1. A volume of gas at standard temperature and pressure undergoes an isothermal expansion to twice the initial volume, i.e. $V_i \rightarrow V_f = 2V_i$.

a. (4) How much heat must flow into the gas to keep the process isothermal?

*Solution:* Since the process is isothermal, by the equipartition theorem the internal energy is constant. Therefore, by the first law, we have: $0 = \Delta U = Q + W$; so $Q = -W$. The work done in the isothermal expansion is:

$$W = -\int PdV = -\int_{V_i}^{V_f} \frac{NkT}{V} dV = -NkT \ln \frac{V_f}{V_i} = -NkT \ln 2.$$  \hspace{1cm} (1)

where the ideal gas law was used. Therefore the heat that must be added is $Q = NkT \ln 2$.

b. (5) The Sackur-Tetrode form for the entropy of an ideal gas is

$$S(N,V,U) = Nk \left( \frac{5}{2} + \ln \left[ \frac{V}{N} \left( \frac{4\pi mU}{3h^2N^2} \right)^{\frac{3}{2}} \right] \right).$$ \hspace{1cm} (2)

What is the change in the Sackur-Tetrode entropy when a volume of gas undergoes an isothermal expansion to twice the initial volume?

*Solution:* Again, since the process is isothermal, the internal energy, $U$ remains constant; so the change in the Sackur-Tetrode entropy is

$$\Delta S = S_f - S_i = S(N,V_f,U) - S(N,V_i,U) = Nk \ln 2.$$ \hspace{1cm} (3)

c. (1) How are the two expressions related?

*Solution:* Examining the two expressions, we find: $\Delta S = \frac{Q}{T}$ which is the classical result for the change in entropy for an isothermal process.