1. (4) State the “equal probability it a priori” hypothesis.

Solution: The equal probability it a priori hypothesis states that the probability of any macrostate is proportional to the number of microstates that share that macrostate.

2. (3) Consider a quincunx (plinko) game with 8 rows. What is the probability that any given ball will land in the center bin? (Explain your answers and show your work.)

Solution: The total number of possible outcomes is $2^8 = 256$. The number of ways a ball can reach the center bin is $\frac{8!}{4!4!} = 70$; therefore the probability of a ball landing in the center bin is $\frac{70}{256} = 0.273$.

3. (2) What is the probability that any given ball will land either to the right one bin away from the center bin?

Solution: The total number of possible outcomes is $2^8 = 256$. A ball can reach the first bin on the right by having a net of 5 right bounces and 3 left bounces. Thus, the number of ways a ball can reach the first bin on the right is: $\frac{8!}{3!5!} = 56$; therefore the probability of a ball landing one bin to the right of the center is $\frac{56}{256} = 0.219$.

4. (1) What is the approximate full width of the distribution of balls collected in the bins?

Solution: As shown in class and worked in your homework, the half width of the binomial distribution for large $N$ is approximately, $\sqrt{\frac{N}{2}} = \sqrt{8/2} = 2$. However, since $N = 8$ is not particularly large, an alternative approach would be to use the binomial distribution to calculate directly the probability for the next couple of bin values until the probability drops to $1/e \approx 0.37$ of the probability for the central bin. Let $P(n)$ be the probability of landing in the $n^{th}$ bin as measured from the central bin. From above we have $P(n = 0) \approx 0.273$ so we seek the value of $n$ such that $P(n) = e^{-1}P(0) \approx 0.1006$. From above we also have, $P(n = 1) \approx 0.219$. Similarly, one finds $P(n = 2) \approx 0.109$ which is very close to the desired value. So, even though $N = 8$ is not so large, the actual half width is very close the approximate one, namely 2 bins.