefficient of δTs

```
vecCδSgOℓ[[1]]
```

```
0
```

▪ Energy Coefficients @ ℓ

Coefficient of δP

```
vecCδPEℓ[[1]]
```

```
9649.34
```

Coefficient of δT

```
vecCδTEℓ[[1]]
```

-6.74818×10^{6}

Coefficient of δSo

```
vecCδSoEℓ[[1]]
```

1.6211×10^{8}

Coefficient of δTs

```
vecCδSgEℓ[[1]]
```

```
0
```

▪ Thermodynamic Constraint Coefficients @ ℓ

Coefficient of δP

```
vecCδPGℓ
```

```
-1
```

Coefficient of δT

```
vecCδTGℓ[[1]]
```

```
0
```

Coefficient of δSo

```
vecCδSoGℓ[[1]]
```

```
-113.334
```

Coefficient of δTs

```
vecCδTsGℓ[[1]]
```

```
4.55308
```

RHS of G

```
vecCRGℓ[[1]]
```

18.9989

Matrices @ n,ℓ

Old LHS, New LHS @ n

This matrix shows the terms which were on the LHS in the original equations and are on the LHS in the matrix solution.

```
MatrixForm[Map[MatrixForm, LHS1, {2}]]
```

$$\begin{pmatrix}
\begin{pmatrix} 0 & 0 & 0 & 0 \\ -0.0432286 & 0 & 0 & 0 \\ -263.318 & -768. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0.0432286 & 0 & 0 & 0 \\ 263.318 & 768. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \\
\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0.0432286 & 0 & 0 & 0 \\ 263.318 & 768. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ -0.0864572 & 0 & 0 & 0 \\ -526.636 & -1536. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0.0432286 & 0 & 0 & 0 \\ 263.318 & 768. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \\
\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0.0432286 & 0 & 0 & 0 \\ 263.318 & 768. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ -0.0432286 & 0 & 0 & 0 \\ -263.318 & -768. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
\end{pmatrix}$$

Old RHS, New LHS @ ℓ

This matrix shows the terms which were on the RHS in the original equations and are on the LHS in the matrix solution. Note these are the block matrices are only on the diagonal.

```
MatrixForm /@ LHS2
```

$$\left\{\begin{pmatrix} -0.591434 & 20.5552 & 211227. & 0 \\ -0.253645 & 6.16012 & -15852.8 & 0 \\ 9649.34 & -6.74818\times 10^{6} & 1.6211\times 10^{8} & 0 \\ -1 & 0 & -113.334 & 4.55308 \end{pmatrix}, \begin{pmatrix} -0.591434 & 20.5552 & 211227. & 0 \\ -0.253645 & 6.16012 & -15852.8 & 0 \\ 9649.34 & -6.74818\times 10^{6} & 1.6211\times 10^{8} & 0 \\ -1 & 0 & -113.334 & 4.55308 \end{pmatrix}, \begin{pmatrix} -0.591434 & 20.5552 & 211227. & 0 \\ -0.253645 & 6.16012 & -15852.8 & 0 \\ 9649.34 & -6.74818\times 10^{6} & 1.6211\times 10^{8} & 0 \\ -1 & 0 & -113.334 & 4.55308 \end{pmatrix}\right\}$$

Combined New LHS @ n, ℓ

This matrix shows the combined LHS.

LHS3⟦1, 1⟧ // MatrixForm

$$\begin{pmatrix} -0.591434 & 20.5552 & 211227. & 0 \\ -0.296873 & 6.16012 & -15852.8 & 0 \\ 9386.03 & -6.74895\times10^{6} & 1.6211\times10^{8} & 0 \\ -1 & 0 & -113.334 & 4.55308 \end{pmatrix}$$

LHS3⟦1, 2⟧ // MatrixForm

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0.0432286 & 0 & 0 & 0 \\ 263.318 & 768. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

LHS3⟦1, 3⟧ // MatrixForm

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

LHS3⟦2, 1⟧ // MatrixForm

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0.0432286 & 0 & 0 & 0 \\ 263.318 & 768. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

LHS3⟦2, 2⟧ // MatrixForm

$$\begin{pmatrix} -0.591434 & 20.5552 & 211227. & 0 \\ -0.340102 & 6.16012 & -15852.8 & 0 \\ 9122.71 & -6.74972\times10^{6} & 1.6211\times10^{8} & 0 \\ -1 & 0 & -113.334 & 4.55308 \end{pmatrix}$$

