1. (10) Short answer:
   a. (2) Define microstate for a system of N spin 1/2 atoms.

   b. (2) Define macrostate for a system of N spin 1/2 atoms.

   c. (2) State the equal probability \textit{a priori} hypothesis.

   d. (2) If equilibrium is defined as the most likely macrostate, what does the equal probability hypothesis imply for the entropy at equilibrium?

   e. (2) If a system is not in its equilibrium macrostate, what does the equal probability hypothesis imply for the change in the entropy as the system spontaneously evolves toward equilibrium?

2. (20) A volume of a monatomic gas at standard temperature and pressure undergoes an isothermal expansion from its initial volume to three times the initial volume, i.e. \( V_i \rightarrow V_f = 3V_i \).

   a. (8) How much heat must flow into the gas to keep the process isothermal?

   b. (8) What is the change in the Sackur-Tetrode entropy when a volume of gas undergoes an isothermal expansion to three times the initial volume? Show all work.

   c. (4) How are the two expressions related?
3. (35) The goal of this problem is to derive the equipartition theorem for a monatomic gas. Consider a single gas monatomic molecule of mass, m, and velocity \( \vec{v} = \{v_x, v_y, v_z\} \), confined to a cylinder of length, L, and radius, R, that lies with its symmetry axis parallel with the x-axis with the left end at x=0. Assume the gas molecule elastically reflects from the walls of the cylinder. Show all work and give your answers in terms of the quantities given in the problem statement. Ignore gravity.

a. (10) Using Newton’s laws find the average force the molecule exerts on the left end of the cylinder.

b. (10) Using your answer from part (a) suppose there are now N molecules in the cylinder. What is average pressure they exert on the left end of the cylinder in terms of the average squared speed?

c. (8) Using your answer from part (b) find the average kinetic energy of all the molecules in the cylinder in terms of the pressure and volume.

d. (7) Using your result from (c) and the ideal gas model, find the temperature of the gas in terms of the average kinetic energy thereby demonstrating the equipartition theorem.
4. A rubber band can be modeled as a one-dimensional collection of $N$ long polymer molecules that can connect at the ends of each other either pointing to the left or to the right. Each molecule has length, $l_0$; so the total length of the rubber band (assumed pointing to the left) is $L \simeq l_0(N_L - N_R)$, where $N_L$ is the number of molecules pointing to the left and $N_R$ is the number of molecules pointing on the right. Show all work. Use the back if you need more working space.

a. What defines a microstate and what defines a macrostate for this rubber band model?

b. For large-$N$ the approximate multiplicity for this model of a rubber band can be written as:

$$
\Omega(N, N_L, N_R) \simeq \left( \frac{N_R}{N} \right)^{-N_R} \left( \frac{N_L}{N} \right)^{-N_L}.
$$

Find the entropy for the rubber band as a function of its net length, $L$, and the total number of molecules, $N$ (and constants).

c. From the entropic definition of force for the rubber band, $F = T \left( \frac{\partial S}{\partial L} \right)_N$, find the force as a function of $\{L, N, T\}$.

d. From your result in part (c), find the effective spring constant, $k_s$, for the rubber band. For small $L$ the effective spring constant is defined as $F \simeq -k_s L$. Comment on how $k_s$ depends on temperature?