LHS3⟦2, 3⟧ // MatrixForm

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0.0432286 & 0 & 0 & 0 \\ 263.318 & 768. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

LHS3⟦3, 1⟧ // MatrixForm

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

LHS3⟦3, 2⟧ // MatrixForm

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0.0432286 & 0 & 0 & 0 \\ 263.318 & 768. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
LHS3⟦3, 3⟧ // MatrixForm
```

$$\begin{pmatrix} -0.591434 & 20.5552 & 211227. & 0 \\ -0.296873 & 6.16012 & -15852.8 & 0 \\ 9386.03 & -6.74895\times 10^{6} & 1.6211\times 10^{8} & 0 \\ -1 & 0 & -113.334 & 4.55308 \end{pmatrix}$$

- **Old LHS, New RHS @ n,ℓ**

This matrix shows the terms which were on the LHS in the original equations and are on the RHS in the matrix solution.

```
MatrixForm /@ RHS1
```

$$\left\{ \begin{pmatrix} 0 \\ 0. \\ 0. \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0. \\ 0. \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0. \\ 0. \\ 0 \end{pmatrix} \right\}$$

- **Old RHS, New RHS @ n,ℓ**

This matrix shows the terms which were on the RHS in the original equations and are on the RHS in the matrix solution.

```
MatrixForm /@ RHS2
```

$$\left\{ \begin{pmatrix} -1944.44 \\ 0. \\ -3.5\times 10^{7} \\ 18.9989 \end{pmatrix}, \begin{pmatrix} 0. \\ 0. \\ 0. \\ 18.9989 \end{pmatrix}, \begin{pmatrix} 0. \\ 145.933 \\ 888920. \\ 18.9989 \end{pmatrix} \right\}$$

- **Combine RHS @ n,ℓ**

This matrix shows the combined RHS.

```
MatrixForm /@ RHS3
```

$$\left\{ \begin{pmatrix} -1944.44 \\ 0. \\ -3.5\times 10^{7} \\ 18.9989 \end{pmatrix}, \begin{pmatrix} 0. \\ 0. \\ 0. \\ 18.9989 \end{pmatrix}, \begin{pmatrix} 0. \\ 145.933 \\ 888920. \\ 18.9989 \end{pmatrix} \right\}$$

■ Block Upper Triangular

(LHS6) // MatrixForm

-12.8213	8814.33	0.	0	-0.343099	-1.00069	0	0	
0.62099	-653.823	0.	0	0.0689786	0.0751031	0	0	
9386.03	-6.74895×10^{6}	1.6211×10^{8}	0	263.318	768.	0	0	
-1	0	-113.334	4.55308	0	0	0	0	
-0.343099	-1.00069	0	0	-12.4782	8815.33	0.	0	-
0.0689786	0.0751031	0	0	0.552012	-653.898	0.	0	0
263.318	768.	0	0	9122.71	-6.74972×10^{6}	1.6211×10^{8}	0	
0	0	0	0	-1	0	-113.334	4.55308	
0	0	0	0	-0.343099	-1.00069	0	0	-
0	0	0	0	0.0689786	0.0751031	0	0	
0	0	0	0	263.318	768.	0	0	
0	0	0	0	0	0	0	0	

(RHS6) // MatrixForm

43 660.
-3422.67
-3.5×10^{7}
18.9989
0.
0.
0.
18.9989
-1158.25
232.861
888 920.
18.9989

■ Normalized Block Upper Triangular

Normalize the diagonal to 1.

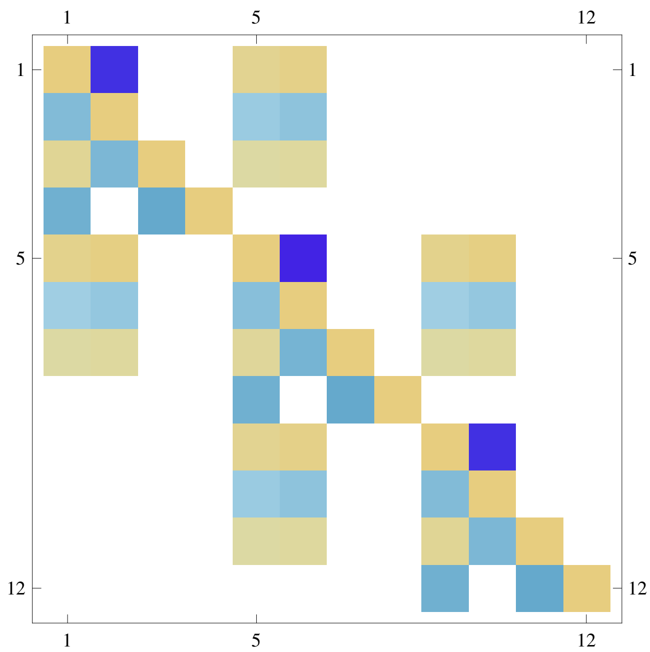
(LHS7) // MatrixForm

1.	-687.477	0.	0	0.0267602	0.0780493	0	0	0
-0.000949784	1.	0.	0	-0.0001055	-0.000114868	0	0	0
0.0000578992	-0.041632	1.	0	1.62432×10^{-6}	4.73753×10^{-6}	0	0	0
-0.219632	0	-24.8917	1.	0	0	0	0	0
0.0274959	0.0801953	0	0	1.	-706.46	0.	0	0.0274959
-0.000105488	-0.000114854	0	0	-0.000844186	1.	0.	0	-0.0001054
1.62432×10^{-6}	4.73753×10^{-6}	0	0	0.0000562748	-0.0416367	1.	0	1.62432 × 10
0	0	0	0	-0.219632	0	-24.8917	1.	0
0	0	0	0	0.0267602	0.0780493	0	0	1.
0	0	0	0	-0.0001055	-0.000114868	0	0	-0.0009497
0	0	0	0	1.62432×10^{-6}	4.73753×10^{-6}	0	0	0.00005789
0	0	0	0	0	0	0	0	-0.219632

```
(RHS7) // MatrixForm
```

$$\begin{pmatrix} -3405.28 \\ 5.23485 \\ -0.215903 \\ 4.17276 \\ 0. \\ 0. \\ 0. \\ 4.17276 \\ 90.338 \\ -0.356153 \\ 0.00548344 \\ 4.17276 \end{pmatrix}$$

```
MatrixPlot[LHS7]
```



■ Extract P,T

Extract P,T

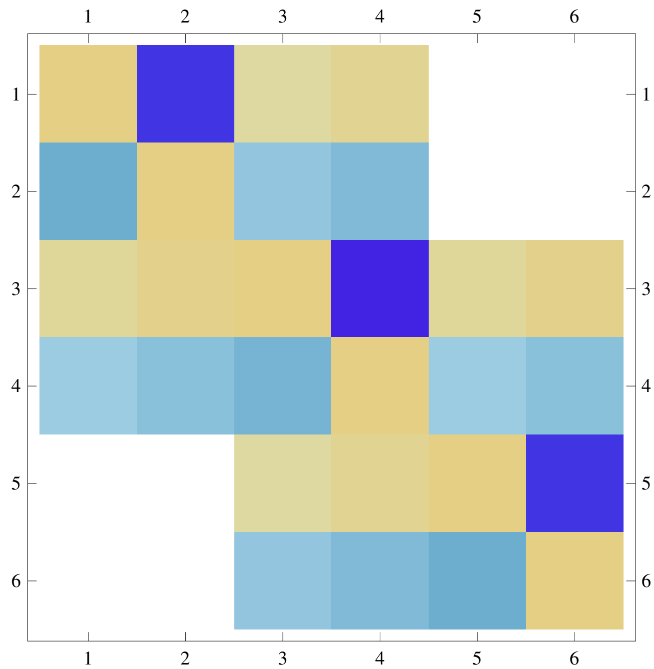
```
(LHS8) // MatrixForm
```

$$\begin{pmatrix} 1. & -687.477 & 0.0267602 & 0.0780493 & 0 & 0 \\ -0.000949784 & 1. & -0.0001055 & -0.000114868 & 0 & 0 \\ 0.0274959 & 0.0801953 & 1. & -706.46 & 0.0274959 & 0.0801953 \\ -0.000105488 & -0.000114854 & -0.000844186 & 1. & -0.000105488 & -0.000114854 \\ 0 & 0 & 0.0267602 & 0.0780493 & 1. & -687.477 \\ 0 & 0 & -0.0001055 & -0.000114868 & -0.000949784 & 1. \end{pmatrix}$$

```
(RHS8) // MatrixForm
```

$$\begin{pmatrix} -3405.28 \\ 5.23485 \\ 0. \\ 0. \\ 90.338 \\ -0.356153 \end{pmatrix}$$

```
MatrixPlot[LHS8]
```



HW #18 Solution, T>=Ts (T=500°F, Δt=1 day)

■ Properties @ n

This section describes the properties which were not defined in HW #17.

■ Conditions at time n

For HW #18, the conditions for all three grid cells for both time n and time ℓ are the same.

Pressure (psia)

```
P1n
```

500

Temperature (°F)

```
T1n
```

```
500
```

So

```
So1n
```

```
0.7
```

Sg

```
Sg1n
```

```
0.1
```

Sw

```
Sw1n
```

```
0.2
```

Satration Temperature (°F)

```
Ts1n
```

```
466.689
```

Use Δt = 1 day for HW #18.

```
Δt
```

```
1
```

- **Transmissibilities @ n**

$T_w = 0$ because krw=0 @ Sw=Swr

```
vecTwn⟦1⟧
```

```
0
```

T_g

```
vecTgn⟦1⟧
```

```
7.59617
```

T_o

```
vecTon⟦1⟧
```

```
37.2111
```

- **Molar Densities (ξ) @ n**

ξ_w

```
vecξwn[[1]]
```

2.83527

ξ_g

```
vecξgn[[1]]
```

0.0589805

ξ_o

```
vecξon[[1]]
```

0.212879

■ Enthalpies @ n

H_w

```
vecHwn[[1]]
```

8195.3

H_g

```
vecHgn[[1]]
```

8938.99

H_o

```
vecHon[[1]]
```

47566.5

■ Internal Energies @ n

U_w

```
vecUwn[[1]]
```

8193.49

U_g

```
vecUgn[[1]]
```

8851.85

U_o

```
vecUon[[1]]
```

47564.8

■ Upstream-weighted Tξ @ n

No harmonic weighting is required since k, Δx, Δy, and Δz are all constant.

$(T\,\xi)_g$

```
Transpose[{transindex, fTξgn /@ transindex}] // Grid[#, Frame → All] &
```

$\frac{1}{2}$	0
$\frac{3}{2}$	0.448026
$\frac{5}{2}$	0.448026
$\frac{7}{2}$	0

$(T\,\xi)_o$

```
Transpose[{transindex, fTξon /@ transindex}] // Grid[#, Frame → All] &
```

$\frac{1}{2}$	0
$\frac{3}{2}$	7.92146
$\frac{5}{2}$	7.92146
$\frac{7}{2}$	0

■ Upstream-weighted TξH @ n

$(T\,\xi\,H)_g$

```
Transpose[{transindex, fTξHgn /@ transindex}] // Grid[#, Frame → All] &
```

$\frac{1}{2}$	0
$\frac{3}{2}$	4004.9
$\frac{5}{2}$	4004.9
$\frac{7}{2}$	0

$(T\,\xi\,H)_o$

```
Transpose[{transindex, fTξHon /@ transindex}] // Grid[#, Frame → All] &
```

$\frac{1}{2}$	0
$\frac{3}{2}$	376 797.
$\frac{5}{2}$	376 797.
$\frac{7}{2}$	0

■ Volumetric Flow

Total volumetric flow (ft^3/day)

```
qtotn
```

685.806

■ Accumulation @ n

Water accumulation term @ n

```
vecACCWn[[1]]
```

```
-34377.1
```

Oil accumulation term @ n

```
vecACCOn[[1]]
```

```
-8940.92
```

Energy accumulation term @ n

```
vecACCEn[[1]]
```

-3.15857×10^{9}

Source Terms @ n

Water source terms @ n

```
{vecSOURCEWn} // Grid[#, Frame → All] &
```

-1944.44	0	6.85733

Oil source terms @ n

```
{vecSOURCEOn} // Grid[#, Frame → All] &
```

0	0	121.243

Energy source terms @ n.
The top & bottom boundary fluxes are 0 since T=TR.

```
{vecSOURCEEn} // Grid[#, Frame → All] &
```

1.30516×10^{8}	1.65516×10^{8}	1.71344×10^{8}

Properties @ ℓ

This section describes the properties which were not defined in HW #17, and are not defined at time n above.

Molar Density Derivatives ($\partial\xi/\partial$P) @ ℓ

$\partial\xi_w/\partial P$

```
vecDξwDPℓ[[1]]
```

```
0.0000113411
```

$\partial\xi_g/\partial P$

```
vecDξgDPℓ[[1]]
```

```
0.000113066
```

$\partial\xi_o/\partial P$

```
vecDξoDPℓ⟦1⟧
```

2.12879×10^{-6}

Molar Density Derivatives (∂ξ/∂T) @ ℓ

$\partial \xi_w / \partial T$

```
vecDξwDTℓ⟦1⟧
```

-0.00171293

$\partial \xi_g / \partial T$

```
vecDξgDTℓ⟦1⟧
```

0

$\partial \xi_o / \partial T$

```
vecDξoDTℓ⟦1⟧
```

-0.000128336

Internal Energy Derivatives (∂U/∂P) @ ℓ

$\partial U_w / \partial P$

```
vecDUwDPℓ⟦1⟧
```

4.8399

$\partial U_g / \partial P$

```
vecDUgDPℓ⟦1⟧
```

4.83628

$\partial U_o / \partial P$

```
vecDUoDPℓ⟦1⟧
```

-0.0034593

Internal Energy Derivatives (∂U/∂T) @ ℓ

$\partial U_w / \partial T$

```
vecDUwDTℓ⟦1⟧
```

-0.00109518

$\partial U_g / \partial T$

```
vecDUgDTℓ⟦1⟧
```

-0.582569

$\partial U_o / \partial T$

```
vecDUoDTℓ[[1]]
```

```
115.747
```

■ Water Coefficients @ ℓ

Coefficient of δP

```
vecCδPWℓ[[1]]
```

```
-1.15826
```

Coefficient of δT

```
vecCδTWℓ[[1]]
```

```
20.5552
```

Coefficient of δSo

```
vecCδSoWℓ[[1]]
```

```
170 116.
```

Coefficient of δSg

```
vecCδSgWℓ[[1]]
```

```
166 577.
```

■ Oil Coefficients @ ℓ

Coefficient of δP

```
vecCδPOℓ[[1]]
```

```
-0.178818
```

Coefficient of δT

```
vecCδTOℓ[[1]]
```

```
5.3901
```

Coefficient of δSo

```
vecCδSoOℓ[[1]]
```

```
-12 772.7
```

Coefficient of δSg

```
vecCδSgOℓ[[1]]
```

```
0
```

■ Energy Coefficients @ ℓ

Coefficient of δP

```
vecCδPEℓ[[1]]
```

```
-164630.
```

Coefficient of δT

```
vecCδTEℓ[[1]]
```

-5.51264×10^{6}

Coefficient of δSo

```
vecCδSoEℓ[[1]]
```

7.86312×10^{8}

Coefficient of δSg

```
vecCδSgEℓ[[1]]
```

1.36252×10^{9}

Thermodynamic Constraint Coefficients @ ℓ

Coefficient of δP

```
vecCδPGℓ
```

```
-1
```

Coefficient of δT

```
vecCδTGℓ[[1]]
```

```
5.78131
```

Coefficient of δSo

```
vecCδSoGℓ[[1]]
```

```
-154.178
```

Coefficient of δSg

```
vecCδSgGℓ[[1]]
```

```
-154.178
```

RHS of G

```
vecCRGℓ[[1]]
```

```
-152.549
```

Matrices @ n,ℓ

Old LHS, New LHS @ n

This matrix shows the terms which were on the LHS in the original equations and are on the LHS in the matrix solution.

```
MatrixForm[Map[MatrixForm, LHS1, {2}]]
```

$$\begin{pmatrix} \begin{pmatrix} -0.448026 & 0 & 0 & 0 \\ -7.92146 & 0 & 0 & 0 \\ -380802. & -768. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0.448026 & 0 & 0 & 0 \\ 7.92146 & 0 & 0 & 0 \\ 380802. & 768. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0.448026 & 0 & 0 & 0 \\ 7.92146 & 0 & 0 & 0 \\ 380802. & 768. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} -0.896051 & 0 & 0 & 0 \\ -15.8429 & 0 & 0 & 0 \\ -761603. & -1536. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0.448026 & 0 & 0 & 0 \\ 7.92146 & 0 & 0 & 0 \\ 380802. & 768. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0.448026 & 0 & 0 & 0 \\ 7.92146 & 0 & 0 & 0 \\ 380802. & 768. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} -0.448026 & 0 & 0 & 0 \\ -7.92146 & 0 & 0 & 0 \\ -380802. & -768. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{pmatrix}$$

■ Old RHS, New LHS @ ℓ

This matrix shows the terms which were on the RHS in the original equations and are on the LHS in the matrix solution. Note these are the block matrices are only on the diagonal.

```
MatrixForm /@ LHS2
```

$$\left\{ \begin{pmatrix} -1.15826 & 20.5552 & 170116. & 166577. \\ -0.178818 & 5.3901 & -12772.7 & 0 \\ -164630. & -5.51264\times10^{6} & 7.86312\times10^{8} & 1.36252\times10^{9} \\ -1 & 5.78131 & -154.178 & -154.178 \end{pmatrix}, \begin{pmatrix} -1.15826 & 20.5552 & 170116. & 166577. \\ -0.178818 & 5.3901 & -12772.7 & 0 \\ -164630. & -5.51264\times10^{6} & 7.86312\times10^{8} & 1.36252\times10^{9} \\ -1 & 5.78131 & -154.178 & -154.178 \end{pmatrix}, \begin{pmatrix} -1.15826 & 20.5552 & 170116. & 166577. \\ -0.178818 & 5.3901 & -12772.7 & 0 \\ -164630. & -5.51264\times10^{6} & 7.86312\times10^{8} & 1.36252\times10^{9} \\ -1 & 5.78131 & -154.178 & -154.178 \end{pmatrix} \right\}$$

■ Combined New LHS @ n, ℓ

This matrix shows the combined LHS.

```
LHS3⟦1, 1⟧ // MatrixForm
```

$$\begin{pmatrix} -1.60628 & 20.5552 & 170116. & 166577. \\ -8.10028 & 5.3901 & -12772.7 & 0 \\ -545431. & -5.51341\times10^{6} & 7.86312\times10^{8} & 1.36252\times10^{9} \\ -1 & 5.78131 & -154.178 & -154.178 \end{pmatrix}$$

```
LHS3⟦1, 2⟧ // MatrixForm
```

$$\begin{pmatrix} 0.448026 & 0 & 0 & 0 \\ 7.92146 & 0 & 0 & 0 \\ 380802. & 768. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
LHS3[[1, 3]] // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
LHS3[[2, 1]] // MatrixForm
```

$$\begin{pmatrix} 0.448026 & 0 & 0 & 0 \\ 7.92146 & 0 & 0 & 0 \\ 380802. & 768. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
LHS3[[2, 2]] // MatrixForm
```

$$\begin{pmatrix} -2.05431 & 20.5552 & 170116. & 166577. \\ -16.0217 & 5.3901 & -12772.7 & 0 \\ -926233. & -5.51418\times10^{6} & 7.86312\times10^{8} & 1.36252\times10^{9} \\ -1 & 5.78131 & -154.178 & -154.178 \end{pmatrix}$$

```
LHS3[[2, 3]] // MatrixForm
```

$$\begin{pmatrix} 0.448026 & 0 & 0 & 0 \\ 7.92146 & 0 & 0 & 0 \\ 380802. & 768. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
LHS3[[3, 1]] // MatrixForm
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
LHS3[[3, 2]] // MatrixForm
```

$$\begin{pmatrix} 0.448026 & 0 & 0 & 0 \\ 7.92146 & 0 & 0 & 0 \\ 380802. & 768. & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
LHS3[[3, 3]] // MatrixForm
```

$$\begin{pmatrix} -1.60628 & 20.5552 & 170116. & 166577. \\ -8.10028 & 5.3901 & -12772.7 & 0 \\ -545431. & -5.51341\times10^{6} & 7.86312\times10^{8} & 1.36252\times10^{9} \\ -1 & 5.78131 & -154.178 & -154.178 \end{pmatrix}$$

▪ Old LHS, New RHS @ n,ℓ

This matrix shows the terms which were on the LHS in the original equations and are on the RHS in the matrix solution.

```
MatrixForm /@ RHS1
```

$$\left\{\begin{pmatrix} 0. \\ 0. \\ 0. \\ 0 \end{pmatrix}, \begin{pmatrix} 0. \\ 0. \\ 0. \\ 0 \end{pmatrix}, \begin{pmatrix} 0. \\ 0. \\ 0. \\ 0 \end{pmatrix}\right\}$$

Old RHS, New RHS @ n,ℓ

This matrix shows the terms which were on the RHS in the original equations and are on the RHS in the matrix solution.

```
MatrixForm /@ RHS2
```

$$\left\{\begin{pmatrix} -1944.44 \\ 0. \\ 1.30516\times 10^{8} \\ -152.549 \end{pmatrix}, \begin{pmatrix} 0. \\ 0. \\ 1.65516\times 10^{8} \\ -152.549 \end{pmatrix}, \begin{pmatrix} 6.85733 \\ 121.243 \\ 1.71344\times 10^{8} \\ -152.549 \end{pmatrix}\right\}$$

Combine RHS @ n,ℓ

This matrix shows the combined RHS.

```
MatrixForm /@ RHS3
```

$$\left\{\begin{pmatrix} -1944.44 \\ 0. \\ 1.30516\times 10^{8} \\ -152.549 \end{pmatrix}, \begin{pmatrix} 0. \\ 0. \\ 1.65516\times 10^{8} \\ -152.549 \end{pmatrix}, \begin{pmatrix} 6.85733 \\ 121.243 \\ 1.71344\times 10^{8} \\ -152.549 \end{pmatrix}\right\}$$

Block Upper Triangular

```
(LHS6) // MatrixForm
```

$$\begin{pmatrix}
-1139.65 & 6546.73 & -4.54747\times 10^{-13} & 0. & 2.78675 & 0.00471674 & 0 \\
199.886 & -1004.93 & 0. & 0. & -0.519727 & -0.0170242 & 0 \\
-9.38274\times 10^{6} & 4.55778\times 10^{7} & -5.76208\times 10^{8} & 0. & 380802. & 768. & 0 \\
-1 & 5.78131 & -154.178 & -154.178 & 0 & 0 & 0 \\
2.78675 & 0.00471674 & 0 & 0 & -1142.44 & 6546.72 & -4.54747\times 10^{-1} \\
-0.519727 & -0.0170242 & 0 & 0 & 200.406 & -1004.91 & 0. \\
380802. & 768. & 0 & 0 & -9.76354\times 10^{6} & 4.5577\times 10^{7} & -5.76208\times 10^{8} \\
0 & 0 & 0 & 0 & -1 & 5.78131 & -154.178 \\
0 & 0 & 0 & 0 & 2.78675 & 0.00471674 & 0 \\
0 & 0 & 0 & 0 & -0.519727 & -0.0170242 & 0 \\
0 & 0 & 0 & 0 & 380802. & 768. & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

(RHS6) // MatrixForm

-174 239.
26 990.5
-1.2176×10^{9}
-152.549
-172 080.
26 214.6
-1.1826×10^{9}
-152.549
-172 037.
26 206.7
-1.17677×10^{9}
-152.549

■ Normalized Block Upper Triangular

Normalize the diagonal to 1.

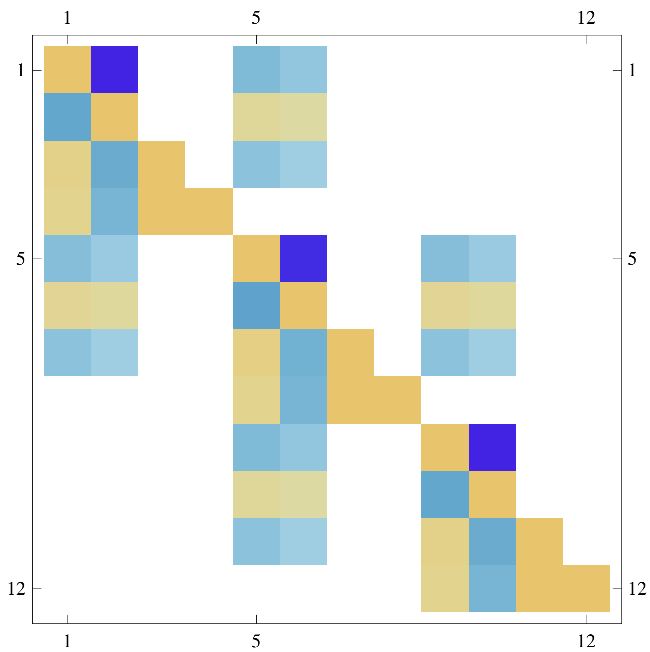
(LHS7) // MatrixForm

1.	-5.7445	3.99023×10^{-16}	0.	-0.00244526	-4.13875×10^{-6}	0	0
-0.198906	1.	0.	0.	0.000517178	0.0000169407	0	0
0.0162836	-0.0790997	1.	0.	-0.000660876	-1.33285×10^{-6}	0	0
0.006486	-0.0374976	1.	1.	0	0	0	0
-0.0024393	-4.12866×10^{-6}	0	0	1.	-5.73048	3.9805×10^{-16}	0.
0.000517187	0.000016941	0	0	-0.199426	1.	0.	0.
-0.000660876	-1.33285×10^{-6}	0	0	0.0169445	-0.0790983	1.	0.
0	0	0	0	0.006486	-0.0374976	1.	1.
0	0	0	0	-0.00244526	-4.13875×10^{-6}	0	0
0	0	0	0	0.000517178	0.0000169407	0	0
0	0	0	0	-0.000660876	-1.33285×10^{-6}	0	0
0	0	0	0	0	0	0	0

(RHS7) // MatrixForm

152.888
-26.8581
2.11313
0.98943
150.625
-26.0865
2.05239
0.98943
150.956
-26.0781
2.04227
0.98943

```
MatrixPlot[LHS7]
```



■ Extract P,T

Extract P,T

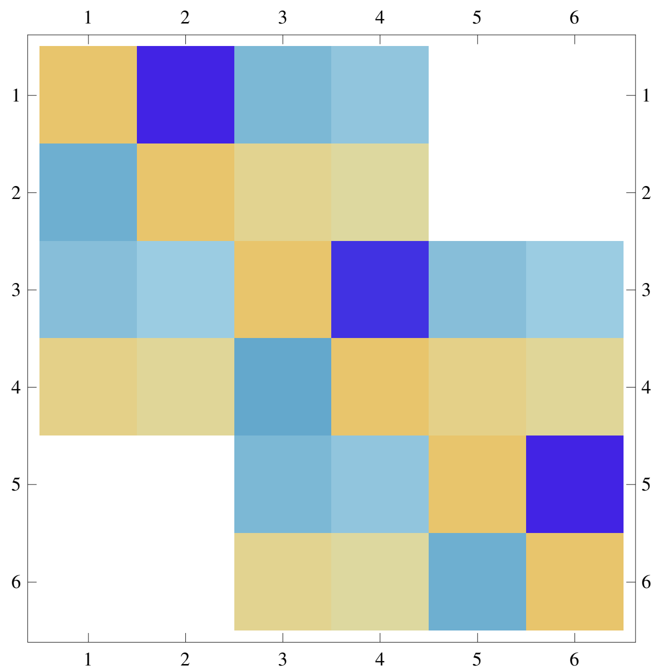
```
(LHS8) // MatrixForm
```

$$\begin{pmatrix} 1. & -5.7445 & -0.00244526 & -4.13875\times10^{-6} & 0 & 0 \\ -0.198906 & 1. & 0.000517178 & 0.0000169407 & 0 & 0 \\ -0.0024393 & -4.12866\times10^{-6} & 1. & -5.73048 & -0.0024393 & -4.12866\times10^{-6} \\ 0.000517187 & 0.000016941 & -0.199426 & 1. & 0.000517187 & 0.000016941 \\ 0 & 0 & -0.00244526 & -4.13875\times10^{-6} & 1. & -5.7445 \\ 0 & 0 & 0.000517178 & 0.0000169407 & -0.198906 & 1. \end{pmatrix}$$

```
(RHS8) // MatrixForm
```

$$\begin{pmatrix} 152.888 \\ -26.8581 \\ 150.625 \\ -26.0865 \\ 150.956 \\ -26.0781 \end{pmatrix}$$

```
MatrixPlot[LHS8]
```



Hint on HW #21 Solution, T<Ts (T=100°F, Δt=0.1 day)

The converged pressure solution is

512.729	500.075	489.843

Hint on HW #21, T>=Ts (T=500°F, Δt=1 day)

The converged pressure solution is

519.389	505.235	504.639